Integrals Ex 7.1 Class 12

Ex 7.1 Class 12 Maths Question 1.
sin 2x
Solution:

The anti derivative of sin 2*x* is a function of *x* whose derivative is sin 2*x*.

It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$
$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

Ex 7.1 Class 12 Maths Question 2.
cos 3x
Solution:

The anti derivative of cos 3*x* is a function of *x* whose derivative is cos 3*x*.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$
$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$
$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$.

Ex 7.1 Class 12 Maths Question 3.
e^{2x}
Solution:

The anti derivative of e^{2x} is the function of *x* whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$
$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}(e^{2x})$$
$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

Ex 7.1 Class 12 Maths Question 4.
$(ax + c)^2$
Solution:

The anti derivative of $(ax+b)^2$ is the function of *x* whose derivative is $(ax+b)^2$

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

Ex 7.1 Class 12 Maths Question 5.
$\sin \quad 2x - 4e^{3x}$
Ex 7.1 Class 12 Maths Solution:

The anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is the function of x whose derivative is $\left(\sin 2x-4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)=\sin 2x-4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)$.

Find the following integrals in Exercises 6 to 20 :

Ex 7.1 Class 12 Maths Question 6.

$\int\left(4e^{3x}+1\right)dx$

Solution:

$$\int\left(4e^{3x}+1\right)dx$$
$$=4\int e^{3x}dx+\int 1dx$$
$$=4\left(\frac{e^{3x}}{3}\right)+x+C$$
$$=\frac{4}{3}e^{3x}+x+C$$

Ex 7.1 Class 12 Maths Question 7.

$\int x^2\left(1-\frac{1}{x^2}\right)dx$

Solution:

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$
$$=\int\left(x^2-1\right)dx$$
$$=\int x^2dx-\int 1dx$$
$$=\frac{x^3}{3}-x+C$$

Ex 7.1 Class 12 Maths Question 8.

$\int\left(ax^2+bx+c\right)dx$

Solution:

$$\int\left(ax^2+bx+c\right)dx$$
$$=a\int x^2dx+b\int xdx+c\int 1.dx$$
$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$
$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

Ex 7.1 Class 12 Maths Question 9.

$\int\left(2x^2+e^x\right)dx$

Solution:

$$\int\left(2x^2+e^x\right)dx$$
$$=2\int x^2dx+\int e^xdx$$
$$=2\left(\frac{x^3}{3}\right)+e^x+C$$
$$=\frac{2}{3}x^3+e^x+C$$

Ex 7.1 Class 12 Maths Question 10.

$\int\left[\sqrt{x}-\frac{1}{\sqrt{x}}\right]^2dx$

Solution:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int\left(x+\frac{1}{x}-2\right)dx$$
$$= \int x dx + \int\frac{1}{x}dx - 2\int 1.dx$$
$$= \frac{x^2}{2} + \log|x| - 2x + C$$

Ex 7.1 Class 12 Maths Question 11.

$$\int \frac{x^3+5x^2-4}{x^2}dx$$

Solution:

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$= \int\left(x+5-4x^{-2}\right)dx$$
$$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$$
$$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$$
$$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

Ex 7.1 Class 12 Maths Question 12.

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

Solution:

$$\int\frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right)dx$$
$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$$
$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$$
$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$$

Ex 7.1 Class 12 Maths Question 13.

$$\int \frac{x^3-x^2+x-1}{x-1}dx$$

Solution:

$$\int\frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$= \int\left(x^2+1\right)dx$$
$$= \int x^2 dx + \int 1 dx$$
$$= \frac{x^3}{3} + x + C$$

Ex 7.1 Class 12 Maths Question 14.

$$\int (1-x)\sqrt{x}dx$$

Solution:

$$\int(1-x)\sqrt{x}dx$$

$$= \int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

Ex 7.1 Class 12 Maths Question 15.

$\int \sqrt{x}\left(3x^2+2x+3\right) dx$

Solution:

$$\int\sqrt{x}\left(3x^2+2x+3\right)dx$$

$$= \int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$$

$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

Ex 7.1 Class 12 Maths Question 16.

$\int (2x - 3\cos x + e^x)dx$

Solution:

$$\int\left(2x-3\cos x+e^x\right)dx$$

$$= 2\int xdx - 3\int\cos xdx + \int e^x dx$$

$$= \frac{2x^2}{2} - 3(\sin x) + e^x + C$$

$$= x^2 - 3\sin x + e^x + C$$

Ex 7.1 Class 12 Maths Question 17.

$\int \left(2x^2 - 3\sin x + 5\sqrt{x}\right) dx$

Solution:

$$\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$$

$$= 2\int x^2dx - 3\int\sin xdx + 5\int x^{\frac{1}{2}}dx$$

$$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

Ex 7.1 Class 12 Maths Question 18.

$\int \sec x(\sec x + \tan x)dx$

Solution:

$$\int\sec x\left(\sec x+\tan x\right)dx$$

$$= \int\left(\sec^2 x + \sec x\tan x\right)dx$$

$$= \int\sec^2 xdx + \int\sec x\tan xdx$$

$$= \tan x + \sec x + C$$

Ex 7.1 Class 12 Maths Question 19.

$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

Solution:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1) dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

Ex 7.1 Class 12 Maths Question 20.

$\int \frac{2-3\sin x}{\cos^2 x} dx$

Solution:

$$\int \frac{2-3\sin x}{\cos^2 x} dx$$

$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

Ex 7.1 Class 12 Maths Question 21.

The antiderivative $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(a) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + c$

(b) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^{2} + c$

(c) \frac { 2 }{ 3 } { x }^{ \frac { 3 }{ 2 } }+{ 2x }^{ \frac { 1 }{ 2 } }+c

$\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

(d) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + c$

Solution:

It is given that,

$$\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$$

$\therefore$Anti derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4} dx$$

$$f(x) = 4\int x^3 dx - 3\int (x^{-4}) dx$$

$$\therefore f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Ex 7.1 Class 12 Maths Question 22.

If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that f(2)=0 then f(x) is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(b) { x }^{ 3 }+\frac { 1 }{ { x }^{ 4 } } +\frac { 129 }{ 8 }

$x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Solution:

Also,

$$f(2)=0$$
$$\therefore f(2)=(2)^4+\frac{1}{(2)^3}+C=0$$
$$\Rightarrow 16+\frac{1}{8}+C=0$$
$$\Rightarrow C=-\left(16+\frac{1}{8}\right)$$
$$\Rightarrow C=\frac{-129}{8}$$
$$\therefore f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$$

Hence, the correct answer is A.

Integrals Ex 7.2 Class 12

Ex 7.2 Class 12 Maths Question 1.

$\frac{2x}{1+x^2}$

Solution:

Let $1+x^2=t$

$\therefore 2x\,dx=dt$

$$\Rightarrow \int\frac{2x}{1+x^2}dx=\int\frac{1}{t}dt$$
$$=\log|t|+C$$
$$=\log\left|1+x^2\right|+C$$
$$=\log\left(1+x^2\right)+C$$

Ex 7.2 Class 12 Maths Question 2.

$\frac{(\log x)^2}{x}$

Solution:

Let $\log|x|=t$

$$\therefore \frac{1}{x}dx=dt$$
$$\Rightarrow \int\frac{(\log|x|)^2}{x}dx=\int t^2dt$$
$$=\frac{t^3}{3}+C$$
$$=\frac{(\log|x|)^3}{3}+C$$

Ex 7.2 Class 12 Maths Question 3.

$\frac{1}{x+x\log x}$

Solution:

$$\frac{1}{x+x\log x}=\frac{1}{x(1+\log x)}$$

Let $1+\log x=t$

$$\therefore \frac{1}{x}dx=dt$$
$$\Rightarrow \int\frac{1}{x(1+\log x)}dx=\int\frac{1}{t}dt$$
$$=\log|t|+C$$
$$=\log|1+\log x|+C$$

Ex 7.2 Class 12 Maths Question 4.

$\sin x\sin(\cos x)$

Solution:

Let $\cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \sin x \cdot \sin(\cos x)\, dx = -\int \sin t\, dt$$
$$= -[-\cos t] + C$$
$$= \cos t + C$$
$$= \cos(\cos x) + C$$

Ex 7.2 Class 12 Maths Question 5.
sin(ax+b) cos(ax+b)
Solution:

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2a\,dx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2}\int \frac{\sin t\ dt}{2a}$$
$$= \frac{1}{4a}[-\cos t] + C$$
$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

Ex 7.2 Class 12 Maths Question 6.
$\sqrt{ax+b}$
Solution:

Let $ax + b = t$

$\Rightarrow a\,dx = dt$

$$\therefore dx = \frac{1}{a}dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

Ex 7.2 Class 12 Maths Question 7.
$x\sqrt{x+2}$
Solution:

Let $(x+2) = t$

$\therefore dx = dt$

$$\Rightarrow \int x\sqrt{x+2}\,dx = \int (t-2)\sqrt{t}\,dt$$
$$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right) dt$$
$$= \int t^{\frac{3}{2}} dt - 2\int t^{\frac{1}{2}} dt$$
$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

Ex 7.2 Class 12 Maths Question 8.
$x\sqrt{1+2x^2}$
Solution:

Let $1 + 2x^2 = t$

$\therefore 4xdx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}\,dx = \int \frac{\sqrt{t}\,dt}{4}$$
$$= \frac{1}{4}\int t^{\frac{1}{2}}dt$$
$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

Ex 7.2 Class 12 Maths Question 9.

$(4x+2)\sqrt{x^2+x+1}$

Solution:

Let $x^2 + x + 1 = t$

$\therefore (2x + 1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\;dx$$
$$= \int 2\sqrt{t}\;dt$$
$$= 2\int \sqrt{t}\;dt$$
$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$$

Ex 7.2 Class 12 Maths Question 10.

$\frac{1}{x-\sqrt{x}}$

Solution:

$$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right) = t$

$$\therefore \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx = \int \frac{2}{t}dt$$

$$= 2\log|t| + C$$
$$= 2\log\left|\sqrt{x}-1\right| + C$$

Ex 7.2 Class 12 Maths Question 11.

$\frac{x}{\sqrt{x+4}}, x > 0$

Solution:

let x+4 = t

$\Rightarrow$ dx = dt, x = t-4

$$\therefore \int \frac{x}{\sqrt{x+4}}dx = \int \frac{t-4}{\sqrt{t}}dt = \int\left(t^{1/2} - 4t^{-\frac{1}{2}}\right)dt$$
$$= \frac{2}{3}t^{3/2} - 4\times 2t^{1/2} + C$$
$$= \frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + C$$

Ex 7.2 Class 12 Maths Question 12.

$(x^3-1)^{\frac{1}{3}}.x^5$

Solution:

Let $x^3-1=t$

$\therefore 3x^2dx=dt$

$$\Rightarrow \int(x^3-1)^{\frac{1}{3}}x^5dx=\int(x^3-1)^{\frac{1}{3}}x^3\cdot x^2dx$$
$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$
$$=\frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$
$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$$
$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$
$$=\frac{1}{7}(x^3-1)^{\frac{7}{3}}+\frac{1}{4}(x^3-1)^{\frac{4}{3}}+C$$

Ex 7.2 Class 12 Maths Question 13.

$\frac{x^2}{(2+3x^3)^3}$

Solution:

Let $2+3x^3=t$

$\therefore 9x^2\,dx=dt$

$$\Rightarrow \int\frac{x^2}{(2+3x^3)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$$
$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$
$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$
$$=\frac{-1}{18(2+3x^3)^2}+C$$

Ex 7.2 Class 12 Maths Question 14.

$\frac{1}{x(\log x)^m}, x>0$

Solution:

Let $\log x=t$

$\therefore \frac{1}{x}dx=dt$

$$\Rightarrow \int\frac{1}{x(\log x)^m}dx=\int\frac{dt}{(t)^m}$$
$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$
$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$

Ex 7.2 Class 12 Maths Question 15.

$\frac{x}{9-4x^2}$

Solution:

Let $\log x=t$

$\therefore \frac{1}{x}dx=dt$

$$\Rightarrow \int\frac{1}{x(\log x)^m}dx=\int\frac{dt}{(t)^m}$$
$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$
$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$

Ex 7.2 Class 12 Maths Question 16.

e^{2x+3}
Solution:
Let $2x+3=t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3}dx = \frac{1}{2}\int e^{t}dt$$
$$= \frac{1}{2}\left(e^{t}\right)+C$$
$$= \frac{1}{2}e^{(2x+3)}+C$$

Ex 7.2 Class 12 Maths Question 17.
$\frac{x}{e^{x^2}}$
Solution:
Let $x^2=t$

$\therefore 2xdx = dt$

$$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^{t}}dt$$

$$= \frac{1}{2}\int e^{-t}dt$$
$$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right)+C$$
$$= -\frac{1}{2}e^{-x^2}+C$$
$$= \frac{-1}{2e^{x^2}}+C$$

Ex 7.2 Class 12 Maths Question 18.
$\frac{e^{\tan^{-1}x}}{1+x^2}$
Solution:
Let $\tan^{-1}x=t$

$$\therefore \frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^{t}dt$$

$$= e^{t}+C$$
$$= e^{\tan^{-1}x}+C$$

Ex 7.2 Class 12 Maths Question 19.
$\frac{e^{2x}-1}{e^{2x}+1}$
Solution:
$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^{x}}}{\frac{\left(e^{2x}+1\right)}{e^{x}}} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Let $e^{x}+e^{-x}=t$

$\therefore \left(e^{x}-e^{-x}\right)dx = dt$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dx$$

$$= \int \frac{dt}{t}$$
$$= \log|t|+C$$
$$= \log\left|e^{x}+e^{-x}\right|+C$$

Ex 7.2 Class 12 Maths Question 20.

$\frac{e^{2x}-e^{2x}}{e^{2x}+e^{-2x}}$
Solution:
Let $e^{2x}+e^{-2x}=t$

$\therefore \left(2e^{2x}-2e^{-2x}\right)dx=dt$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

Ex 7.2 Class 12 Maths Question 21.
$\tan^2(2x-3)$
Solution:
$\int\tan^2(2x-3)dx = \int[\sec^2(2x-3)-1]dx = I$
put $2x-3 = t$
so that $2dx = dt$
$I = \frac{1}{2}\int\sec^2 t\,dt-x+c$
$= \frac{1}{2}t - x + c$
$= \frac{1}{2}\tan(2x-3) - x + c$
Ex 7.2 Class 12 Maths Question 22.
$\sec^2(7-4x)$
Solution:
Let $7 - 4x = t$

$\therefore -4dx = dt$

$$\therefore \int\sec^2(7-4x)dx=\frac{-1}{4}\int\sec^2 t\,dt$$
$$=\frac{-1}{4}(\tan t)+C$$
$$=\frac{-1}{4}\tan(7-4x)+C$$

Ex 7.2 Class 12 Maths Question 23.
$\frac{\sin^{-1}x}{\sqrt{1-x^2}}$
Solution:
Let $\sin^{-1}x=t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$

$\Rightarrow \int\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\int t\,dt$

$$=\frac{t^2}{2}+C$$
$$=\frac{\left(\sin^{-1}x\right)^2}{2}+C$$

Ex 7.2 Class 12 Maths Question 24.
$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$
Solution:

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let $3\cos x+2\sin x=t$

$\therefore (-3\sin x+2\cos x)dx=dt$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log|2\sin x+3\cos x|+C$$

Ex 7.2 Class 12 Maths Question 25.

$$\frac{1}{\cos^2 x(1-\tan x)^2}$$

Solution:

$$\frac{1}{\cos^2 x(1-\tan x)^2}=\frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x)=t$

$\therefore -\sec^2 x dx=dt$

$$\Rightarrow \int\frac{\sec^2 x}{(1-\tan x)^2}dx=\int\frac{-dt}{t^2}$$
$$=-\int t^{-2}dt$$
$$=+\frac{1}{t}+C$$
$$=\frac{1}{(1-\tan x)}+C$$

Ex 7.2 Class 12 Maths Question 26.

$$\frac{\cos\sqrt{x}}{\sqrt{x}}$$

Solution:

Let $\sqrt{x}=t$

$\therefore \frac{1}{2\sqrt{x}}dx=dt$

$$\Rightarrow \int\frac{\cos\sqrt{x}}{\sqrt{x}}dx=2\int\cos t\,dt$$
$$=2\sin t+C$$
$$=2\sin\sqrt{x}+C$$

Ex 7.2 Class 12 Maths Question 27.

$$\sqrt{\sin 2x}\cos 2x$$

Solution:

Let $\sin 2x = t$

$\therefore 2\cos 2x\,dx=dt$

$$\Rightarrow \int\sqrt{\sin 2x}\ \cos 2x\,dx=\frac{1}{2}\int\sqrt{t}\,dt$$
$$=\frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$
$$=\frac{1}{3}t^{\frac{3}{2}}+C$$
$$=\frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C$$

Ex 7.2 Class 12 Maths Question 28.

$$\frac{\cos x}{\sqrt{1+\sin x}}$$

Solution:

Let $1+\sin x=t$

$\therefore \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}}dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

Ex 7.2 Class 12 Maths Question 29.
cotx log sinx
Solution:

Let log sin $x = t$

$$\Rightarrow \frac{1}{\sin x}\cdot \cos x\ dx = dt$$
$$\therefore \cot x\ dx = dt$$

$$\Rightarrow \int \cot x\ \log \sin x\ dx = \int t\, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Ex 7.2 Class 12 Maths Question 30.
$\frac{\sin x}{1+\cos x}$
Solution:

Let 1 + cos $x = t$

$\therefore$ - sin x $dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x}\ dx = \int -\frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|1+\cos x| + C$$

Ex 7.2 Class 12 Maths Question 31.
$\frac{\sin x}{(1+\cos x)^2}$
Solution:

Let 1 + cos $x = t$

$\therefore$ - sin x $dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}\ dx = \int -\frac{dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

Ex 7.2 Class 12 Maths Question 32.
$\frac{1}{1+\cot x}$
Solution:

$$\text{Let } I=\int\frac{1}{1+\cot x}dx$$
$$=\int\frac{1}{1+\frac{\cos x}{\sin x}}dx$$
$$=\int\frac{\sin x}{\sin x+\cos x}dx$$
$$=\frac{1}{2}\int\frac{2\sin x}{\sin x+\cos x}dx$$
$$=\frac{1}{2}\int\frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x+\cos x)}dx$$
$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$$
$$=\frac{1}{2}(x)+\frac{1}{2}\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$$

Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore I=\frac{x}{2}+\frac{1}{2}\int\frac{-(dt)}{t}$$
$$=\frac{x}{2}-\frac{1}{2}\log|t|+C$$
$$=\frac{x}{2}-\frac{1}{2}\log|\sin x+\cos x|+C$$

Ex 7.2 Class 12 Maths Question 33.

$\frac{1}{1-\tan x}$

Solution:

$$\text{Let } I=\int\frac{1}{1-\tan x}dx$$
$$=\int\frac{1}{1-\frac{\sin x}{\cos x}}dx$$
$$=\int\frac{\cos x}{\cos x-\sin x}dx$$
$$=\frac{1}{2}\int\frac{2\cos x}{\cos x-\sin x}dx$$
$$=\frac{1}{2}\int\frac{(\cos x-\sin x)+(\cos x+\sin x)}{(\cos x-\sin x)}dx$$
$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int\frac{\cos x+\sin x}{\cos x-\sin x}dx$$
$$=\frac{x}{2}+\frac{1}{2}\int\frac{\cos x+\sin x}{\cos x-\sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\, dx = dt$

$$\therefore I=\frac{x}{2}+\frac{1}{2}\int\frac{-(dt)}{t}$$
$$=\frac{x}{2}-\frac{1}{2}\log|t|+C$$
$$=\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$$

Ex 7.2 Class 12 Maths Question 34.

$\frac{\sqrt{\tan x}}{\sin x\cos x}$

Solution:

$$\text{Let } I=\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx$$
$$=\int\frac{\sqrt{\tan x}\times\cos x}{\sin x\cos x\times\cos x}dx$$
$$=\int\frac{\sqrt{\tan x}}{\tan x\cos^2 x}dx$$
$$=\int\frac{\sec^2 x\,dx}{\sqrt{\tan x}}$$
$$\text{Let } \tan x=t \Rightarrow \sec^2 x\,dx=dt$$
$$\therefore I=\int\frac{dt}{\sqrt{t}}$$
$$=2\sqrt{t}+C$$
$$=2\sqrt{\tan x}+C$$

Ex 7.2 Class 12 Maths Question 35.

$\frac{(1+\log x)^2}{x}$

Solution:

Let $1+\log x=t$

$$\therefore \frac{1}{x}dx=dt$$
$$\Rightarrow \int\frac{(1+\log x)^2}{x}dx=\int t^2dt$$
$$=\frac{t^3}{3}+C$$
$$=\frac{(1+\log x)^3}{3}+C$$

Ex 7.2 Class 12 Maths Question 36.

$\frac{(x+1)(x+\log x)^2}{x}$

Solution:

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$$\therefore\left(1+\frac{1}{x}\right)dx=dt$$
$$\Rightarrow\int\left(1+\frac{1}{x}\right)(x+\log x)^2dx=\int t^2dt$$
$$=\frac{t^3}{3}+C$$
$$=\frac{1}{3}(x+\log x)^3+C$$

Ex 7.2 Class 12 Maths Question 37.

$\frac{x^3\sin(\tan^{-1}x^4)}{1+x^8}dx$

Solution:

Let $x^4 = t$

$\therefore 4x^3 dx = dt$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt \qquad ...(1)$$

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2} dt = du$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4} \int \sin u \, du$$
$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4} \cos\left(\tan^{-1} t\right) + C$$
$$= \frac{-1}{4} \cos\left(\tan^{-1} x^4\right) + C$$

Ex 7.2 Class 12 Maths Question 38.

$\int \frac{10x^9+10^x \log e^{10}}{x^{10}+10^x} dx$

(a) 10x – x10 + C
(b) 10x + x10 + C
(c) (10x – x10) + C
(d) log (10x + x10) + C

Solution:

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct answer is D.

Ex 7.2 Class 12 Maths Question 39.

$\int \frac{dx}{\sin^2 x \quad \cos^2 x} =$

(a) tanx + cotx + c
(b) tanx – cotx + c
(c) tanx cotx + c
(d) tanx – cot2x + c

Solution:

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct answer is D.

Integrals Ex 7.3 Class 12

Find the integrals of the functions in Exercises 1 to 22.

Ex 7.3 Class 12 Maths Question 1.

$\sin^2(2x+5)$

Solution:

$$\sin^2(2x+5)=\frac{1-\cos 2(2x+5)}{2}=\frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx=\int\frac{1-\cos(4x+10)}{2}dx$$

$$=\frac{1}{2}\int 1\,dx-\frac{1}{2}\int\cos(4x+10)\,dx$$

$$=\frac{1}{2}x-\frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+C$$

$$=\frac{1}{2}x-\frac{1}{8}\sin(4x+10)+C$$

Ex 7.3 Class 12 Maths Question 2.

sin3x cos4x

Solution:

It is known that, $\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\therefore \int \sin 3x\cos 4x\,dx=\frac{1}{2}\int\{\sin(3x+4x)+\sin(3x-4x)\}\,dx$$

$$=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}\,dx$$

$$=\frac{1}{2}\int\{\sin 7x-\sin x\}\,dx$$

$$=\frac{1}{2}\int\sin 7x\,dx-\frac{1}{2}\int\sin x\,dx$$

$$=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C$$

$$=\frac{-\cos 7x}{14}+\frac{\cos x}{2}+C$$

Ex 7.3 Class 12 Maths Question 3.

∫cos2x cos4x cos6x dx

Solution:

It is known that, $\cos A\cos B=\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\therefore \int\cos 2x(\cos 4x\cos 6x)dx=\int\cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\}\,dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos^2 2x\}\,dx$$

$$=\frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)\,dx$$

$$=\frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

Ex 7.3 Class 12 Maths Question 4.

$\int\sin^3(2x+1)dx$

Solution:

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$
$$= \int\left(1-\cos^2(2x+1)\right)\sin(2x+1)\,dx$$

Let $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1)\,dx = dt$$
$$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int\left(1-t^2\right)dt$$
$$= \frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$
$$= \frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos^3(2x+1)}{3}\right\}$$
$$= \frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

Ex 7.3 Class 12 Maths Question 5.

$\sin^3 x \cos^3 x$

Solution:

$$\text{Let } I = \int \sin^3 x\cos^3 x\cdot dx$$
$$= \int \cos^3 x\cdot\sin^2 x\cdot\sin x\cdot dx$$
$$= \int \cos^3 x\left(1-\cos^2 x\right)\sin x\cdot dx$$

Let $\cos x = t$

$$\Rightarrow -\sin x\cdot dx = dt$$
$$\Rightarrow I = -\int t^3\left(1-t^2\right)dt$$
$$= -\int\left(t^3-t^5\right)dt$$
$$= -\left\{\frac{t^4}{4}-\frac{t^6}{6}\right\}+C$$
$$= -\left\{\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}\right\}+C$$
$$= \frac{\cos^6 x}{6}-\frac{\cos^4 x}{4}+C$$

Ex 7.3 Class 12 Maths Question 6.

sinx sin2x sin3x

Solution:

It is known that, $\sin A\sin B = \frac{1}{2}\left\{\cos(A-B)-\cos(A+B)\right\}$

$$\therefore \int \sin x\sin 2x\sin 3x\ dx = \int\left[\sin x\cdot\frac{1}{2}\left\{\cos(2x-3x)-\cos(2x+3x)\right\}\right]dx$$
$$= \frac{1}{2}\int\left(\sin x\cos(-x)-\sin x\cos 5x\right)dx$$
$$= \frac{1}{2}\int\left(\sin x\cos x-\sin x\cos 5x\right)dx$$
$$= \frac{1}{2}\int\frac{\sin 2x}{2}\,dx-\frac{1}{2}\int\sin x\cos 5x\,dx$$
$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\}dx$$
$$= \frac{-\cos 2x}{8}-\frac{1}{4}\int\left(\sin 6x+\sin(-4x)\right)dx$$
$$= \frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{4}\right]+C$$
$$= \frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{2}\right]+C$$
$$= \frac{1}{8}\left[\frac{\cos 6x}{3}-\frac{\cos 4x}{2}-\cos 2x\right]+C$$

Ex 7.3 Class 12 Maths Question 7.

sin 4x sin 8x

Solution:

$\frac{1}{2}\int \sin 4x \sin 8x dx$
$= \frac{1}{2}\int (\cos 4x - \cos 12x) dx$
$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + c$

Ex 7.3 Class 12 Maths Question 8.

$\frac{1-\cos x}{1+\cos x}$

Solution:

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$
$$= \left(\frac{1+\cos 4x}{2}\right)^2$$
$$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$\therefore \int \cos^4 2x\, dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$
$$= \frac{3}{8}x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

Ex 7.3 Class 12 Maths Question 9.

$\frac{\cos x}{1+\cos x}$

Solution:

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$
$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$
$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$
$$= \frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$
$$= \frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$
$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$
$$= x - \tan\frac{x}{2} + C$$

Ex 7.3 Class 12 Maths Question 10.

$\int \sin x^4\, dx$

Solution:

$$\begin{aligned}
\sin^4 x &= \sin^2 x \sin^2 x \\
&= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) \\
&= \frac{1}{4}(1-\cos 2x)^2 \\
&= \frac{1}{4}\left[1+\cos^2 2x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] \\
&= \frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] \\
&= \frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right]
\end{aligned}$$

$$\begin{aligned}
\therefore \int \sin^4 x \, dx &= \frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x - 2\cos 2x\right] dx \\
&= \frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C \\
&= \frac{1}{8}\left[3x+\frac{\sin 4x}{4} - 2\sin 2x\right] + C \\
&= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C
\end{aligned}$$

Ex 7.3 Class 12 Maths Question 11.

$\cos^4 2x$

Solution:

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[2\sin^2 \frac{x}{2} = 1-\cos x \text{ and } 2\cos^2 \frac{x}{2} = 1+\cos x\right]$$

$$\begin{aligned}
&= \tan^2 \frac{x}{2} \\
&= \left(\sec^2 \frac{x}{2} - 1\right)
\end{aligned}$$

$$\begin{aligned}
\therefore \int \frac{1-\cos x}{1+\cos x} dx &= \int\left(\sec^2 \frac{x}{2} - 1\right) dx \\
&= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x\right] + C \\
&= 2\tan \frac{x}{2} - x + C
\end{aligned}$$

Ex 7.3 Class 12 Maths Question 12.

$\frac{\sin^2 x}{1+\cos x}$

Solution:

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin \frac{x}{2}\cos \frac{x}{2}\right)^2}{2\cos^2 \frac{x}{2}} \qquad \left[\sin x = 2\sin \frac{x}{2}\cos \frac{x}{2};\ \cos x = 2\cos^2 \frac{x}{2} - 1\right]$$

$$\begin{aligned}
&= \frac{4\sin^2 \frac{x}{2}\cos^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \\
&= 2\sin^2 \frac{x}{2} \\
&= 1-\cos x
\end{aligned}$$

$$\begin{aligned}
\therefore \int \frac{\sin^2 x}{1+\cos x} dx &= \int (1-\cos x) dx \\
&= x - \sin x + C
\end{aligned}$$

Ex 7.3 Class 12 Maths Question 13.

$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

Solution:

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+C$$

Ex 7.3 Class 12 Maths Question 14.

$\frac{\cos x-\sin x}{1+\sin 2x}$

Solution:

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$$

Let $\sin x+\cos x=t$

$\therefore(\cos x-\sin x)\,dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+C$$

$$=-\frac{1}{t}+C$$

$$=\frac{-1}{\sin x+\cos x}+C$$

Ex 7.3 Class 12 Maths Question 15.

$\int\tan^3 2x \quad \sec 2x \quad dx=I$

Solution:

$$\tan^3 2x\sec 2x=\tan^2 2x\tan 2x\sec 2x$$

$$=\left(\sec^2 2x-1\right)\tan 2x\sec 2x$$

$$=\sec^2 2x\cdot\tan 2x\sec 2x-\tan 2x\sec 2x$$

$$\therefore\int\tan^3 2x\sec 2x\ dx=\int\sec^2 2x\tan 2x\sec 2x\ dx-\int\tan 2x\sec 2x\ dx$$

$$=\int\sec^2 2x\tan 2x\sec 2x\ dx-\frac{\sec 2x}{2}+C$$

Let $\sec 2x=t$

$\therefore 2\sec 2x\tan 2x\ dx=dt$

$$\therefore\int\tan^3 2x\sec 2x\ dx=\frac{1}{2}\int t^2dt-\frac{\sec 2x}{2}+C$$

$$=\frac{t^3}{6}-\frac{\sec 2x}{2}+C$$

$$=\frac{(\sec 2x)^3}{6}-\frac{\sec 2x}{2}+C$$

Ex 7.3 Class 12 Maths Question 16.

$\tan^4 x$

Solution:

$$\tan^4 x$$
$$= \tan^2 x \cdot \tan^2 x$$
$$= \left(\sec^2 x - 1\right)\tan^2 x$$
$$= \sec^2 x \tan^2 x - \tan^2 x$$
$$= \sec^2 x \tan^2 x - \left(\sec^2 x - 1\right)$$
$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

$$\therefore \int \tan^4 x \, dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad \ldots(1)$$

Consider $\int \sec^2 x \tan^2 x \, dx$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x \, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

Ex 7.3 Class 12 Maths Question 17.

$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Solution:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$
$$= \tan x \sec x + \cot x \operatorname{cosec} x$$
$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\tan x \sec x + \cot x \operatorname{cosec} x\right) dx$$
$$= \sec x - \operatorname{cosec} x + C$$

Ex 7.3 Class 12 Maths Question 18.

$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Solution:

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos 2x + \left(1 - \cos 2x\right)}{\cos^2 x} \qquad \left[\cos 2x = 1 - 2\sin^2 x\right]$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$
$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + C$$

Ex 7.3 Class 12 Maths Question 19.

$\frac{1}{\sin x \cos^3 x}$

Solution:

$$I = \int \left(tanx + \frac{1}{tanx}\right) sec^2 x dx$$

put tanx = t

so that sec²x dx = dt

$$I = \int \left(t + \frac{1}{t}\right) dt \quad = \frac{t^2}{2} + log|t| + c$$
$$= log|tanx| + \frac{1}{2} tan^2 x + c$$

Ex 7.3 Class 12 Maths Question 20.

$\frac{\cos 2x}{(\cos x + \sin x)^2}$

Solution:

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2}\, dx=\int \frac{\cos 2x}{(1+\sin 2x)}\, dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\ dx=dt$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2} dx=\frac{1}{2}\int \frac{1}{t} dt$$

$$=\frac{1}{2}\log|t|+\mathrm{C}$$

$$=\frac{1}{2}\log|1+\sin 2x|+\mathrm{C}$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+\mathrm{C}$$

$$=\log|\sin x+\cos x|+\mathrm{C}$$

Ex 7.3 Class 12 Maths Question 21.

$\sin^{-1}(\cos x)$

Solution:

$\sin^{-1}(\cos x)$

Let $\cos x=t$

Then, $\sin x=\sqrt{1-t^2}$

$\Rightarrow(-\sin x)dx=dt$

$$dx=\frac{-dt}{\sin x}$$

$$dx=\frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx=\int \sin^{-1}t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$=-\int \frac{\sin^{-1}t}{\sqrt{1-t^2}}dt$$

Let $\sin^{-1}t=u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}}dt=du$$

$$\therefore \int \sin^{-1}(\cos x)dx=\int 4du$$

$$=-\frac{u^2}{2}+\mathrm{C}$$

$$=\frac{-(\sin^{1}t)^2}{2}+\mathrm{C}$$

$$=\frac{-\left[\sin^{-1}(\cos x)\right]^2}{2}+\mathrm{C} \qquad \ldots(1)$$

It is known that,

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x)=\frac{\pi}{2}-\cos^{-1}(\cos x)=\left(\frac{\pi}{2}-x\right)$$

Substituting in equation (1), we obtain

$$\begin{aligned}\int \sin^{-1}(\cos x)\,dx &= \frac{-\left[\frac{\pi}{2}-x\right]^2}{2}+C\\ &= -\frac{1}{2}\left(\frac{\pi^2}{2}+x^2-\pi x\right)+C\\ &= -\frac{\pi^2}{8}-\frac{x^2}{2}+\frac{1}{2}\pi x+C\\ &= \frac{\pi x}{2}-\frac{x^2}{2}+\left(C-\frac{\pi^2}{8}\right)\\ &= \frac{\pi x}{2}-\frac{x^2}{2}+C_1\end{aligned}$$

Ex 7.3 Class 12 Maths Question 22.

$\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

Solution:

$$\begin{aligned}\frac{1}{\cos(x-a)\cos(x-b)} &= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]\\ &= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]\\ &= \frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}\\ &= \frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]\end{aligned}$$

$$\begin{aligned}\Rightarrow \int\frac{1}{\cos(x-a)\cos(x-b)}dx &= \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx\\ &= \frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)|+\log|\cos(x-a)|\right]\\ &= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C\end{aligned}$$

Ex 7.3 Class 12 Maths Question 23.

$\int \frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx$ is equal to

(a) tanx+cotx+c
(b) tanx+cosecx+c
(c) -tanx+cotx+c
(d) tanx+secx+c

Solution:

$$\begin{aligned}\int\frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx &= \int\left(\frac{\sin^2x}{\sin^2x\cos^2x}-\frac{\cos^2x}{\sin^2x\cos^2x}\right)dx\\ &= \int(\sec^2x-\operatorname{cosec}^2x)\,dx\\ &= \tan x+\cot x+C\end{aligned}$$

Hence, the correct answer is A.

Ex 7.3 Class 12 Maths Question 24.

$\int \frac{e^x(1+x)}{\cos^2(e^x.x)}dx$ is equal to

(a) $-\cot(e.x^x)+c$
(b) $\tan(xe^x)+c$
(c) $\tan(e^x)+c$
(d) $\cot e^x+c$

Solution:

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

Let $e^x x = t$

$$\Rightarrow (e^x \cdot x + e^x \cdot 1)dx = dt$$
$$e^x(x+1)dx = dt$$

$$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t\ dt$$
$$= \tan t + C$$
$$= \tan(e^x \cdot x) + C$$

Hence, the correct answer is B.

Integrals Class 12 Ex 7.4

Ex 7.4 Class 12 Maths Question 1.

$\frac{3x^2}{x^6+1}$

Solution:

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1}$$
$$= \tan^1 t + C$$
$$= \tan^{-1}(x^3) + C$$

Ex 7.4 Class 12 Maths Question 2.

$\frac{1}{\sqrt{1+4x^2}}$

Solution:

Let $2x = t$

$\therefore 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$$
$$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$
$$= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + C$$

Ex 7.4 Class 12 Maths Question 3.

$\frac{1}{\sqrt{(2-x)^2+1}}$

Solution:

Let $2 - x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$
$$= -\log\left|t+\sqrt{t^2+1}\right| + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$
$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C$$
$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

Ex 7.4 Class 12 Maths Question 4.

$\frac{1}{\sqrt{9-25x^2}}$

Solution:

Let $5x = t$

$\therefore 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}\,dx = \frac{1}{5}\int \frac{1}{9-t^2}\,dt$$
$$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}\,dt$$
$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right)+C$$
$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+C$$

Ex 7.4 Class 12 Maths Question 5.

$\frac{3x}{1+2x^4}$

Solution:

Let $\sqrt{2}x^2 = t$

$\therefore 2\sqrt{2}x\ dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$$
$$= \frac{3}{2\sqrt{2}}\left[\tan^{-1} t\right]+C$$
$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right)+C$$

Ex 7.4 Class 12 Maths Question 6.

$\frac{x^2}{1-x^6}$

Solution:

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$$
$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right]+C$$
$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+C$$

Ex 7.4 Class 12 Maths Question 7.

$\frac{x-1}{\sqrt{x^2-1}}$

Solution:

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx \qquad \ldots(1)$$

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2-1 = t \Rightarrow 2x\ dx = dt$

$$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$$
$$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

From (1), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \qquad \left[\int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$
$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right|+C$$

Ex 7.4 Class 12 Maths Question 8.

$\frac{x^2}{\sqrt{x^6+a^6}}$

Solution:

Let $x^3 = t$

$\Rightarrow 3x^2\,dx = dt$

$$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$$
$$= \frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right| + C$$
$$= \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right| + C$$

Ex 7.4 Class 12 Maths Question 9.

$\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

Solution:

Let $\tan x = t$

$\therefore \sec^2 x\,dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx = \int \frac{dt}{\sqrt{t^2+2^2}}$$
$$= \log\left|t+\sqrt{t^2+4}\right| + C$$
$$= \log\left|\tan x+\sqrt{\tan^2 x+4}\right| + C$$

Ex 7.4 Class 12 Maths Question 10.

$\frac{1}{\sqrt{x^2+2x+2}}$

Solution:

$$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$$

Let $x+1=t$

$\therefore\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$$
$$= \log\left|t+\sqrt{t^2+1}\right| + C$$
$$= \log\left|(x+1)+\sqrt{(x+1)^2+1}\right| + C$$
$$= \log\left|(x+1)+\sqrt{x^2+2x+2}\right| + C$$

Ex 7.4 Class 12 Maths Question 11.

$\frac{1}{9x^2+6x+5}$

Solution:

$I = \int \frac{dx}{\sqrt{4^2-(x+3)^2}} \quad = sin^{-1}\left(\frac{x+3}{4}\right) + c$

Ex 7.4 Class 12 Maths Question 12.

$\frac{1}{\sqrt{7-6x-x^2}}$

Solution:

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-(x+3)^2$$
$$=(4)^2-(x+3)^2$$

$$\therefore \int \frac{1}{\sqrt{7-6x-x^2}}dx=\int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\int \frac{1}{\sqrt{(4)^2-(t)^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$
$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Ex 7.4 Class 12 Maths Question 13.

$$\frac{1}{\sqrt{(x-1)(x-2)}}$$

Solution:

$(x-1)(x-2)$ can be written as x^2-3x+2.

Therefore,

$$x^2-3x+2$$
$$=x^2-3x+\frac{9}{4}-\frac{9}{4}+2$$
$$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$$
$$=\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}}dx=\int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx=\int \frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}dt$$
$$=\log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right|+C$$
$$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

Ex 7.4 Class 12 Maths Question 14.

$$\frac{1}{\sqrt{8+3x-x^2}}$$

Solution:

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}dx=\int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

Ex 7.4 Class 12 Maths Question 15.

$\frac{1}{\sqrt{(x-a)(x-b)}}$

Solution:

$(x-a)(x-b)$ can be written as $x^2-(a+b)x+ab$.

Therefore,

$$x^2-(a+b)x+ab$$

$$=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$

$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$

$$\Rightarrow \int \frac{1}{\sqrt{(x-a)(x-b)}}dx=\int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx=\int \frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|+C$$

$$=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

Ex 7.4 Class 12 Maths Question 16.

$\frac{4x+1}{\sqrt{2x^2+x-3}}$

Solution:

$$\text{Let } 4x+1 = A\frac{d}{dx}\left(2x^2+x-3\right)+B$$
$$\Rightarrow 4x+1 = A\left(4x+1\right)+B$$
$$\Rightarrow 4x+1 = 4Ax+A+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)\, dx = dt$

$$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int \frac{1}{\sqrt{t}}dt$$
$$= 2\sqrt{t}+C$$
$$= 2\sqrt{2x^2+x-3}+C$$

Ex 7.4 Class 12 Maths Question 17.

$\frac{x+2}{\sqrt{x^2-1}}$

Solution:

$$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$
$$\Rightarrow x+2 = A\left(2x\right)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x)+2$$

$$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$
$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$

$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2x\,dx = dt$$

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\left[2\sqrt{t}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$$

Ex 7.4 Class 12 Maths Question 18.

$\frac{5x-2}{1+2x+3x^2}$

Solution:

put $5x-2=A\frac{d}{dx}(1+2x+3x^2)+B$

$\Rightarrow 6A=5,\ A=\frac{5}{6} - 2 = 2A + B,\ B=-\frac{11}{3}$

$$I=\int \frac{\frac{5}{6}(6x+2)}{3x^2+2x+1}\,dx-\frac{11}{3}\int \frac{dx}{3x^2+2x+1}$$

$$=I_1-\frac{11}{3}I_2\text{ ; put }3x^2+2x+1=t\therefore\ (6x+2)\,dx=dt$$

$$I_1=\frac{5}{6}\int\frac{dt}{t}=\frac{5}{6}\log t=\frac{5}{6}\log(3x^2+2x+1)+c_1$$

$$\text{and }I_2=\int\frac{dx}{3x^2+2x+1}=\frac{1}{3}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}$$

$$\Rightarrow I_2=\frac{1}{\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}}+c$$

$$\therefore I=\frac{5}{6}\log(3x^2+2x+1)-\frac{11}{3}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}}+c$$

Ex 7.4 Class 12 Maths Question 19.

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}}$$

Solution:

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}}=\frac{6x+7}{\sqrt{x^2-9x+20}}$$

$$\text{Let }6x+7=A\frac{d}{dx}\left(x^2-9x+20\right)+B$$
$$\Rightarrow 6x+7=A(2x-9)+B$$

Equating the coefficients of x and constant term, we obtain

$2A=6\Rightarrow A=3$

$-9A+B=7\Rightarrow B=34$

$\therefore 6x+7=3(2x-9)+34$

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}=\int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$
$$=3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$\text{Let }I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx\text{ and }I_2=\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$$\therefore\int\frac{6x+7}{\sqrt{x^2-9x+20}}=3I_1+34I_2\qquad\ldots(1)$$

Then,

$$I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$$

$$\text{Let }x^2-9x+20=t$$
$$\Rightarrow(2x-9)dx=dt$$
$$\Rightarrow I_1=\frac{dt}{\sqrt{t}}$$
$$I_1=2\sqrt{t}$$
$$I_1=2\sqrt{x^2-9x+20}\qquad\ldots(2)$$

$$\text{and }I_2=\int\frac{1}{\sqrt{x^2-9x+20}}dx$$

$x^2-9x+20$ can be written as $x^2-9x+20+\frac{81}{4}-\frac{81}{4}$.

Therefore,

$$x^2-9x+20+\frac{81}{4}-\frac{81}{4}$$

$$=\left(x-\frac{9}{2}\right)^2-\frac{1}{4}$$

$$=\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2=\int\frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

$$I_2=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right| \qquad ...(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}dx=3\left[2\sqrt{x^2-9x+20}\right]+34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$

$$=6\sqrt{x^2-9x+20}+34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$

Ex 7.4 Class 12 Maths Question 20.

$\frac{x+2}{\sqrt{4x-x^2}}$

Solution:

$$\text{Let } x+2=A\frac{d}{dx}\left(4x-x^2\right)+B$$

$$\Rightarrow x+2=A(4-2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$-2A=1\Rightarrow A=-\frac{1}{2}$$

$$4A+B=2\Rightarrow B=4$$

$$\Rightarrow (x+2)=-\frac{1}{2}(4-2x)+4$$

$$\therefore \int\frac{x+2}{\sqrt{4x-x^2}}dx=\int\frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$$

$$=-\frac{1}{2}\int\frac{4-2x}{\sqrt{4x-x^2}}dx+4\int\frac{1}{\sqrt{4x-x^2}}dx$$

$$\text{Let } I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}dx \text{ and } I_2\int\frac{1}{\sqrt{4x-x^2}}dx$$

$$\therefore \int\frac{x+2}{\sqrt{4x-x^2}}dx=-\frac{1}{2}I_1+4I_2 \qquad ...(1)$$

$$\text{Then, } I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}dx$$

$$\text{Let } 4x-x^2=t$$

$$\Rightarrow (4-2x)dx=dt$$

$$\Rightarrow I_1=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{4x-x^2} \qquad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{4x - x^2}} dx$$

$$\Rightarrow 4x - x^2 = -(-4x + x^2)$$

$$= (-4x + x^2 + 4 - 4)$$

$$= 4 - (x-2)^2$$

$$= (2)^2 - (x-2)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}} dx = \sin^{-1}\left(\frac{x-2}{2}\right) \quad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2}\left(2\sqrt{4x-x^2}\right) + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

$$= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$$

Ex 7.4 Class 12 Maths Question 21.

$$\frac{x+2}{\sqrt{x^2+2x+3}}$$

Solution:

$$\int \frac{(x+2)}{\sqrt{x^2+2x+3}} dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} I_1 + I_2 \quad ...(1)$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x + 2)\, dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad ...(2)$$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$\Rightarrow x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x+1)^2 + \left(\sqrt{2}\right)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2 + \left(\sqrt{2}\right)^2}} dx = \log\left|(x+1) + \sqrt{x^2+2x+3}\right| \quad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$$

$$= \sqrt{x^2+2x+3} + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$$

Ex 7.4 Class 12 Maths Question 22.

$$\frac{x+3}{x^2-2x-5}$$

Solution:

$$\text{Let } (x+3) = A\frac{d}{dx}(x^2-2x-5)+B$$
$$(x+3) = A(2x-2)+B$$

Equating the coefficients of *x* and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$-2A+B = 3 \Rightarrow B = 4$$
$$\therefore (x+3) = \frac{1}{2}(2x-2)+4$$
$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$
$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

$$\text{Let } I_1 = \int \frac{2x-2}{x^2-2x-5}dx \text{ and } I_2 = \int \frac{1}{x^2-2x-5}dx$$
$$\therefore \int \frac{x+3}{(x^2-2x-5)}dx = \frac{1}{2}I_1 + 4I_2 \quad \text{...(1)}$$
$$\text{Then, } I_1 = \int \frac{2x-2}{x^2-2x-5}dx$$

$$\text{Let } x^2-2x-5 = t$$
$$\Rightarrow (2x-2)dx = dt$$
$$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5| \quad \text{...(2)}$$

$$I_2 = \int \frac{1}{x^2-2x-5}dx$$
$$= \int \frac{1}{(x^2-2x+1)-6}dx$$
$$= \int \frac{1}{(x-1)^2+(\sqrt{6})^2}dx$$
$$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \quad \text{...(3)}$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log|x^2-2x-5| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$
$$= \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

Ex 7.4 Class 12 Maths Question 23.

$$\frac{5x+3}{\sqrt{x^2+4x+10}}$$

Solution:

$$\text{Let } 5x+3 = A\frac{d}{dx}\left(x^2+4x+10\right)+B$$
$$\Rightarrow 5x+3 = A(2x+4)+B$$

Equating the coefficients of *x* and constant term, we obtain

$$2A = 5 \Rightarrow A = \frac{5}{2}$$
$$4A+B = 3 \Rightarrow B = -7$$
$$\therefore 5x+3 = \frac{5}{2}(2x+4)-7$$
$$\Rightarrow \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int\frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$
$$= \frac{5}{2}\int\frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\text{Let } I_1 = \int\frac{2x+4}{\sqrt{x^2+4x+10}}dx \text{ and } I_2 = \int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\therefore \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \qquad ...(1)$$
$$\text{Then, } I_1 = \int\frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

$$\text{Let } x^2+4x+10 = t$$
$$\therefore (2x+4)dx = dt$$
$$\Rightarrow I_1 = \int\frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \qquad ...(2)$$
$$I_2 = \int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$= \int\frac{1}{\sqrt{\left(x^2+4x+4\right)+6}}dx$$
$$= \int\frac{1}{(x+2)^2+\left(\sqrt{6}\right)^2}dx$$
$$= \log\left|(x+2)\sqrt{x^2+4x+10}\right| \qquad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\left[2\sqrt{x^2+4x+10}\right] - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$
$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$

Ex 7.4 Class 12 Maths Question 24.

$\int \frac{dx}{x^2+2x+2}$ equals

(a) $x\tan^{-1}(x+1)+c$

(b) $(x+1)\tan^{-1}x+c$

(c) $\tan^{-1}(x+1)+c$

(d) $\tan^{-1}x+c$

Solution:

$$\int\frac{dx}{x^2+2x+2} = \int\frac{dx}{\left(x^2+2x+1\right)+1}$$
$$= \int\frac{1}{(x+1)^2+(1)^2}dx$$
$$= \left[\tan^{-1}(x+1)\right]+C$$

Hence, the correct answer is B.

Ex 7.4 Class 12 Maths Question 25.

$\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

(a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$

(b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+c$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$

(d) $\sin^{-1}\left(\frac{9x-8}{9}\right)+c$

Solution:

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$= \int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$= \int \frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}dx$$

$$= \int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1}\frac{y}{a}+C\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Hence, the correct answer is B.

Integrals Class 12 Ex 7.5

Ex 7.5 Class 12 Maths Question 1.

$\frac{x}{(x+1)(x+2)}$

Solution:

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2)+B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C$$

$$= \log\frac{(x+2)^2}{(x+1)} + C$$

Ex 7.5 Class 12 Maths Question 2.

$\frac{1}{x^2-9}$

Solution:

Let $\frac{1}{(x+3)(x-3)}=\frac{A}{(x+3)}+\frac{B}{(x-3)}$

$1=A(x-3)+B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A=-\frac{1}{6}$ and $B=\frac{1}{6}$

$$\therefore \frac{1}{(x+3)(x-3)}=\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}$$

$$\Rightarrow \int\frac{1}{(x^2-9)}dx=\int\left(\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}\right)dx$$

$$=-\frac{1}{6}\log|x+3|+\frac{1}{6}\log|x-3|+C$$

$$=\frac{1}{6}\log\left|\frac{(x-3)}{(x+3)}\right|+C$$

Ex 7.5 Class 12 Maths Question 3.

$\frac{3x-1}{(x-1)(x-2)(x-3)}$

Solution:

Let $\frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$

$$3x-1=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad ...(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

$A = 1$, $B = -5$, and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}$$

$$\Rightarrow \int\frac{3x-1}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}\right\}dx$$

$$=\log|x-1|-5\log|x-2|+4\log|x-3|+C$$

Ex 7.5 Class 12 Maths Question 4.

$\frac{x}{(x-1)(x-2)(x-3)}$

Solution:

Let $\frac{x}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$

$$x=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad ...(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain $A=\frac{1}{2}$, $B=-2$, and $C=\frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}$$

$$\Rightarrow \int\frac{x}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{2(x-3)}\right\}dx$$

$$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$$

Ex 7.5 Class 12 Maths Question 5.

$\frac{2x}{x^2+3x+2}$

Solution:

Let $\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$

$$2x=A(x+2)+B(x+1) \quad ...(1)$$

Substituting x = - 1 and - 2 in equation (1), we obtain

A = - 2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)}=\frac{-2}{(x+1)}+\frac{4}{(x+2)}$$

$$\Rightarrow \int\frac{2x}{(x+1)(x+2)}dx=\int\left\{\frac{4}{(x+2)}-\frac{2}{(x+1)}\right\}dx$$

$$=4\log|x+2|-2\log|x+1|+C$$

Ex 7.5 Class 12 Maths Question 6.

$\frac{1-x^2}{x(1-2x)}$

Solution:

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing (1 - x^2) by x(1 - 2x), we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)}=\frac{A}{x}+\frac{B}{(1-2x)}$

$$\Rightarrow(2-x)=A(1-2x)+Bx \quad ...(1)$$

Substituting x = 0 and $\frac{1}{2}$ in equation (1), we obtain

A = 2 and B = 3

$$\therefore \frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left\{\frac{2}{x}+\frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{1-2x}\right)\right\}dx$$

$$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$$

Ex 7.5 Class 12 Maths Question 7.

$\frac{x}{(x^2+1)(x-1)}$

Solution:

Let $\frac{x}{(x^2+1)(x-1)}=\frac{Ax+B}{(x^2+1)}+\frac{C}{(x-1)}$

$$x=(Ax+B)(x-1)+C(x^2+1)$$

$$x=Ax^2-Ax+Bx-B+Cx^2+C$$

Equating the coefficients of x^2, x, and constant term, we obtain

A + C = 0

- A + B = 1

- B + C = 0

On solving these equations, we obtain

$$A=-\frac{1}{2}, B=\frac{1}{2}, \text{ and } C=\frac{1}{2}$$

From equation (1), we obtain

$$\therefore \frac{x}{\left(x^2+1\right)(x-1)}=\frac{\left(-\frac{1}{2}x+\frac{1}{2}\right)}{x^2+1}+\frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int\frac{x}{\left(x^2+1\right)(x-1)}=-\frac{1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=-\frac{1}{4}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$

Consider $\int\frac{2x}{x^2+1}dx$, let $\left(x^2+1\right)=t \Rightarrow 2x\,dx=dt$

$$\Rightarrow \int\frac{2x}{x^2+1}dx=\int\frac{dt}{t}=\log|t|=\log\left|x^2+1\right|$$

$$\therefore \int\frac{x}{\left(x^2+1\right)(x-1)}=-\frac{1}{4}\log\left|x^2+1\right|+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$

$$=\frac{1}{2}\log|x-1|-\frac{1}{4}\log\left|x^2+1\right|+\frac{1}{2}\tan^{-1}x+C$$

Ex 7.5 Class 12 Maths Question 8.

$\frac{x}{(x-1)^2(x+2)}$

Solution:

Let $\frac{x}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$

$$x=A(x-1)(x+2)+B(x+2)+C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B=\frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$$A=\frac{2}{9} \text{ and } C=\frac{-2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)}=\frac{2}{9(x-1)}+\frac{1}{3(x-1)^2}-\frac{2}{9(x+2)}$$

$$\Rightarrow \int\frac{x}{(x-1)^2(x+2)}dx=\frac{2}{9}\int\frac{1}{(x-1)}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{(x+2)}dx$$

$$=\frac{2}{9}\log|x-1|+\frac{1}{3}\left(\frac{-1}{x-1}\right)-\frac{2}{9}\log|x+2|+C$$

$$=\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+C$$

Ex 7.5 Class 12 Maths Question 9.

$\frac{3x+5}{x^3-x^2-x+1}$

Solution:

$$\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+1)}$

$$3x+5=A(x-1)(x+1)+B(x+1)+C(x-1)^2$$
$$3x+5=A(x^2-1)+B(x+1)+C(x^2+1-2x) \qquad ...(1)$$

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)}=\frac{-1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$$
$$\Rightarrow \int\frac{3x+5}{(x-1)^2(x+1)}dx=-\frac{1}{2}\int\frac{1}{x-1}dx+4\int\frac{1}{(x-1)^2}dx+\frac{1}{2}\int\frac{1}{(x+1)}dx$$
$$=-\frac{1}{2}\log|x-1|+4\left(\frac{-1}{x-1}\right)+\frac{1}{2}\log|x+1|+C$$
$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{(x-1)}+C$$

Ex 7.5 Class 12 Maths Question 10.

$\frac{2x-3}{(x^2-1)(2x+3)}$

Solution:

$$\frac{2x-3}{(x^2-1)(2x+3)}=\frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{C}{(2x+3)}$

$$\Rightarrow (2x-3)=A(x-1)(2x+3)+B(x+1)(2x+3)+C(x+1)(x-1)$$
$$\Rightarrow (2x-3)=A(2x^2+x-3)+B(2x^2+5x+3)+C(x^2-1)$$
$$\Rightarrow (2x-3)=(2A+2B+C)x^2+(A+5B)x+(-3A+3B-C)$$

Equating the coefficients of x^2 and x, we obtain

$B=-\frac{1}{10}$, $A=\frac{5}{2}$, and $C=-\frac{24}{5}$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)}=\frac{5}{2(x+1)}-\frac{1}{10(x-1)}-\frac{24}{5(2x+3)}$$
$$\Rightarrow \int\frac{2x-3}{(x^2-1)(2x+3)}dx=\frac{5}{2}\int\frac{1}{(x+1)}dx-\frac{1}{10}\int\frac{1}{x-1}dx-\frac{24}{5}\int\frac{1}{(2x+3)}dx$$
$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{24}{5\times 2}\log|2x+3|$$
$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+C$$

Ex 7.5 Class 12 Maths Question 11.

$\frac{5x}{(x-1)(x^2-4)}$

Solution:

$$\frac{5x}{(x+1)(x^2-4)}=\frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x-2)}$

$$5x=A(x+2)(x-2)+B(x+1)(x-2)+C(x+1)(x+2) \qquad ...(1)$$

Substituting x = - 1, - 2, and 2 respectively in equation (1), we obtain

$A=\frac{5}{3}$, $B=-\frac{5}{2}$, and $C=\frac{5}{6}$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)}=\frac{5}{3(x+1)}-\frac{5}{2(x+2)}+\frac{5}{6(x-2)}$$

$$\Rightarrow \int\frac{5x}{(x+1)(x^2-4)}dx=\frac{5}{3}\int\frac{1}{(x+1)}dx-\frac{5}{2}\int\frac{1}{(x+2)}dx+\frac{5}{6}\int\frac{1}{(x-2)}dx$$

$$=\frac{5}{3}\log|x+1|-\frac{5}{2}\log|x+2|+\frac{5}{6}\log|x-2|+C$$

Ex 7.5 Class 12 Maths Question 12.

$\frac{x^3+x+1}{x^2-1}$

Solution:

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1}=x+\frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1}=\frac{A}{(x+1)}+\frac{B}{(x-1)}$

$$2x+1=A(x-1)+B(x+1) \qquad ...(1)$$

Substituting x = 1 and - 1 in equation (1), we obtain

$A=\frac{1}{2}$ and $B=\frac{3}{2}$

$$\therefore \frac{x^3+x+1}{x^2-1}=x+\frac{1}{2(x+1)}+\frac{3}{2(x-1)}$$

$$\Rightarrow \int\frac{x^3+x+1}{x^2-1}dx=\int x\,dx+\frac{1}{2}\int\frac{1}{(x+1)}dx+\frac{3}{2}\int\frac{1}{(x-1)}dx$$

$$=\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+C$$

Ex 7.5 Class 12 Maths Question 13.

$\frac{2}{(1-x)(1+x^2)}$

Solution:

Let $\frac{2}{(1-x)(1+x^2)}=\frac{A}{(1-x)}+\frac{Bx+C}{(1+x^2)}$

$2=A(1+x^2)+(Bx+C)(1-x)$

$2=A+Ax^2+Bx-Bx^2+C-Cx$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)}=\frac{1}{1-x}+\frac{x+1}{1+x^2}$$

$$\Rightarrow \int\frac{2}{(1-x)(1+x^2)}dx=\int\frac{1}{1-x}dx+\int\frac{x}{1+x^2}dx+\int\frac{1}{1+x^2}dx$$

$$=-\int\frac{1}{x-1}dx+\frac{1}{2}\int\frac{2x}{1+x^2}dx+\int\frac{1}{1+x^2}dx$$

$$=-\log|x-1|+\frac{1}{2}\log|1+x^2|+\tan^{-1}x+C$$

Ex 7.5 Class 12 Maths Question 14.

$\frac{3x-1}{(x+2)^2}$

Solution:

Let $\frac{3x-1}{(x+2)^2}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}$

$\Rightarrow 3x-1=A(x+2)+B$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2}=\frac{3}{(x+2)}-\frac{7}{(x+2)^2}$$

$$\Rightarrow \int\frac{3x-1}{(x+2)^2}dx=3\int\frac{1}{(x+2)}dx-7\int\frac{x}{(x+2)^2}dx$$

$$=3\log|x+2|-7\left(\frac{-1}{(x+2)}\right)+C$$

$$=3\log|x+2|+\frac{7}{(x+2)}+C$$

Ex 7.5 Class 12 Maths Question 15.

$\frac{1}{x^4-1}$

Solution:

$$\frac{1}{(x^4-1)}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(1+x^2)}$$

$$\text{Let } \frac{1}{(x+1)(x-1)(1+x^2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+1)}$$

$$1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)$$

$$1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$$

$$1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$$A+B+C=0$$
$$-A+B+D=0$$
$$A+B-C=0$$
$$-A+B-D=1$$

On solving these equations, we obtain

$$A=-\frac{1}{4}, B=\frac{1}{4}, C=0, \text{ and } D=-\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2(x^2+1)}$$

$$\Rightarrow \int\frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$$

Ex 7.5 Class 12 Maths Question 16.

$\frac{1}{x(x^n+1)}$

[Hint : multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Solution:

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)}=\frac{x^{n-1}}{x^{n-1}x(x^n+1)}=\frac{x^{n-1}}{x^n(x^n+1)}$$

$$\text{Let } x^n=t \Rightarrow x^{n-1}dx=dt$$

$$\therefore \int\frac{1}{x(x^n+1)}dx=\int\frac{x^{n-1}}{x^n(x^n+1)}dx=\frac{1}{n}\int\frac{1}{t(t+1)}dt$$

$$\text{Let } \frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{(t+1)}$$

$$1=A(1+t)+Bt \qquad \ldots(1)$$

Substituting t = 0, - 1 in equation (1), we obtain

A = 1 and B = - 1

$$\therefore \frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(1+t)}$$

$$\Rightarrow \int\frac{1}{x(x^n+1)}dx=\frac{1}{n}\int\left\{\frac{1}{t}-\frac{1}{(t+1)}\right\}dx$$

$$=\frac{1}{n}\left[\log|t|-\log|t+1|\right]+C$$

$$=-\frac{1}{n}\left[\log|x^n|-\log|x^n+1|\right]+C$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$$

Ex 7.5 Class 12 Maths Question 17.

$\frac{\cos x}{(1-\sin x)(2-\sin x)}$

Solution:

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t \Rightarrow \cos x\,dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$$1 = A(2-t) + B(1-t) \qquad ...(1)$$

Substituting t = 2 and then t = 1 in equation (1), we obtain

A = 1 and B = - 1

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int \left\{\frac{1}{1-t} - \frac{1}{(2-t)}\right\}dt$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

Ex 7.5 Class 12 Maths Question 18.

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Solution:

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

Let $\frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$

$$4x^2+10 = (Ax+B)(x^2+4) + (Cx+D)(x^2+3)$$

$$4x^2+10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$4x^2+10 = (A+C)x^3 + (B+D)x^2 + (4A+3C)x + (4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}dx$$

$$= \int \left\{1 + \frac{2}{x^2+(\sqrt{3})^2} - \frac{6}{x^2+2^2}\right\}$$

$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

Ex 7.5 Class 12 Maths Question 19.

$\frac{2x}{(x^2+1)(x^2+3)}$

Solution:

$$\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}$$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \int \frac{dt}{(t+1)(t+3)} \qquad ...(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$$

$$1 = A(t+3) + B(t+1) \qquad ...(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\begin{aligned}
\Rightarrow \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx &= \int \left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\}dt \\
&= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|t+3| + C \\
&= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C \\
&= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C
\end{aligned}$$

Ex 7.5 Class 12 Maths Question 20.

$\frac{1}{x(x^4-1)}$

Solution:

$$\frac{1}{x\left(x^4-1\right)}$$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x\left(x^4-1\right)}=\frac{x^3}{x^4\left(x^4-1\right)}$$

$$\therefore \int\frac{1}{x\left(x^4-1\right)}dx=\int\frac{x^3}{x^4\left(x^4-1\right)}dx$$

Let $x^4 = t \Rightarrow 4x^3dx = dt$

$$\therefore \int\frac{1}{x\left(x^4-1\right)}dx=\frac{1}{4}\int\frac{dt}{t\left(t-1\right)}$$

Let $\frac{1}{t\left(t-1\right)}=\frac{A}{t}+\frac{B}{\left(t-1\right)}$

$1=A\left(t-1\right)+Bt \qquad ...(1)$

Substituting $t = 0$ and 1 in (1), we obtain

$A = -1$ and $B = 1$

$$\Rightarrow \frac{1}{t\left(t+1\right)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int\frac{1}{x\left(x^4-1\right)}dx=\frac{1}{4}\int\left\{\frac{-1}{t}+\frac{1}{t-1}\right\}dt$$

$$=\frac{1}{4}\left[-\log|t|+\log|t-1|\right]+C$$

$$=\frac{1}{4}\log\left|\frac{t-1}{t}\right|+C$$

$$=\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+C$$

Ex 7.5 Class 12 Maths Question 21.

$\frac{1}{e^x-1}$

Solution:

$$\frac{1}{\left(e^x-1\right)}$$

Let $e^x = t \Rightarrow e^x\, dx = dt$

$$\Rightarrow \int\frac{1}{e^x-1}dx=\int\frac{1}{t-1}\times\frac{dt}{t}=\int\frac{1}{t\left(t-1\right)}dt$$

Let $\frac{1}{t\left(t-1\right)}=\frac{A}{t}+\frac{B}{t-1}$

$1=A\left(t-1\right)+Bt \qquad ...(1)$

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$A = -1$ and $B = 1$

$$\therefore \frac{1}{t\left(t-1\right)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int\frac{1}{t\left(t-1\right)}dt=\log\left|\frac{t-1}{t}\right|+C$$

$$=\log\left|\frac{e^x-1}{e^x}\right|+C$$

Ex 7.5 Class 12 Maths Question 22.

choose the correct answer in each of the following :

$\int\frac{x\,dx}{(x-1)(x-2)}$ equals

(a) $\log\left|\frac{(x-1)^2}{x-2}\right|+c$

(b) $\log\left|\frac{(x-2)^2}{x-1}\right|+c$

(c) $\log\left|\left(\frac{x-1^2}{x-2}\right)\right|+c$
(d) log|(x-1)(x-2)|+c
Solution:

$$\text{Let } \frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$$
$$x=A(x-2)+B(x-1) \qquad ...(1)$$

Substituting x = 1 and 2 in (1), we obtain

A = - 1 and B = 2

$$\therefore \frac{x}{(x-1)(x-2)}=-\frac{1}{(x-1)}+\frac{2}{(x-2)}$$
$$\Rightarrow \int\frac{x}{(x-1)(x-2)}dx=\int\left\{\frac{-1}{(x-1)}+\frac{2}{(x-2)}\right\}dx$$
$$=-\log|x-1|+2\log|x-2|+C$$
$$=\log\left|\frac{(x-2)^2}{x-1}\right|+C$$

Hence, the correct answer is B.

Ex 7.5 Class 12 Maths Question 23.

$\int\frac{dx}{x(x^2+1)}$ equals
(a) $\log|x|-\frac{1}{2}\log(x^2+1)+c$
(b) $\log|x|+\frac{1}{2}\log(x^2+1)+c$
(c) $-\log|x|+\frac{1}{2}\log(x^2+1)+c$
(d) $\frac{1}{2}\log|x|+\log(x^2+1)+c$
Solution:

$$\text{Let } \frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$$
$$1=A(x^2+1)+(Bx+C)x$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

A = 1, B = - 1, and C = 0

$$\therefore \frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-x}{x^2+1}$$
$$\Rightarrow \int\frac{1}{x(x^2+1)}dx=\int\left\{\frac{1}{x}-\frac{x}{x^2+1}\right\}dx$$
$$=\log|x|-\frac{1}{2}\log|x^2+1|+C$$

Hence, the correct answer is A.

Integrals Class 12 Ex 7.6

Ex 7.6 Class 12 Maths Question 1.
x sinx
Solution:

Let $I = \int x\sin x\,dx$

Taking x as first function and sin x as second function and integrating by parts, we obtain

$$I=x\int\sin x\,dx-\int\left\{\left(\frac{d}{dx}x\right)\int\sin x\,dx\right\}dx$$
$$=x(-\cos x)-\int 1\cdot(-\cos x)dx$$
$$=-x\cos x+\sin x+C$$

Ex 7.6 Class 12 Maths Question 2.

x sin3x
Solution:

Let $I = \int x \sin 3x \, dx$

Taking x as first function and sin $3x$ as second function and integrating by parts, we obtain

$$I = x\int \sin 3x \; dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x \; dx\right\}$$
$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right) dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \; dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

Ex 7.6 Class 12 Maths Question 3.
$x^2 e^x$
Solution:

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2\int e^x \; dx - \int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\} dx$$
$$= x^2 e^x - \int 2x \cdot e^x dx$$
$$= x^2 e^x - 2\int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2\left[x\cdot\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^x dx\right\} dx\right]$$
$$= x^2 e^x - 2\left[xe^x - \int e^x dx\right]$$
$$= x^2 e^x - 2\left[xe^x - e^x\right]$$
$$= x^2 e^x - 2xe^x + 2e^x + C$$
$$= e^x\left(x^2 - 2x + 2\right) + C$$

Ex 7.6 Class 12 Maths Question 4.
x logx
Solution:

Let $I = \int x \log x dx$

Taking log x as first function and x as second function and integrating by parts, we obtain

$$I = \log x\int x \; dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x \; dx\right\} dx$$
$$= \log x\cdot\frac{x^2}{2} - \int\frac{1}{x}\cdot\frac{x^2}{2} dx$$
$$= \frac{x^2\log x}{2} - \int\frac{x}{2} dx$$
$$= \frac{x^2\log x}{2} - \frac{x^2}{4} + C$$

Ex 7.6 Class 12 Maths Question 5.
x log2x
Solution:

Let $I = \int x \log 2x dx$

Taking log $2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x\int x \; dx - \int\left\{\left(\frac{d}{dx}2\log x\right)\int x \; dx\right\} dx$$
$$= \log 2x\cdot\frac{x^2}{2} - \int\frac{2}{2x}\cdot\frac{x^2}{2} dx$$
$$= \frac{x^2\log 2x}{2} - \int\frac{x}{2} dx$$
$$= \frac{x^2\log 2x}{2} - \frac{x^2}{4} + C$$

Ex 7.6 Class 12 Maths Question 6.

$x^2 \log x$

Solution:

Let $I = \int x^2 \log x \, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2 \, dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x^2 dx\right\}dx$$

$$= \log x\left(\frac{x^3}{3}\right) - \int \frac{1}{x}\cdot\frac{x^3}{3}dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3}dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Ex 7.6 Class 12 Maths Question 7.

$x \quad \sin^{-1} x$

Solution:

Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x \, dx - \int\left\{\left(\frac{d}{dx}\sin^{-1} x\right)\int x \, dx\right\}dx$$

$$= \sin^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\int\left\{\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\left\{\int\sqrt{1-x^2}dx - \int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x - \sin^{-1} x\right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1} x - \frac{1}{2}\sin^{-1} x + C$$

$$= \frac{1}{4}\left(2x^2-1\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + C$$

Ex 7.6 Class 12 Maths Question 8.

$x \quad \tan^{-1} x$

Solution:

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x \, dx - \int\left\{\left(\frac{d}{dx}\tan^{-1} x\right)\int x \, dx\right\}dx$$

$$= \tan^{-1} x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + C$$

Ex 7.6 Class 12 Maths Question 9.

$x \quad \cos^{-1} x$

Solution:

Let $I=\int x\cos^{-1}x\,dx$

Taking $\cos^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$
$$=\cos^{-1}x\frac{x^2}{2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$
$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$
$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}+\left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$
$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\sqrt{1-x^2}\,dx-\frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$
$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}x \qquad \ldots(1)$$

where, $I_1=\int\sqrt{1-x^2}\,dx$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$
$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$
$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-x^2}{\sqrt{1-x^2}}dx$$
$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$
$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{\int\sqrt{1-x^2}\,dx+\int\frac{-dx}{\sqrt{1-x^2}}\right\}$$
$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{I_1+\cos^{-1}x\right\}$$
$$\Rightarrow 2I_1=x\sqrt{1-x^2}-\cos^{-1}x$$
$$\therefore I_1=\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I=\frac{x\cos^{-1}x}{2}-\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x\right)-\frac{1}{2}\cos^{-1}x$$
$$=\frac{\left(2x^2-1\right)}{4}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$$

Ex 7.6 Class 12 Maths Question 10.

$(\sin^{-1}x)^2$

Solution:

Let $I=\int\left(\sin^{-1}x\right)^2\cdot 1\,dx$

Taking $\left(\sin^{-1}x\right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I=\left(\sin^{-1}x\right)\int 1\,dx-\int\left\{\frac{d}{dx}\left(\sin^{-1}x\right)^2\cdot\int 1\cdot dx\right\}dx$$
$$=\left(\sin^{-1}x\right)^2\cdot x-\int\frac{2\sin^{-1}x}{\sqrt{1-x^2}}\cdot x\,dx$$
$$=x\left(\sin^{-1}x\right)^2+\int\sin^{-1}x\cdot\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$
$$=x\left(\sin^{-1}x\right)^2+\left[\sin^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$=x\left(\sin^{-1}x\right)^2+\left[\sin^{-1}x\cdot 2\sqrt{1-x^2}-\int\frac{1}{\sqrt{1-x^2}}\cdot 2\sqrt{1-x^2}\,dx\right]$$
$$=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-\int 2\,dx$$
$$=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-2x+C$$

Ex 7.6 Class 12 Maths Question 11.

$\frac{x \quad \cos^{-1}x}{\sqrt{1-x^2}}$

Solution:

Let $I=\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

$$I=\frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\cdot \cos^{-1}x dx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I=\frac{-1}{2}\left[\cos^{-1}x\int \frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$=\frac{-1}{2}\left[\cos^{-1}x\cdot 2\sqrt{1-x^2}-\int \frac{-1}{\sqrt{1-x^2}}\cdot 2\sqrt{1-x^2}\,dx\right]$$

$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+\int 2\,dx\right]$$

$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+2x\right]+C$$

$$=-\left[\sqrt{1-x^2}\cos^{-1}x+x\right]+C$$

Ex 7.6 Class 12 Maths Question 12.

x $\sec^2$x

Solution:

Let $I=\int x\sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I=x\int \sec^2 x\,dx-\int\left\{\left\{\frac{d}{dx}x\right\}\int \sec^2 x\,dx\right\}dx$$

$$=x\tan x-\int 1\cdot \tan x dx$$

$$=x\tan x+\log|\cos x|+C$$

Ex 7.6 Class 12 Maths Question 13.

$\tan^{-1}$x

Solution:

Let $I=\int 1\cdot \tan^{-1}x dx$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we obtain

$$I=\tan^{-1}x\int 1dx-\int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$$

$$=\tan^{-1}x\cdot x-\int \frac{1}{1+x^2}\cdot x\,dx$$

$$=x\tan^{-1}x-\frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$=x\tan^{-1}x-\frac{1}{2}\log\left|1+x^2\right|+C$$

$$=x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+C$$

Ex 7.6 Class 12 Maths Question 14.

x(logx)^2

Solution:

$I=\int x(\log x)^2 dx$

Taking $(\log x)^2$ as first function and x as second function and integrating by parts, we obtain

Ex 7.6 Class 12 Maths Question 15.

(x^2+1)logx

Solution:

Let $I = \int(x^2+1)\log x\,dx = \int x^2\log x\,dx + \int \log x\,dx$

Let $I = I_1 + I_2$ … (1)

Where, $I_1 = \int x^2 \log x\,dx$ and $I_2 = \int \log x\,dx$

$I_1 = \int x^2 \log x dx$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x - \int x^2 dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x^2 dx\right\}dx$$
$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{x^3}{3}dx$$
$$= \frac{x^3}{3}\log x - \frac{1}{3}\left(\int x^2 dx\right)$$
$$= \frac{x^3}{3}\log x - \frac{x^3}{9} + C_1 \qquad \ldots (2)$$

$I_2 = \int \log x\,dx$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1\cdot dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int 1\cdot dx\right\}$$
$$= \log x \cdot x - \int \frac{1}{x}\cdot x dx$$
$$= x\log x - \int 1 dx$$
$$= x\log x - x + C_2 \qquad \ldots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3}\log x - \frac{x^3}{9} + C_1 + x\log x - x + C_2$$
$$= \frac{x^3}{3}\log x - \frac{x^3}{9} + x\log x - x + (C_1 + C_2)$$
$$= \left(\frac{x^3}{3} + x\right)\log x - \frac{x^3}{9} - x + C$$

Ex 7.6 Class 12 Maths Question 16.

$e^x(\sin x + \cos x)$

Solution:

Let $I = \int e^x(\sin x + \cos x)dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x\{f(x) + f'(x)\}dx$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

Ex 7.6 Class 12 Maths Question 17.

$\frac{xe^x}{(1+x)^2}$

Solution:

Let $I = \int \frac{xe^x}{(1+x)^2}dx = \int e^x \left\{\frac{x}{(1+x)^2}\right\}dx$

$= \int e^x \left\{\frac{1+x-1}{(1+x)^2}\right\}dx$

$= \int e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\}dx$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2}dx = \int e^x \{f(x) + f'(x)\}dx$

It is known that, $\int e^x \{f(x) + f'(x)\}dx = e^x f(x) + C$

$\therefore \int \frac{xe^x}{(1+x)^2}dx = \frac{e^x}{1+x} + C$

Ex 7.6 Class 12 Maths Question 18.

$\frac{e^x(1+\sin x)}{1+\cos x}$

Solution:

$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

$= e^x\left(\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$= \frac{e^x\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$

$= \frac{1}{2}e^x \cdot \left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$

$= \frac{1}{2}e^x\left[\tan\frac{x}{2} + 1\right]^2$

$= \frac{1}{2}e^2\left(1 + \tan\frac{x}{2}\right)^2$

$= \frac{1}{2}e^x\left[1 + \tan^2\frac{x}{2} + 2\tan\frac{x}{2}\right]$

$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2} + 2\tan\frac{x}{2}\right]$

$\frac{e^x(1+\sin x)dx}{(1+\cos x)} = e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] \quad \ldots(1)$

Let $\tan\frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x \{f(x) + f'(x)\}dx = e^x f(x) + C$

From equation (1), we obtain

$\int \frac{e^x(1+\sin x)}{(1+\cos x)}dx = e^x \tan\frac{x}{2} + C$

Ex 7.6 Class 12 Maths Question 19.

$e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)$

Solution:

Let $I = \int e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx$

Also, let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{x} + C$

Ex 7.6 Class 12 Maths Question 20.

$\frac{(x-2)e^x}{(x-1)^3}$

Solution:

$$\int e^x \left\{\frac{x-3}{(x-1)^3}\right\} dx = \int e^x \left\{\frac{x-1-2}{(x-1)^3}\right\} dx$$

$$= \int e^x \left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\} dx$$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore \int e^x \left\{\frac{(x-3)}{(x-1)^2}\right\} dx = \frac{e^x}{(x-1)^2} + C$$

Ex 7.6 Class 12 Maths Question 21.

$e^{2x} \sin x$

Solution:

Let $I = \int e^{2x} \sin x \, dx \quad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{\left(\frac{d}{dx} \sin x\right) \int e^{2x} dx\right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\cos x \int e^{2x} dx - \int \left\{\left(\frac{d}{dx} \cos x\right) \int e^{2x} dx\right\} dx\right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx\right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx\right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad \text{[From (1)]}$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}\right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5}[2 \sin x - \cos x] + C$$

Ex 7.6 Class 12 Maths Question 22.

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution:

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta \cdot \sec^2\theta\, d\theta = 2\int \theta \cdot \sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta \cdot \int \sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int \sec^2\theta d\theta\right\}d\theta\right]$$

$$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log(1+x^2)^{-\frac{1}{2}} + C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C$$

$$= 2x\tan^{-1}x - \log(1+x^2) + C$$

Ex 7.6 Class 12 Maths Question 23.

$\int x^2 e^{x^3} dx$ equals

(a) $\frac{1}{3}e^{x^3} + c$

(b) $\frac{1}{3} + e^{x^2} + c$

(c) $\frac{1}{2}e^{x^3} + c$

(d) $\frac{1}{2}e^{x^2} + c$

Solution:

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3}\int e^t dt$$

$$= \frac{1}{3}(e^t) + C$$

$$= \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is A.

Ex 7.6 Class 12 Maths Question 24.

$\int e^x \sec x(1 + \tan x)dx$ equals

(a) $e^x \cos x + c$

(b) $e^x \sec x + c$

(c) $e^x \sin x + c$

(d) $e^x \tan x + c$

Solution:

$\int e^x \sec x(1+\tan x)dx$

Let $I = \int e^x \sec x(1+\tan x)dx = \int e^x(\sec x + \sec x\tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x\tan x = f'(x)$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct answer is B.

Class 12 Integrals Ex 7.7

Ex 7.7 Class 12 Maths Question 1.

$\sqrt{4 - x^2}$

Solution:

Let $I = \int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2-(x)^2}\,dx$

It is known that, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\therefore I = \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$$
$$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$$

Ex 7.7 Class 12 Maths Question 2.

$\sqrt{1-4x^2}$

Solution:

Let $I = \int \sqrt{1-4x^2}\,dx = \int \sqrt{(1)^2-(2x)^2}\,dx$

Let $2x = t \Rightarrow 2\,dx = dt$

$$\therefore I = \frac{1}{2}\int \sqrt{(1)^2-(t)^2}\,dt$$

It is known that, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right] + C$$
$$= \frac{t}{4}\sqrt{1-t^2} + \frac{1}{4}\sin^{-1}t + C$$
$$= \frac{2x}{4}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$$
$$= \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$$

Ex 7.7 Class 12 Maths Question 3.

$\sqrt{x^2+4x+6}$

Solution:

$$\text{Let } I = \int \sqrt{x^2+4x+6}\,dx$$
$$= \int \sqrt{x^2+4x+4+2}\,dx$$
$$= \int \sqrt{(x^2+4x+4)+2}\,dx$$
$$= \int \sqrt{(x+2)^2+(\sqrt{2})^2}\,dx$$

It is known that, $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right| + C$$
$$= \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log\left|(x+2)+\sqrt{x^2+4x+6}\right| + C$$

Ex 7.7 Class 12 Maths Question 4.

$\sqrt{x^2+4x+1}$

Solution:

$$\text{Let } I = \int \sqrt{x^2+4x+1}\,dx$$
$$= \int \sqrt{(x^2+4x+4)-3}\,dx$$
$$= \int \sqrt{(x+2)^2-(\sqrt{3})^2}\,dx$$

It is known that, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$$

Ex 7.7 Class 12 Maths Question 5.

$\sqrt{1-4x-x^2}$

Solution:

$$\text{Let } I = \int\sqrt{1-4x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$= \int\sqrt{1+4-(x+2)^2}\,dx$$
$$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

Ex 7.7 Class 12 Maths Question 6.

$\sqrt{x^2+4x-5}$

Solution:

$$\text{Let } I = \int\sqrt{x^2+4x-5}\,dx$$
$$= \int\sqrt{\left(x^2+4x+4\right)-9}\,dx$$
$$= \int\sqrt{(x+2)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

Ex 7.7 Class 12 Maths Question 7.

$\sqrt{1+3x-x^2}$

Solution:

$$\text{Let } I = \int\sqrt{1+3x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$
$$= \int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$$
$$= \int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$
$$= \frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

Ex 7.7 Class 12 Maths Question 8.

$\sqrt{x^2+3x}$

Solution:

$$\text{Let } I = \int\sqrt{x^2+3x}\,dx$$
$$= \int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$
$$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$
$$= \frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

Ex 7.7 Class 12 Maths Question 9.

$\sqrt{1+\frac{x^2}{9}}$

Solution:

$$\text{Let } I = \int\sqrt{1+\frac{x^2}{9}}\,dx = \frac{1}{3}\int\sqrt{9+x^2}\,dx = \frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$$

It is known that, $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$
$$= \frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

Ex 7.7 Class 12 Maths Question 10.

$\int\sqrt{1+x^2}dx$ is equal to

(a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

(b) $\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}}+c$

(c) $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+c$

(d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+c$

Solution:

It is known that, $\int\sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore \int\sqrt{1+x^2}\,dx = \frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

Hence, the correct answer is A.

Ex 7.7 Class 12 Maths Question 11.

$\int\sqrt{x^2-8x+7}dx$ is equal to

(a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+\log|x-4+\sqrt{x^2-8x+7}|+C$

(b) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log|x+4+\sqrt{x^2-8x+7}|+C$

(c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

(d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

Solution:

Let $I = \int \sqrt{x^2 - 8x + 7}\, dx$

$$= \int \sqrt{\left(x^2 - 8x + 16\right) - 9}\, dx$$

$$= \int \sqrt{(x-4)^2 - (3)^2}\, dx$$

It is known that, $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$\therefore I = \frac{(x-4)}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|(x-4) + \sqrt{x^2 - 8x + 7}\right| + C$$

Hence, the correct answer is D.

Class 12 Integrals Ex 7.8

Ex 7.8 Class 12 Maths Question 1.

$\int_a^b x \quad dx$

Solution:

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + \ldots + f\left(a + (n-1)h\right)\right], \text{ where } h = \frac{b-a}{n}$$

Here, $a = a$, $b = b$, and $f(x) = x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a + (a+h)\ldots(a+2h)\ldots a + (n-1)h\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a + a + a + \ldots + a}_{n \text{ times}}\right) + \left(h + 2h + 3h + \ldots + (n-1)h\right)\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h\left(1 + 2 + 3 + \ldots + (n-1)\right)\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + \frac{n(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{n}{n}\left[a + \frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)(b-a)}{2n}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{\left(1 - \frac{1}{n}\right)(b-a)}{2}\right]$$

$$= (b-a)\left[a + \frac{(b-a)}{2}\right]$$

$$= (b-a)\left[\frac{2a + b - a}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2}$$

$$= \frac{1}{2}\left(b^2 - a^2\right)$$

Ex 7.8 Class 12 Maths Question 2.

$\int_0^5 (x+1)dx$

Solution:

Let $I=\int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=5,$ and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\begin{aligned}
\therefore \int_0^5 (x+1)\,dx &= (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]\\
&=5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]\\
&=5\left[1+\frac{5}{2}\right]\\
&=5\left[\frac{7}{2}\right]\\
&=\frac{35}{2}
\end{aligned}$$

Ex 7.8 Class 12 Maths Question 3.

$\int_2^3 x^2\,dx$

Solution:

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=2,\ b=3,$ and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot 2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot 2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot 2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Ex 7.8 Class 12 Maths Question 4.

$\int_1^4 (x^2-x)dx$

Solution:

$$\text{Let } I=\int_1^4\left(x^2-x\right)dx$$

$$=\int_1^4 x^2\,dx-\int_1^4 x\,dx$$

Let $I=I_1-I_2$, where $I_1=\int_1^4 x^2dx$ and $I_2=\int_1^4 x\,dx$ ...(1)

It is known that,

$$\int_a^b f(x)dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

For $I_1=\int_1^4 x^2dx$,

$a=1, b=4,$ and $f(x)=x^2$

$$\therefore h=\frac{4-1}{n}=\frac{3}{n}$$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1)+f(1+h)+\ldots+f(1+(n-1)h)\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2+\left(1+\frac{3}{n}\right)^2+\left(1+2\cdot\frac{3}{n}\right)^2+\ldots\left(1+\frac{(n-1)3}{n}\right)^2\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2+\left\{1^2+\left(\frac{3}{n}\right)^2+2\cdot\frac{3}{n}\right\}+\ldots+\left\{1^2+\left(\frac{(n-1)3}{n}\right)^2+\frac{2\cdot(n-1)\cdot 3}{n}\right\}\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1^2+\ldots+1^2}_{n\text{ times}}\right)+\left(\frac{3}{n}\right)^2\left\{1^2+2^2+\ldots+(n-1)^2\right\}+2\cdot\frac{3}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\}+\frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{6n-6}{2}\right]$$
$$= 3\lim_{n\to\infty}\left[1+\frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+3-\frac{3}{n}\right]$$
$$= 3[1+3+3]$$
$$= 3[7]$$
$$I_1 = 21 \qquad \ldots(2)$$

For $I_2 = \int_1^4 x dx$,

$a=1, b=4,$ and $f(x)=x$

$$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$$

$$\therefore I_2 = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1)+f(1+h)+\ldots f(a+(n-1)h)\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+(1+h)+\ldots+(1+(n-1)h)\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\ldots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+\ldots+1}_{n\text{ times}}\right)+\frac{3}{n}(1+2+\ldots+(n-1))\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$
$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$
$$= 3\left[1+\frac{3}{2}\right]$$
$$= 3\left[\frac{5}{2}\right]$$
$$I_2 = \frac{15}{2} \qquad \ldots(3)$$

From equations (2) and (3), we obtain

$$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$$

Ex 7.8 Class 12 Maths Question 5.

$\int_{-1}^{1} e^x dx$

Solution:

Let $I = \int_{-1}^{1} e^x dx \qquad \ldots(1)$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=-1$, $b=1$, and $f(x)=e^x$

$$\therefore h = \frac{1+1}{n} = \frac{2}{n}$$

$$\therefore I=(1+1)\lim_{n\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2\cdot\frac{2}{n}\right)+...+f\left(-1+\frac{(n-1)2}{n}\right)\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}+e^{\left(-1+\frac{2}{n}\right)}+e^{\left(-1+2\cdot\frac{2}{n}\right)}+...e^{\left(-1+(n-1)\frac{2}{n}\right)}\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$=2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}}-1}{e^{\frac{2}{n}}-1}\right]$$

$$=e^{-1}\times2\lim_{n\to\infty}\frac{1}{n}\left[\frac{e^{2}-1}{e^{\frac{2}{n}}-1}\right]$$

$$=\frac{e^{-1}\times2\left(e^{2}-1\right)}{\lim\limits_{\frac{2}{n}\to0}\left(\frac{e^{\frac{2}{n}}-1}{\frac{2}{n}}\right)\times2}$$

$$=e^{-1}\left[\frac{2\left(e^{2}-1\right)}{2}\right] \qquad \left[\lim_{h\to0}\left(\frac{e^{h}-1}{h}\right)=1\right]$$

$$=\frac{e^{2}-1}{e}$$

$$=\left(e-\frac{1}{e}\right)$$

Ex 7.8 Class 12 Maths Question 6.

$\int_0^4 (x+e^{2x})dx$

Solution:

It is known that,

$$\int_a^b f(x)dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+...+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow\int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^{0}\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot2h}\right)+...+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+...+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+...+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+...+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+...(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^{8}-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^{8}-1\right)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^{8}-1\right)}{8} \qquad \left(\lim_{x\to0}\frac{e^{x}-1}{x}=1\right)$$

$$=8+\frac{e^{8}-1}{2}$$

$$=\frac{15+e^{8}}{2}$$

Class 12 Integrals Ex 7.9

Ex 7.9 Class 12 Maths Question 1.

$\int_{-1}^{1} (x+1)dx$
Solution:

Let $I = \int_{-1}^{1}(x+1)dx$

$\int(x+1)\,dx = \frac{x^2}{2} + x = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(-1)$$
$$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)$$
$$= \frac{1}{2}+1-\frac{1}{2}+1$$
$$= 2$$

Ex 7.9 Class 12 Maths Question 2.

$\int_{2}^{3} \frac{1}{x}dx$
Solution:

Let $I = \int_{2}^{3}\frac{1}{x}\,dx$

$\int\frac{1}{x}\,dx = \log|x| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$
$$= \log|3| - \log|2| = \log\frac{3}{2}$$

Ex 7.9 Class 12 Maths Question 3.

$\int_{1}^{2} \left(4x^3 - 5x^2 + 6x + 9\right) dx$
Solution:

Let $I = \int_{1}^{2}\left(4x^3 - 5x^2 + 6x + 9\right)dx$

$$\int\left(4x^3 - 5x^2 + 6x + 9\right)dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$
$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$
$$I = \left\{2^4 - \frac{5\cdot(2)^3}{3} + 3(2)^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$
$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$
$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$
$$= 33 - \frac{35}{3}$$
$$= \frac{99-35}{3}$$
$$= \frac{64}{3}$$

Ex 7.9 Class 12 Maths Question 4.

$\int_{0}^{\frac{\pi}{4}} \sin 2x \quad dx$
Solution:

Let $I=\int_0^{\frac{\pi}{4}}\sin 2x\,dx$

$\int \sin 2x\,dx=\left(\frac{-\cos 2x}{2}\right)=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$
\begin{aligned}
I&=\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}(0)\\
&=-\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos 0\right]\\
&=-\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right)-\cos 0\right]\\
&=-\frac{1}{2}[0-1]\\
&=\frac{1}{2}
\end{aligned}
$$

Ex 7.9 Class 12 Maths Question 5.

$\int_0^{\frac{\pi}{2}}\cos 2x\quad dx$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}\cos 2x\,dx$

$\int \cos 2x\,dx=\left(\frac{\sin 2x}{2}\right)=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$
\begin{aligned}
I&=\mathrm{F}\left(\frac{\pi}{2}\right)-\mathrm{F}(0)\\
&=\frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right)-\sin 0\right]\\
&=\frac{1}{2}[\sin\pi-\sin 0]\\
&=\frac{1}{2}[0-0]=0
\end{aligned}
$$

Ex 7.9 Class 12 Maths Question 6.

$\int_4^5 e^x\,dx$

Solution:

Let $I=\int_4^5 e^x\,dx$

$\int e^x\,dx=e^x=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$
\begin{aligned}
I&=\mathrm{F}(5)-\mathrm{F}(4)\\
&=e^5-e^4\\
&=e^4(e-1)
\end{aligned}
$$

Ex 7.9 Class 12 Maths Question 7.

$\int_0^{\frac{\pi}{4}}\tan x\quad dx$

Solution:

Let $I=\int_0^{\frac{\pi}{4}}\tan x\,dx$

$\int \tan x\,dx=-\log|\cos x|=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}(0)$$
$$=-\log\left|\cos\frac{\pi}{4}\right|+\log|\cos 0|$$
$$=-\log\left|\frac{1}{\sqrt{2}}\right|+\log|1|$$
$$=-\log(2)^{-\frac{1}{2}}$$
$$=\frac{1}{2}\log 2$$

Ex 7.9 Class 12 Maths Question 8.

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} \quad xdx$

Solution:

Let $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\operatorname{cosec} x\,dx$

$\int \operatorname{cosec} x\ dx=\log|\operatorname{cosec} x-\cot x|=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(\frac{\pi}{6}\right)$$
$$=\log\left|\operatorname{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right|-\log\left|\operatorname{cosec}\frac{\pi}{6}-\cot\frac{\pi}{6}\right|$$
$$=\log\left|\sqrt{2}-1\right|-\log\left|2-\sqrt{3}\right|$$
$$=\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$$

Ex 7.9 Class 12 Maths Question 9.

$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Solution:

Let $I=\int_0^1\frac{dx}{\sqrt{1-x^2}}$

$\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}(1)-\mathrm{F}(0)$$
$$=\sin^{-1}(1)-\sin^{-1}(0)$$
$$=\frac{\pi}{2}-0$$
$$=\frac{\pi}{2}$$

Ex 7.9 Class 12 Maths Question 10.

$\int_0^1 \frac{dx}{1+x^2}$

Solution:

Let $I=\int_0^1\frac{dx}{1+x^2}$

$\int\frac{dx}{1+x^2}=\tan^{-1}x=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}(1)-\mathrm{F}(0)$$
$$=\tan^{-1}(1)-\tan^{-1}(0)$$
$$=\frac{\pi}{4}$$

Ex 7.9 Class 12 Maths Question 11.

$\int_2^3 \frac{dx}{x^2-1}$

Solution:

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$
$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$
$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$
$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$
$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

Ex 7.9 Class 12 Maths Question 12.

$\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Solution:

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

$\int \cos^2 x\, dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = \left[F\left(\frac{\pi}{2}\right) - F(0)\right]$$
$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$
$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$
$$= \frac{\pi}{4}$$

Ex 7.9 Class 12 Maths Question 13.

$\int_2^3 \frac{x}{x^2+1} dx$

Solution:

Let $I = \int_2^3 \frac{x}{x^2+1} dx$

$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \frac{1}{2}\log\left(1+x^2\right) = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$
$$= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right]$$
$$= \frac{1}{2}\left[\log(10) - \log(5)\right]$$
$$= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$$

Ex 7.9 Class 12 Maths Question 14.

$\int_0^1 \frac{2x+3}{5x^2+1} dx$

Solution:

$$\text{Let } I=\int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\begin{aligned}\int\frac{2x+3}{5x^2+1}dx &= \frac{1}{5}\int\frac{5(2x+3)}{5x^2+1}dx\\ &= \frac{1}{5}\int\frac{10x+15}{5x^2+1}dx\\ &= \frac{1}{5}\int\frac{10x}{5x^2+1}dx+3\int\frac{1}{5x^2+1}dx\\ &= \frac{1}{5}\int\frac{10x}{5x^2+1}dx+3\int\frac{1}{5\left(x^2+\frac{1}{5}\right)}dx\\ &= \frac{1}{5}\log\left(5x^2+1\right)+\frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}\\ &= \frac{1}{5}\log\left(5x^2+1\right)+\frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)\\ &= \mathrm{F}(x)\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I &= \mathrm{F}(1)-\mathrm{F}(0)\\ &= \left\{\frac{1}{5}\log(5+1)+\frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\}-\left\{\frac{1}{5}\log(1)+\frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}\\ &= \frac{1}{5}\log 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}\end{aligned}$$

Ex 7.9 Class 12 Maths Question 15.

$\int_0^1 xe^{x^2}dx$

Solution:

$$\text{Let } I=\int_0^1 xe^{x^2}dx$$

Put $x^2=t \Rightarrow 2x\ dx=dt$

As $x\to 0, t\to 0$ and as $x\to 1, t\to 1$,

$$\therefore I=\frac{1}{2}\int_0^1 e^t dt$$

$$\frac{1}{2}\int e^t dt=\frac{1}{2}e^t=\mathrm{F}(t)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I &= \mathrm{F}(1)-\mathrm{F}(0)\\ &= \frac{1}{2}e-\frac{1}{2}e^0\\ &= \frac{1}{2}(e-1)\end{aligned}$$

Ex 7.9 Class 12 Maths Question 16.

$\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Solution:

Let $I=\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Dividing $5x^2$ by x^2+4x+3, we obtain

$$I=\int_1^2\left\{5-\frac{20x+15}{x^2+4x+3}\right\}dx$$
$$=\int_1^2 5dx-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$
$$=[5x]_1^2-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$
$$I=5-I_1,\text{ where } I=\int_1^2\frac{20x+15}{x^2+4x+3}dx \qquad ...(1)$$

$$\text{Consider } I_1=\int_1^2\frac{20x+15}{x^2+4x+8}dx$$
$$\text{Let } 20x+15=\mathrm{A}\frac{d}{dx}\left(x^2+4x+3\right)+\mathrm{B}$$
$$=2\mathrm{A}x+\left(4\mathrm{A}+\mathrm{B}\right)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = - 25

$$\Rightarrow I_1=10\int_1^2\frac{2x+4}{x^2+4x+3}dx-25\int_1^2\frac{dx}{x^2+4x+3}$$
$$\text{Let } x^2+4x+3=t$$
$$\Rightarrow (2x+4)dx=dt$$

$$\Rightarrow I_1=10\int\frac{dt}{t}-25\int\frac{dx}{(x+2)^2-1^2}$$
$$=10\log t-25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$
$$=\left[10\log\left(x^2+4x+3\right)\right]_1^2-25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$
$$=[10\log 15-10\log 8]-25\left[\frac{1}{2}\log\frac{3}{5}-\frac{1}{2}\log\frac{2}{4}\right]$$
$$=\left[10\log(5\times3)-10\log(4\times2)\right]-\frac{25}{2}[\log 3-\log 5-\log 2+\log 4]$$

$$=[10\log 5+10\log 3-10\log 4-10\log 2]-\frac{25}{2}[\log 3-\log 5-\log 2+\log 4]$$
$$=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]\log 3+\left[-10+\frac{25}{2}\right]\log 2$$
$$=\frac{45}{2}\log 5-\frac{45}{2}\log 4-\frac{5}{2}\log 3+\frac{5}{2}\log 2$$
$$=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in (1), we obtain

$$I=5-\left[\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}\right]$$
$$=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]$$

Ex 7.9 Class 12 Maths Question 17.

$\int_0^{\frac{\pi}{4}}\left(2\sec^2 x+x^3+2\right)dx$

Solution:

Let $I=\int_0^{\frac{\pi}{4}}\left(2\sec^2 x+x^3+2\right)dx$

$\int\left(2\sec^2 x+x^3+2\right)dx=2\tan x+\frac{x^4}{4}+2x=\mathrm{F}(x)$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}(0)$$
$$=\left\{\left(2\tan\frac{\pi}{4}+\frac{1}{4}\left(\frac{\pi}{4}\right)^4+2\left(\frac{\pi}{4}\right)\right)-\left(2\tan 0+0+0\right)\right\}$$
$$=2\tan\frac{\pi}{4}+\frac{\pi^4}{4^5}+\frac{\pi}{2}$$
$$=2+\frac{\pi}{2}+\frac{\pi^4}{1024}$$

Ex 7.9 Class 12 Maths Question 18.

$\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

Solution:

$$\text{Let } I=\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$$
$$=-\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)dx$$
$$=-\int_0^{\pi}\cos x\,dx$$
$$\int\cos x\,dx=\sin x=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}(\pi)-\mathrm{F}(0)$$
$$=\sin\pi-\sin 0$$
$$=0$$

Ex 7.9 Class 12 Maths Question 19.

$\int_0^{2}\frac{6x+3}{x^2+4}dx$

Solution:

$$\text{Let } I=\int_0^{2}\frac{6x+3}{x^2+4}dx$$
$$\int\frac{6x+3}{x^2+4}dx=3\int\frac{2x+1}{x^2+4}dx$$
$$=3\int\frac{2x}{x^2+4}dx+3\int\frac{1}{x^2+4}dx$$
$$=3\log\left(x^2+4\right)+\frac{3}{2}\tan^{-1}\frac{x}{2}=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}(2)-\mathrm{F}(0)$$
$$=\left\{3\log\left(2^2+4\right)+\frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\}-\left\{3\log(0+4)+\frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$
$$=3\log 8+\frac{3}{2}\tan^{-1}1-3\log 4-\frac{3}{2}\tan^{-1}0$$
$$=3\log 8+\frac{3}{2}\left(\frac{\pi}{4}\right)-3\log 4-0$$
$$=3\log\left(\frac{8}{4}\right)+\frac{3\pi}{8}$$
$$=3\log 2+\frac{3\pi}{8}$$

Ex 7.9 Class 12 Maths Question 20.

$\int_0^{1}\left(xe^x+\sin\frac{\pi x}{4}\right)dx$

Solution:

$$\text{Let } I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$$

$$\int \left(xe^x + \sin\frac{\pi x}{4} \right) dx = x\int e^x dx - \int \left\{ \left(\frac{d}{dx}x \right) \int e^x dx \right\} dx + \left\{ \frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}} \right\}$$

$$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4} \right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0 \right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}} \right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

Ex 7.9 Class 12 Maths Question 21.

$\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{12}$

Solution:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \mathrm{F}(\sqrt{3}) - \mathrm{F}(1)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence, the correct answer is D.

Ex 7.9 Class 12 Maths Question 22.

$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$

(d) $\frac{\pi}{4}$

Solution:

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2+t^2}$$
$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$
$$= \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \mathrm{F}\left(\frac{2}{3}\right) - \mathrm{F}(0)$$
$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1}0$$
$$= \frac{1}{6}\tan^{-1}1 - 0$$
$$= \frac{1}{6}\times\frac{\pi}{4}$$
$$= \frac{\pi}{24}$$

Hence, the correct answer is C.

Integration Class 12 Ex 7.10

Ex 7.10 Class 12 Maths Question 1:

$$\int_0^1 \frac{x}{x^2+1}dx$$

Solution:

$$\int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1 = t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$
$$= \frac{1}{2}\left[\log|t|\right]_1^2$$
$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$
$$= \frac{1}{2}\log 2$$

Ex 7.10 Class 12 Maths Question 2:

$$\int_0^1 \frac{x}{x^2+1}dx$$

Solution:

$$\int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1 = t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$
$$= \frac{1}{2}\left[\log|t|\right]_1^2$$
$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$
$$= \frac{1}{2}\log 2$$

Ex 7.10 Class 12 Maths Question 3:

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi d\phi$$

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$
$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$
$$= \int_0^1 \left[t^{\frac{1}{2}}+t^{\frac{9}{2}}-2t^{\frac{5}{2}}\right]dt$$
$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{t^{\frac{11}{2}}}{\frac{11}{2}}-\frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$$
$$= \frac{2}{3}+\frac{2}{11}-\frac{4}{7}$$
$$= \frac{154+42-132}{231}$$
$$= \frac{64}{231}$$

Ex 7.10 Class 12 Maths Question 4:

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi d\phi$$

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$
$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$
$$= \int_0^1 \left[t^{\frac{1}{2}}+t^{\frac{9}{2}}-2t^{\frac{5}{2}}\right]dt$$
$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{t^{\frac{11}{2}}}{\frac{11}{2}}-\frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$$
$$= \frac{2}{3}+\frac{2}{11}-\frac{4}{7}$$
$$= \frac{154+42-132}{231}$$
$$= \frac{64}{231}$$

Ex 7.10 Class 12 Maths Question 5:

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

Solution:

Let $I=\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta=\frac{\pi}{4}$

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta \\
&= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta)\sec^2\theta\, d\theta \\
&= \int_0^{\frac{\pi}{4}} 2\theta\cdot\sec^2\theta\, d\theta \\
&= 2\int_0^{\frac{\pi}{4}} \theta\cdot\sec^2\theta\, d\theta
\end{aligned}$$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$\begin{aligned}
I &= 2\left[\theta\int\sec^2\theta\, d\theta-\int\left\{\left(\frac{d}{dx}\theta\right)\int\sec^2\theta\, d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}} \\
&= 2\left[\theta\tan\theta-\int\tan\theta\, d\theta\right]_0^{\frac{\pi}{4}} \\
&= 2\left[\theta\tan\theta+\log|\cos\theta|\right]_0^{\frac{\pi}{4}} \\
&= 2\left[\frac{\pi}{4}\tan\frac{\pi}{4}+\log\left|\cos\frac{\pi}{4}\right|-\log|\cos 0|\right] \\
&= 2\left[\frac{\pi}{4}+\log\left(\frac{1}{\sqrt{2}}\right)-\log 1\right] \\
&= 2\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right] \\
&= \frac{\pi}{2}-\log 2
\end{aligned}$$

Ex 7.10 Class 12 Maths Question 6:

$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Solution:

Let $I=\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

Also, let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

When $x = 0$, $\theta = 0$ and when $x = 1$, $\theta=\frac{\pi}{4}$

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta \\
&= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta)\sec^2\theta\, d\theta \\
&= \int_0^{\frac{\pi}{4}} 2\theta\cdot\sec^2\theta\, d\theta \\
&= 2\int_0^{\frac{\pi}{4}} \theta\cdot\sec^2\theta\, d\theta
\end{aligned}$$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$$I=2\left[\theta\int \sec^2\theta\,d\theta-\int\left\{\left(\frac{d}{dx}\theta\right)\int\sec^2\theta\,d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}}$$
$$=2\left[\theta\tan\theta-\int\tan\theta\,d\theta\right]_0^{\frac{\pi}{4}}$$
$$=2\left[\theta\tan\theta+\log|\cos\theta|\right]_0^{\frac{\pi}{4}}$$
$$=2\left[\frac{\pi}{4}\tan\frac{\pi}{4}+\log\left|\cos\frac{\pi}{4}\right|-\log|\cos 0|\right]$$
$$=2\left[\frac{\pi}{4}+\log\left(\frac{1}{\sqrt{2}}\right)-\log 1\right]$$
$$=2\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right]$$
$$=\frac{\pi}{2}-\log 2$$

Ex 7.10 Class 12 Maths Question 7:

$\int_0^2 x\sqrt{x+2}\ \left(\text{Put } x+2=t^2\right)$

Solution:

$$\int_0^2 x\sqrt{x+2}dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t=\sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}dx=\int_{\sqrt{2}}^2\left(t^2-2\right)\sqrt{t^2}\,2tdt$$
$$=2\int_{\sqrt{2}}^2\left(t^2-2\right)t^2dt$$
$$=2\int_{\sqrt{2}}^2\left(t^4-2t^2\right)dt$$
$$=2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$
$$=2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$
$$=2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$
$$=2\left[\frac{16+8\sqrt{2}}{15}\right]$$
$$=\frac{16\left(2+\sqrt{2}\right)}{15}$$
$$=\frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

Ex 7.10 Class 12 Maths Question 8:

$\int_0^2 x\sqrt{x+2}\ \left(\text{Put } x+2=t^2\right)$

Solution:

$\int_0^2 x\sqrt{x+2}dx$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t=\sqrt{2}$ and when $x = 2$, $t = 2$

$$\begin{aligned}
\therefore \int_0^2 x\sqrt{x+2}dx &= \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt \\
&= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt \\
&= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt \\
&= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2 \\
&= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right] \\
&= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right] \\
&= 2\left[\frac{16+8\sqrt{2}}{15}\right] \\
&= \frac{16\left(2+\sqrt{2}\right)}{15} \\
&= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}
\end{aligned}$$

Ex 7.10 Class 12 Maths Question 9:

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

Solution:

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

When $x = 0$, $t = 1$ and when $x=\frac{\pi}{2}$, $t=0$

$$\begin{aligned}
\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx &= -\int_1^0 \frac{dt}{1+t^2} \\
&= -\left[\tan^{-1} t\right]_1^0 \\
&= -\left[\tan^{-1} 0-\tan^{-1} 1\right] \\
&= -\left[-\frac{\pi}{4}\right] \\
&= \frac{\pi}{4}
\end{aligned}$$

Ex 7.10 Class 12 Maths Question 10:

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

Solution:

$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

When $x = 0$, $t = 1$ and when $x=\frac{\pi}{2}$, $t=0$

$$\begin{aligned}
\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx &= -\int_1^0 \frac{dt}{1+t^2} \\
&= -\left[\tan^{-1} t\right]_1^0 \\
&= -\left[\tan^{-1} 0-\tan^{-1} 1\right] \\
&= -\left[-\frac{\pi}{4}\right] \\
&= \frac{\pi}{4}
\end{aligned}$$

Ex 7.10 Class 12 Maths Question 11:

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Solution:

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Ex 7.10 Class 12 Maths Question 12:

$$\int_0^2 \frac{dx}{x+4-x^2}$$

Solution:

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0,\ t=-\frac{1}{2}$ and when $x=2,\ t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Ex 7.10 Class 12 Maths Question 13:

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5}$$

Solution:

$$\int_{-1}^{1}\frac{dx}{x^2+2x+5}=\int_{-1}^{1}\frac{dx}{\left(x^2+2x+1\right)+4}=\int_{-1}^{1}\frac{dx}{\left(x+1\right)^2+\left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1}\frac{dx}{\left(x+1\right)^2+\left(2\right)^2}=\int_{0}^{2}\frac{dt}{t^2+2^2}$$

$$=\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$=\frac{1}{2}\tan^{-1}1-\frac{1}{2}\tan^{-1}0$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{8}$$

Ex 7.10 Class 12 Maths Question 14:

$$\int_{-1}^{1}\frac{dx}{x^2+2x+5}$$

Solution:

$$\int_{-1}^{1}\frac{dx}{x^2+2x+5}=\int_{-1}^{1}\frac{dx}{\left(x^2+2x+1\right)+4}=\int_{-1}^{1}\frac{dx}{\left(x+1\right)^2+\left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1}\frac{dx}{\left(x+1\right)^2+\left(2\right)^2}=\int_{0}^{2}\frac{dt}{t^2+2^2}$$

$$=\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$=\frac{1}{2}\tan^{-1}1-\frac{1}{2}\tan^{-1}0$$

$$=\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{8}$$

Ex 7.10 Class 12 Maths Question 15:

$$\int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Solution:

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$$

$$= \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt = \int_2^4 e^t\left[f(t)+f'(t)\right]dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4}-\frac{e^2}{2}$$

$$= \frac{e^2\left(e^2-2\right)}{4}$$

Ex 7.10 Class 12 Maths Question 16:

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Solution:

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$$

$$= \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt = \int_2^4 e^t\left[f(t)+f'(t)\right]dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4}-\frac{e^2}{2}$$

$$= \frac{e^2\left(e^2-2\right)}{4}$$

Ex 7.10 Class 12 Maths Question 17:

The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$ **is**

Solution:

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}} \left(1 - \sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}} (\cos\theta)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}} (\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \sin^2\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \operatorname{cosec}^2\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot\theta)^{\frac{5}{3}} \operatorname{cosec}^2\theta\, d\theta$$

Let $\cot\theta = t \Rightarrow -\operatorname{cosec}2\theta\, d\theta = dt$

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$$

$$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}[16]$$

$$= 3 \times 2$$

$$= 6$$

Hence, the correct answer is A.

Ex 7.10 Class 12 Maths Question 18:

The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ **is**

Solution:

$$\text{Let } I=\int_{\frac{1}{3}}^{1}\frac{\left(x-x^{3}\right)^{\frac{1}{3}}}{x^{4}}dx$$

Also, let $x=\sin\theta \Rightarrow dx=\cos\theta\, d\theta$

When $x=\frac{1}{3},\ \theta=\sin^{-1}\left(\frac{1}{3}\right)$ and when $x=1,\ \theta=\frac{\pi}{2}$

$$\Rightarrow I=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta-\sin^{3}\theta\right)^{\frac{1}{3}}}{\sin^{4}\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}\left(1-\sin^{2}\theta\right)^{\frac{1}{3}}}{\sin^{4}\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^{4}\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^{2}\theta\sin^{2}\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}}\operatorname{cosec}^{2}\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}(\cot\theta)^{\frac{5}{3}}\operatorname{cosec}^{2}\theta\, d\theta$$

Let $\cot\theta = t \Rightarrow$ - cosec2θ $d\theta= dt$

When $\theta=\sin^{-1}\left(\frac{1}{3}\right),\ t=2\sqrt{2}$ and when $\theta=\frac{\pi}{2},\ t=0$

$$\therefore I=-\int_{2\sqrt{2}}^{0}(t)^{\frac{5}{3}}dt$$

$$=-\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$=-\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$=-\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$=\frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$=\frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$=\frac{3}{8}[16]$$

$$=3\times 2$$

$$=6$$

Hence, the correct answer is A.

Ex 7.10 Class 12 Maths Question 19:

If $f(x)=\int_{0}^{x}t\sin t\, dt$, then $f'(x)$ is

A. cos x + x sin x

B. x sin$x$

C. x cos x

D. sin x + x cos x

Solution:

$$f(x)=\int_0^x t\sin t dt$$

Integrating by parts, we obtain

$$\begin{aligned}f(x)&=t\int_0^x \sin t\,dt-\int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt\\&=\left[t(-\cos t)\right]_0^x-\int_0^x(-\cos t)dt\\&=\left[-t\cos t+\sin t\right]_0^x\\&=-x\cos x+\sin x\end{aligned}$$

$$\begin{aligned}\Rightarrow f'(x)&=-\left[\{x(-\sin x)\}+\cos x\right]+\cos x\\&=x\sin x-\cos x+\cos x\\&=x\sin x\end{aligned}$$

Hence, the correct answer is B.

Ex 7.10 Class 12 Maths Question 20:

If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. cos *x* + *x* sin *x*

B. *x* sin*x*

C. *x* cos *x*

D. sin *x* + *x* cos *x*

Solution:

$$f(x)=\int_0^x t\sin t dt$$

Integrating by parts, we obtain

$$\begin{aligned}f(x)&=t\int_0^x \sin t\,dt-\int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt\\&=\left[t(-\cos t)\right]_0^x-\int_0^x(-\cos t)dt\\&=\left[-t\cos t+\sin t\right]_0^x\\&=-x\cos x+\sin x\end{aligned}$$

$$\begin{aligned}\Rightarrow f'(x)&=-\left[\{x(-\sin x)\}+\cos x\right]+\cos x\\&=x\sin x-\cos x+\cos x\\&=x\sin x\end{aligned}$$

Hence, the correct answer is B.

NCERT Solutions for Class 2 Maths Integration Class 12 Ex 7.11

$$\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$I=\int_0^{\frac{\pi}{2}}\cos^2 x\,dx \quad \text{...(1)}$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\cos^2\left(\frac{\pi}{2}-x\right)dx \quad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\sin^2 x dx \quad \text{...(2)}$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\left(\sin^2 x+\cos^2 x\right)dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos}+\sqrt{\sin x}} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x+\cos^{\frac{3}{2}} x}$$

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}}\frac{\cos^5 x\,dx}{\sin^5 x+\cos^5 x}$$

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}}\frac{\cos^5 x dx}{\sin^5 x+\cos^5 x}$$

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

$$\int_{-5}^{5}|x+2|\,dx$$

$$\text{Let } I=\int_{-5}^{5}|x+2|dx$$

It can be seen that $(x+2)\le 0$ on $[-5,-2]$ and $(x+2)\ge 0$ on $[-2,5]$.

$$\therefore I=\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$

$$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$

$$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$

$$=29$$

$$\int_{-5}^{5}|x+2|\,dx$$

Let $I = \int_{-5}^{5} |x+2| dx$

It can be seen that $(x + 2) \le 0$ on $[-5, -2]$ and $(x + 2) \ge 0$ on $[-2, 5]$.

$$\therefore I = \int_{-5}^{-2} -(x+2) dx + \int_{-2}^{5} (x+2) dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$$

$$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4$$

$$= 29$$

$\int_{2}^{8} |x-5| dx$

Let $I = \int_{2}^{8} |x-5| dx$

It can be seen that $(x - 5) \le 0$ on $[2, 5]$ and $(x - 5) \ge 0$ on $[5, 8]$.

$$I = \int_{2}^{5} -(x-5) dx + \int_{2}^{8} (x-5) dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= -\left[\frac{x^2}{2} - 5x\right]_{2}^{5} + \left[\frac{x^2}{2} - 5x\right]_{5}^{8}$$

$$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$$

$$= 9$$

$\int_0^1 x(1-x)^n dx$

Let $I = \int_0^1 x(1-x)^n dx$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$$

$$= \int_0^1 (1-x)(x)^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 \qquad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

$\int_0^1 x(1-x)^n dx$

Let $I = \int_0^1 x(1-x)^n dx$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n dx$$

$$= \int_0^1 (1-x)(x)^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 \qquad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

$\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$

Let $I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$ $\quad\ldots(1)$

$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$ $\quad\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{\left(1+\tan x\right)}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I$ $\quad\left[\text{From }(1)\right]$

$\Rightarrow 2I = \left[x\log 2\right]_0^{\frac{\pi}{4}}$

$\Rightarrow 2I = \frac{\pi}{4}\log 2$

$\Rightarrow I = \frac{\pi}{8}\log 2$

$\int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$

Let $I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$ $\quad\ldots(1)$

$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$ $\quad\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{\left(1+\tan x\right)}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log\left(1+\tan x\right)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I$ $\quad\left[\text{From }(1)\right]$

$\Rightarrow 2I = \left[x\log 2\right]_0^{\frac{\pi}{4}}$

$\Rightarrow 2I = \frac{\pi}{4}\log 2$

$\Rightarrow I = \frac{\pi}{8}\log 2$

$\int_0^2 x\sqrt{2-x}dx$

Let $I = \int_0^2 x\sqrt{2-x}\,dx$

$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$

$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \frac{16\sqrt{2}}{15}$

$\int_0^2 x\sqrt{2-x}\,dx$

Let $I = \int_0^2 x\sqrt{2-x}\,dx$

$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

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$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \frac{16\sqrt{2}}{15}$

$\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)\,dx$

Let $I = \int_0^{\frac{\pi}{2}} \left(2\log \sin x - \log \sin 2x\right) dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{2\log \sin x - \log\left(2\sin x \cos x\right)\right\} dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{2\log \sin x - \log \sin x - \log \cos x - \log 2\right\} dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{\log \sin x - \log \cos x - \log 2\right\} dx \qquad ...(1)$

It is known that, $\left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{\log \cos x - \log \sin x - \log 2\right\} dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}} \left(-\log 2 - \log 2\right) dx$

$\Rightarrow 2I = -2\log 2 \int_0^{\frac{\pi}{2}} 1 \cdot dx$

$\Rightarrow I = -\log 2 \left[\frac{\pi}{2}\right]$

$\Rightarrow I = \frac{\pi}{2}\left(-\log 2\right)$

$\Rightarrow I = \frac{\pi}{2}\left[\log \frac{1}{2}\right]$

$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$

$\int_0^{\frac{\pi}{2}} \left(2\log \sin x - \log \sin 2x\right) dx$

Let $I = \int_0^{\frac{\pi}{2}} \left(2\log \sin x - \log \sin 2x\right) dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{2\log \sin x - \log\left(2\sin x \cos x\right)\right\} dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{2\log \sin x - \log \sin x - \log \cos x - \log 2\right\} dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{\log \sin x - \log \cos x - \log 2\right\} dx \qquad ...(1)$

It is known that, $\left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \left\{\log \cos x - \log \sin x - \log 2\right\} dx \qquad ...(2)$

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$\Rightarrow I = \frac{\pi}{2}\left(-\log 2\right)$

$\Rightarrow I = \frac{\pi}{2}\left[\log \frac{1}{2}\right]$

$\Rightarrow I = \frac{\pi}{2} \log \frac{1}{2}$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$

Let $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\,dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$$
\begin{aligned}
I &= 2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx\\
&= 2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}\,dx\\
&= \int_0^{\frac{\pi}{2}}(1-\cos 2x)\,dx\\
&= \left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}\\
&= \frac{\pi}{2}
\end{aligned}
$$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\,dx$

Let $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\,dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

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I &= 2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx\\
&= 2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}\,dx\\
&= \int_0^{\frac{\pi}{2}}(1-\cos 2x)\,dx\\
&= \left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}\\
&= \frac{\pi}{2}
\end{aligned}
$$

$\int_0^{\pi}\frac{x\,dx}{1+\sin x}$

$$\text{Let } I=\int_0^{\pi}\frac{x\,dx}{1+\sin x} \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)}{1+\sin(\pi-x)}\,dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)}{1+\sin x}\,dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$
\begin{aligned}
2I &= \int_0^{\pi}\frac{\pi}{1+\sin x}\,dx\\
\Rightarrow 2I &= \pi\int_0^{\pi}\frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}\,dx\\
\Rightarrow 2I &= \pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}\,dx\\
\Rightarrow 2I &= \pi\int_0^{\pi}\left\{\sec^2 x-\tan x\sec x\right\}dx\\
\Rightarrow 2I &= \pi\left[\tan x-\sec x\right]_0^{\pi}\\
\Rightarrow 2I &= \pi[2]\\
\Rightarrow I &= \pi
\end{aligned}
$$

$\int_0^{\pi}\frac{x\,dx}{1+\sin x}$

Let $I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$ ...(1)

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$ $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x} dx$ ...(2)

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} dx$$

$$\Rightarrow 2I = \pi [\tan x - \sec x]_0^{\pi}$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ ...(1)

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ ...(1)

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$$

$\int_0^{2\pi} \cos^5 x dx$

Let $I = \int_0^{2\pi} \cos^5 x dx$ ...(1)

$\cos^5(2\pi - x) = \cos^5 x$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$\Rightarrow I = 2(0) = 0$ $\left[\cos^5(\pi - x) = -\cos^5 x\right]$

$\int_0^{2\pi} \cos^5 x dx$

$$\text{Let } I = \int_0^{2\pi} \cos^5 x dx \quad ...(1)$$

$$\cos^5(2\pi - x) = \cos^5 x$$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$
$$\Rightarrow I = 0$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$
$$\Rightarrow I = 0$$

$$\int_0^{\pi} \log(1 + \cos x) dx$$

$$\text{Let } I=\int_0^{\pi}\log(1+\cos x)\,dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\pi}\log(1+\cos(\pi-x))\,dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\pi}\log(1-\cos x)\,dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\pi}\{\log(1+\cos x)+\log(1-\cos x)\}\,dx$$

$$\Rightarrow 2I=\int_0^{\pi}\log(1-\cos^2 x)\,dx$$

$$\Rightarrow 2I=\int_0^{\pi}\log\sin^2 x\,dx$$

$$\Rightarrow 2I=2\int_0^{\pi}\log\sin x\,dx$$

$$\Rightarrow I=\int_0^{\pi}\log\sin x\,dx \qquad ...(3)$$

sin (π - *x*) = sin *x*

$$\therefore I=2\int_0^{\frac{\pi}{2}}\log\sin x\,dx \qquad ...(4)$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2}-x\right)dx=2\int_0^{\frac{\pi}{2}}\log\cos x\,dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I=2\int_0^{\frac{\pi}{2}}(\log\sin x+\log\cos x)\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}(\log\sin x+\log\cos x+\log 2-\log 2)\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}(\log 2\sin x\cos x-\log 2)\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\sin 2x\,dx-\int_0^{\frac{\pi}{2}}\log 2\,dx$$

Let 2*x* = *t* ⇒ 2*dx* = *dt*

When *x* = 0, *t* = 0
and when
∴

$$\int_0^{\pi}\log(1+\cos x)\,dx$$

$$\text{Let } I=\int_0^{\pi}\log(1+\cos x)\,dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\pi}\log(1+\cos(\pi-x))\,dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\pi}\log(1-\cos x)\,dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\pi}\{\log(1+\cos x)+\log(1-\cos x)\}\,dx$$

$$\Rightarrow 2I=\int_0^{\pi}\log(1-\cos^2 x)\,dx$$

$$\Rightarrow 2I=\int_0^{\pi}\log\sin^2 x\,dx$$

$$\Rightarrow 2I=2\int_0^{\pi}\log\sin x\,dx$$

$$\Rightarrow I=\int_0^{\pi}\log\sin x\,dx \qquad ...(3)$$

sin (π - *x*) = sin *x*

$$\therefore I=2\int_0^{\frac{\pi}{2}}\log\sin x\,dx \qquad ...(4)$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2}-x\right)dx=2\int_0^{\frac{\pi}{2}}\log\cos x\,dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I = 2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x\,dx - \int_0^{\frac{\pi}{2}} \log 2\,dx$$

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad \ldots(1)$$

It is known that, $\left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad \ldots(1)$$

It is known that, $\left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

$$\int_0^4 |x-1|\,dx$$

$I = \int_0^4 |x-1|\, dx$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1|\, dx + \int_1^4 |x-1|\, dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1)\, dx + \int_1^4 (x-1)\, dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

$\int_0^4 |x-1|\, dx$

$I = \int_0^4 |x-1|\, dx$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1|\, dx + \int_1^4 |x-1|\, dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1)\, dx + \int_1^4 (x-1)\, dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Show that $\int_0^a f(x) g(x)\, dx = 2\int_0^a f(x)\, dx$, if *f* and *g* are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

$$\text{Let } I = \int_0^a f(x) g(x)\, dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^a f(a-x) g(a-x)\, dx \qquad \left(\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right)$$

$$\Rightarrow I = \int_0^a f(x) g(a-x)\, dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \{f(x) g(x) + f(x) g(a-x)\}\, dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a-x)\}\, dx$$

$$\Rightarrow 2I = \int_0^a f(x) \times 4\, dx \qquad [g(x) + g(a-x) = 4]$$

$$\Rightarrow I = 2\int_0^a f(x)\, dx$$

Show that $\int_0^a f(x) g(x)\, dx = 2\int_0^a f(x)\, dx$, if *f* and *g* are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

$$\text{Let } I = \int_0^a f(x) g(x)\, dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^a f(a-x) g(a-x)\, dx \qquad \left(\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right)$$

$$\Rightarrow I = \int_0^a f(x) g(a-x)\, dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \{f(x) g(x) + f(x) g(a-x)\}\, dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x) + g(a-x)\}\, dx$$

$$\Rightarrow 2I = \int_0^a f(x) \times 4\, dx \qquad [g(x) + g(a-x) = 4]$$

$$\Rightarrow I = 2\int_0^a f(x)\, dx$$

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ **is**

A. 0

B. 2

C. π

D. 1

$$\text{Let } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x^3dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan^5 xdx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\cdot dx$$

It is known that if *f*(*x*) is an even function, then $\int_{-a}^{a}f(x)dx=2\int_0^a f(x)dx$ and

if *f*(*x*) is an odd function, then $\int_{-a}^{a}f(x)dx=0$

$$I=0+0+0+2\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$=2[x]_0^{\frac{\pi}{2}}$$

$$=\frac{2\pi}{2}$$

$$\pi=$$

Hence, the correct answer is C.

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ **is**

$$\text{Let } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}x^3dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan^5 xdx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\cdot dx$$

It is known that if *f*(*x*) is an even function, then $\int_{-a}^{a}f(x)dx=2\int_0^a f(x)dx$ and

if *f*(*x*) is an odd function, then $\int_{-a}^{a}f(x)dx=0$

$$I=0+0+0+2\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$=2[x]_0^{\frac{\pi}{2}}$$

$$=\frac{2\pi}{2}$$

$$\pi=$$

Hence, the correct answer is C.

The value of $\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ **is**

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right\}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log 1\,dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}0\,dx$$

$$\Rightarrow I=0$$

Hence, the correct answer is C.

The value of $\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ **is**

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\left\{\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right\}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\log 1\,dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}0\,dx$$

$$\Rightarrow I=0$$

Hence, the correct answer is C.

NCERT Solutions for Class 12 Maths Chapter 7 Integrals Miscellaneous Exercise

Miscellaneous Exercise Class 11 Maths Question 1:

$$\frac{1}{x-x^3}$$

Solution:

$$\frac{1}{x-x^3}=\frac{1}{x\left(1-x^2\right)}=\frac{1}{x(1-x)(1+x)}$$

$$\text{Let } \frac{1}{x(1-x)(1+x)}=\frac{A}{x}+\frac{B}{(1-x)}+\frac{C}{1+x} \qquad ...(1)$$

$$\Rightarrow 1=A\left(1-x^2\right)+Bx(1+x)+Cx(1-x)$$

$$\Rightarrow 1=A-Ax^2+Bx+Bx^2+Cx-Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$-A+B-C=0$

$B+C=0$

$A=1$

On solving these equations, we obtain

$$A=1, B=\frac{1}{2}, \text{ and } C=-\frac{1}{2}$$

From equation (1), we obtain

$$\frac{1}{x(1-x)(1+x)}=\frac{1}{x}+\frac{1}{2(1-x)}-\frac{1}{2(1+x)}$$

$$\Rightarrow \int\frac{1}{x(1-x)(1+x)}dx=\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{1-x}dx-\frac{1}{2}\int\frac{1}{1+x}dx$$

$$=\log|x|-\frac{1}{2}\log|(1-x)|-\frac{1}{2}\log|(1+x)|$$

$$=\log|x|-\log\left|(1-x)^{\frac{1}{2}}\right|-\log\left|(1+x)^{\frac{1}{2}}\right|$$

$$=\log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right|+C$$

$$=\log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right|+C$$

$$=\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$$

Miscellaneous Exercise Class 11 Maths Question 2:

$$\frac{1}{\sqrt{x+a}+\sqrt{(x+b)}}$$

Solution:

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}}=\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\times\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$=\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}$$

$$=\frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)}{a-b}$$

$$\Rightarrow \int\frac{1}{\sqrt{x+a}-\sqrt{x+b}}dx=\frac{1}{a-b}\int\left(\sqrt{x+a}-\sqrt{x+b}\right)dx$$

$$=\frac{1}{(a-b)}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$=\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+C$$

Miscellaneous Exercise Class 11 Maths Question 3:

$\frac{1}{x\sqrt{ax-x^2}}$ **[Hint: Put** $x=\frac{a}{t}$**]**

Solution:

$$\frac{1}{x\sqrt{ax-x^2}}$$

Let $x=\frac{a}{t} \Rightarrow dx=-\frac{a}{t^2}dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}}dx=\int \frac{1}{\frac{a}{t}\sqrt{a\cdot\frac{a}{t}-\left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}dt\right)$$

$$=-\int \frac{1}{at}\cdot\frac{1}{\sqrt{\frac{1}{t}-\frac{1}{t^2}}}dt$$

$$=-\frac{1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}dt$$

$$=-\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$$

$$=-\frac{1}{a}\left[2\sqrt{t-1}\right]+C$$

$$=-\frac{1}{a}\left[2\sqrt{\frac{a}{x}-1}\right]+C$$

$$=-\frac{2}{a}\left(\frac{\sqrt{a-x}}{\sqrt{x}}\right)+C$$

$$=-\frac{2}{a}\left(\sqrt{\frac{a-x}{x}}\right)+C$$

vMiscellaneous Exercise Class 11 Maths Question 4:

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Solution:

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{-\frac{3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx = \int \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$= -\frac{1}{4}\int (1+t)^{-\frac{3}{4}}dt$$

$$= -\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$= -\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

Miscellaneous Exercise Class 11 Maths Question 5:

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} \quad \left[\text{Hint: } \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)} \text{ Put } x = t^6\right]$$

Solution:

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$$

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\therefore \int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = \int \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}dx$$

$$= \int \frac{6t^5}{t^2(1+t)}dt$$

$$= 6\int \frac{t^3}{(1+t)}dt$$

On dividing, we obtain

$$\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = 6\int\left\{\left(t^2-t+1\right)-\frac{1}{1+t}\right\}dt$$

$$= 6\left[\left(\frac{t^3}{3}\right)-\left(\frac{t^2}{2}\right)+t-\log|1+t|\right]$$

$$= 2x^{\frac{1}{2}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

$$= 2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

Miscellaneous Exercise Class 11 Maths Question 6:

$$\frac{5x}{(x+1)\left(x^2+9\right)}$$

Solution:

Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+9)}$ ...(1)

$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$

$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{9}{2}$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)} + \frac{\frac{x}{2}+\frac{9}{2}}{(x^2+9)}$$

$$\int \frac{5x}{(x+1)(x^2+9)}dx = \int\left\{\frac{-1}{2(x+1)} + \frac{(x+9)}{2(x^2+9)}\right\}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{2}\int\frac{x}{x^2+9}dx + \frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\int\frac{2x}{x^2+9}dx + \frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + \mathrm{C}$$

Miscellaneous Exercise Class 11 Maths Question 7:

$$\frac{\sin x}{\sin(x-a)}$$

Solution:

$$\frac{\sin x}{\sin(x-a)}$$

Let $x - a = t \Rightarrow dx = dt$

$$\int\frac{\sin x}{\sin(x-a)}dx = \int\frac{\sin(t+a)}{\sin t}dt$$

$$= \int\frac{\sin t\cos a + \cos t\sin a}{\sin t}dt$$

$$= \int(\cos a + \cot t\sin a)dt$$

$$= t\cos a + \sin a\log|\sin t| + C_1$$

$$= (x-a)\cos a + \sin a\log|\sin(x-a)| + \mathrm{C}_1$$

$$= x\cos a + \sin a\log|\sin(x-a)| - a\cos a + \mathrm{C}_1$$

$$= \sin a\log|\sin(x-a)| + x\cos a + \mathrm{C}$$

Miscellaneous Exercise Class 11 Maths Question 8:

$$\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$$

Solution:

$$\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} = \frac{e^{4\log x}(e^{\log x}-1)}{e^{2\log x}(e^{\log x}-1)}$$

$$= e^{2\log x}$$

$$= e^{\log x^2}$$

$$= x^2$$

$$\therefore \int\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}dx = \int x^2 dx = \frac{x^3}{3} + \mathrm{C}$$

Miscellaneous Exercise Class 11 Maths Question 9:

$$\frac{\cos x}{\sqrt{4-\sin^2 x}}$$

Solution:

$$\frac{\cos x}{\sqrt{4-\sin^2 x}}$$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{4-\sin^2 x}} dx = \int \frac{dt}{\sqrt{(2)^2-(t)^2}}$$
$$= \sin^{-1}\left(\frac{t}{2}\right) + C$$
$$= \sin^{-1}\left(\frac{\sin x}{2}\right) + C$$

Miscellaneous Exercise Class 11 Maths Question 10:

$$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} = \frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$$
$$= \frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{(\sin^2 x-\sin^2 x\cos^2 x)+(\cos^2 x-\sin^2 x\cos^2 x)}$$
$$= \frac{(\sin^4 x+\cos^4 x)(\sin^2 x-\cos^2 x)}{\sin^2 x(1-\cos^2 x)+\cos^2 x(1-\sin^2 x)}$$
$$= \frac{-(\sin^4 x+\cos^4 x)(\cos^2 x-\sin^2 x)}{(\sin^4 x+\cos^4 x)}$$
$$= -\cos 2x$$
$$\therefore \int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx = \int -\cos 2x\, dx = -\frac{\sin 2x}{2} + C$$

Solution:

$$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} = \frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$$
$$= \frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{(\sin^2 x-\sin^2 x\cos^2 x)+(\cos^2 x-\sin^2 x\cos^2 x)}$$
$$= \frac{(\sin^4 x+\cos^4 x)(\sin^2 x-\cos^2 x)}{\sin^2 x(1-\cos^2 x)+\cos^2 x(1-\sin^2 x)}$$
$$= \frac{-(\sin^4 x+\cos^4 x)(\cos^2 x-\sin^2 x)}{(\sin^4 x+\cos^4 x)}$$
$$= -\cos 2x$$
$$\therefore \int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx = \int -\cos 2x\, dx = -\frac{\sin 2x}{2} + C$$

Miscellaneous Exercise Class 11 Maths Question 11:

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Solution:

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by $\sin(a-b)$, we obtain

$$\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x+a)-(x+b)\right]}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cdot\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)}-\frac{\sin(x+b)}{\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\tan(x+a)-\tan(x+b)\right]$$

$$\int\frac{1}{\cos(x+a)\cos(x+b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x+a)-\tan(x+b)\right]dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log\left|\cos(x+a)\right|+\log\left|\cos(x+b)\right|\right]+C$$

$$=\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right|+C$$

Miscellaneous Exercise Class 11 Maths Question 12:

$$\frac{x^3}{\sqrt{1-x^8}}$$

Solution:

$$\frac{x^3}{\sqrt{1-x^8}}$$

Let $x^4 = t \Rightarrow 4x^3\,dx = dt$

$$\Rightarrow \int\frac{x^3}{\sqrt{1-x^8}}dx=\frac{1}{4}\int\frac{dt}{\sqrt{1-t^2}}$$

$$=\frac{1}{4}\sin^{-1}t+C$$

$$=\frac{1}{4}\sin^{-1}\left(x^4\right)+C$$

Miscellaneous Exercise Class 11 Maths Question 13:

$$\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$$

Solution:

$$\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$$

Let $e^x = t \Rightarrow e^x\,dx = dt$

$$\Rightarrow \int\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}dx=\int\frac{dt}{(t+1)(t+2)}$$

$$=\int\left[\frac{1}{(t+1)}-\frac{1}{(t+2)}\right]dt$$

$$=\log|t+1|-\log|t+2|+C$$

$$=\log\left|\frac{t+1}{t+2}\right|+C$$

$$=\log\left|\frac{1+e^x}{2+e^x}\right|+C$$

Miscellaneous Exercise Class 11 Maths Question 14:

$$\frac{1}{\left(x^2+1\right)\left(x^2+4\right)}$$

Solution:

$$\frac{1}{(x^2+1)(x^2+4)}$$

$$\therefore \frac{1}{(x^2+1)(x^2+4)}=\frac{Ax+B}{(x^2+1)}+\frac{Cx+D}{(x^2+4)}$$

$$\Rightarrow 1=(Ax+B)(x^2+4)+(Cx+D)(x^2+1)$$

$$\Rightarrow 1=Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx^2+D$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$$A=0,\ B=\frac{1}{3},\ C=0, \text{and } D=-\frac{1}{3}$$

From equation (1), we obtain

$$\frac{1}{(x^2+1)(x^2+4)}=\frac{1}{3(x^2+1)}-\frac{1}{3(x^2+4)}$$

$$\int\frac{1}{(x^2+1)(x^2+4)}dx=\frac{1}{3}\int\frac{1}{x^2+1}dx-\frac{1}{3}\int\frac{1}{x^2+4}dx$$

$$=\frac{1}{3}\tan^{-1}x-\frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2}+C$$

$$=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+C$$

Miscellaneous Exercise Class 11 Maths Question 15:

$\cos^3 x e^{\log\sin x}$

Solution:

$\cos^3 x e^{\log\sin x} = \cos^3 x \times \sin x$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\Rightarrow \int\cos^3 x\, e^{\log\sin x}dx=\int\cos^3 x\sin x dx$$

$$=-\int t\cdot dt$$

$$=-\frac{t^4}{4}+C$$

$$=-\frac{\cos^4 x}{4}+C$$

Miscellaneous Exercise Class 11 Maths Question 16:

$e^{3\log x}(x^4+1)^{-1}$

Solution:

$$e^{3\log x}(x^4+1)^{-1}=e^{\log x^3}(x^4+1)^{-1}=\frac{x^3}{(x^4+1)}$$

Let $x^4+1=t \Rightarrow 4x^3\,dx=dt$

$$\Rightarrow \int e^{3\log x}(x^4+1)^{-1}dx=\int\frac{x^3}{(x^4+1)}dx$$

$$=\frac{1}{4}\int\frac{dt}{t}$$

$$=\frac{1}{4}\log|t|+C$$

$$=\frac{1}{4}\log|x^4+1|+C$$

$$=\frac{1}{4}\log(x^4+1)+C$$

Miscellaneous Exercise Class 11 Maths Question 17:

$$f'(ax+b)\left[f(ax+b)\right]^n$$

Let $f(ax+b)=t \Rightarrow af'(ax+b)dx=dt$

$$\Rightarrow \int f'(ax+b)\left[f(ax+b)\right]^n dx=\frac{1}{a}\int t^n dt$$
$$=\frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$$
$$=\frac{1}{a(n+1)}\left(f(ax+b)\right)^{n+1}+\mathrm{C}$$

Solution:

$$f'(ax+b)\left[f(ax+b)\right]^n$$

Let $f(ax+b)=t \Rightarrow af'(ax+b)dx=dt$

$$\Rightarrow \int f'(ax+b)\left[f(ax+b)\right]^n dx=\frac{1}{a}\int t^n dt$$
$$=\frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$$
$$=\frac{1}{a(n+1)}\left(f(ax+b)\right)^{n+1}+\mathrm{C}$$

Miscellaneous Exercise Class 11 Maths Question 18:

$$\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}$$

Solution:

$$\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}=\frac{1}{\sqrt{\sin^3 x(\sin x\cos\alpha+\cos x\sin\alpha)}}$$
$$=\frac{1}{\sqrt{\sin^4 x\cos\alpha+\sin^3 x\cos x\sin\alpha}}$$
$$=\frac{1}{\sin^2 x\sqrt{\cos\alpha+\cot x\sin\alpha}}$$
$$=\frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha+\cot x\sin\alpha}}$$

Let $\cos\alpha+\cot x\sin\alpha=t \;\Rightarrow\; -\operatorname{cosec}^2 x\sin\alpha\, dx=dt$

$$\therefore \int\frac{1}{\sin^3 x\sin(x+\alpha)}dx=\int\frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha+\cot x\sin\alpha}}dx$$
$$=\frac{-1}{\sin\alpha}\int\frac{dt}{\sqrt{t}}$$
$$=\frac{-1}{\sin\alpha}\left[2\sqrt{t}\right]+\mathrm{C}$$
$$=\frac{-1}{\sin\alpha}\left[2\sqrt{\cos\alpha+\cot x\sin\alpha}\right]+\mathrm{C}$$
$$=\frac{-2}{\sin\alpha}\sqrt{\cos\alpha+\frac{\cos x\sin\alpha}{\sin x}}+\mathrm{C}$$
$$=\frac{-2}{\sin\alpha}\sqrt{\frac{\sin x\cos\alpha+\cos x\sin\alpha}{\sin x}}+\mathrm{C}$$
$$=-\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+\mathrm{C}$$

Miscellaneous Exercise Class 11 Maths Question 20:

$$\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$$

Solution:

$$I = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$$

Let $x = \cos^2\theta \Rightarrow dx = -2\sin\theta\cos\theta\,d\theta$

$$I = \int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-2\sin\theta\cos\theta)d\theta$$

$$= -\int\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin 2\theta\,d\theta$$

$$= -\int\tan\frac{\theta}{2}\cdot 2\sin\theta\cos\theta\,d\theta$$

$$= -2\int\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta\,d\theta$$

$$= -2\int\sin^2\theta\,d\theta + 4\int\sin^2\frac{\theta}{2}d\theta$$

$$= -2\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta + 4\int\frac{1-\cos\theta}{2}d\theta$$

$$= -2\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right] + 4\left[\frac{\theta}{2} - \frac{\sin\theta}{2}\right] + C$$

$$= -\theta + \frac{\sin 2\theta}{2} + 2\theta - 2\sin\theta + C$$

$$= \theta + \frac{\sin 2\theta}{2} - 2\sin\theta + C$$

$$= \theta + \frac{2\sin\theta\cos\theta}{2} - 2\sin\theta + C$$

$$= \theta + \sqrt{1-\cos^2\theta}\cdot\cos\theta - 2\sqrt{1-\cos^2\theta} + C$$

$$= \cos^{-1}\sqrt{x} + \sqrt{1-x}\cdot\sqrt{x} - 2\sqrt{1-x} + C$$

$$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x(1-x)} + C$$

$$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + C$$

Miscellaneous Exercise Class 11 Maths Question 21:

$$\frac{2+\sin 2x}{1+\cos 2x}e^x$$

Solution:

$$I = \int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x$$

$$= \int\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)e^x$$

$$= \int\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)e^x$$

$$= \int(\sec^2 x + \tan x)e^x$$

Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$$\therefore I = \int(f(x) + f'(x)]e^x dx$$

$$= e^x f(x) + C$$

$$= e^x\tan x + C$$

Miscellaneous Exercise Class 11 Maths Question 22:

$$\frac{x^2+x+1}{(x+1)^2(x+2)}$$

Solution:

Let $\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x+2)}$ ...(1)

$\Rightarrow x^2+x+1=A(x+1)(x+2)+B(x+2)+C(x^2+2x+1)$

$\Rightarrow x^2+x+1=A(x^2+3x+2)+B(x+2)+C(x^2+2x+1)$

$\Rightarrow x^2+x+1=(A+C)x^2+(3A+B+2C)x+(2A+2B+C)$

Equating the coefficients of x^2, x,and constant term, we obtain

$A + C = 1$

$3A + B + 2C = 1$

$2A + 2B + C = 1$

On solving these equations, we obtain

$A = -2$, $B = 1$, and $C = 3$

From equation (1), we obtain

$$\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{-2}{(x+1)}+\frac{3}{(x+2)}+\frac{1}{(x+1)^2}$$

$$\int\frac{x^2+x+1}{(x+1)^2(x+2)}dx=-2\int\frac{1}{x+1}dx+3\int\frac{1}{(x+2)}dx+\int\frac{1}{(x+1)^2}dx$$

$$=-2\log|x+1|+3\log|x+2|-\frac{1}{(x+1)}+C$$

Miscellaneous Exercise Class 11 Maths Question 23:

$\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Solution:

$$I=\tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

Let $x=\cos\theta \Rightarrow dx=-\sin\theta d\theta$

$$I=\int\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta d\theta)$$

$$=-\int\tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin\theta d\theta$$

$$=-\int\tan^{-1}\tan\frac{\theta}{2}\cdot\sin\theta d\theta$$

$$=-\frac{1}{2}\int\theta\cdot\sin\theta d\theta$$

$$=-\frac{1}{2}\left[\theta\cdot(-\cos\theta)-\int1\cdot(-\cos\theta)d\theta\right]$$

$$=-\frac{1}{2}[-\theta\cos\theta+\sin\theta]$$

$$=+\frac{1}{2}\theta\cos\theta-\frac{1}{2}\sin\theta$$

$$=\frac{1}{2}\cos^{-1}x\cdot x-\frac{1}{2}\sqrt{1-x^2}+C$$

$$=\frac{x}{2}\cos^{-1}x-\frac{1}{2}\sqrt{1-x^2}+C$$

$$=\frac{1}{2}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)+C$$

Miscellaneous Exercise Class 11 Maths Question 24:

$\frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}$

Solution:

$$\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}=\frac{\sqrt{x^2+1}}{x^4}\left[\log\left(x^2+1\right)-\log x^2\right]$$
$$=\frac{\sqrt{x^2+1}}{x^4}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$
$$=\frac{\sqrt{x^2+1}}{x^4}\log\left(1+\frac{1}{x^2}\right)$$
$$=\frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$
$$=\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

Let $1+\frac{1}{x^2}=t \Rightarrow \frac{-2}{x^3}dx=dt$

$$\therefore I=\int\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)dx$$
$$=-\frac{1}{2}\int\sqrt{t}\log t\,dt$$
$$=-\frac{1}{2}\int t^{\frac{1}{2}}\cdot\log t\,dt$$

Integrating by parts, we obtain

$$I=-\frac{1}{2}\left[\log t\cdot\int t^{\frac{1}{2}}dt-\int\left\{\left(\frac{d}{dt}\log t\right)\int t^{\frac{1}{2}}dt\right\}dt\right]$$
$$=-\frac{1}{2}\left[\log t\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\int\frac{1}{t}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt\right]$$
$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{2}{3}\int t^{\frac{1}{2}}dt\right]$$
$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{4}{9}t^{\frac{3}{2}}\right]$$
$$=-\frac{1}{3}t^{\frac{3}{2}}\log t+\frac{2}{9}t^{\frac{3}{2}}$$
$$=-\frac{1}{3}t^{\frac{3}{2}}\left[\log t-\frac{2}{3}\right]$$
$$=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$$

Miscellaneous Exercise Class 11 Maths Question 25:

$$\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$$

Solution:

$$I=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$$
$$=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^{2}\frac{x}{2}}\right)dx$$
$$=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{\operatorname{cosec}^{2}\frac{x}{2}}{2}-\cot\frac{x}{2}\right)dx$$
Let $f(x)=-\cot\frac{x}{2}$

$\Rightarrow f'(x)=-\left(-\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}\right)=\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}$

$$\therefore I=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(f(x)+f'(x)\right]dx$$
$$=\left[e^{x}\cdot f(x)dx\right]_{\frac{\pi}{2}}^{\pi}$$
$$=-\left[e^{x}\cdot\cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$
$$=-\left[e^{\pi}\times\cot\frac{\pi}{2}-e^{\frac{\pi}{2}}\times\cot\frac{\pi}{4}\right]$$
$$=-\left[e^{\pi}\times 0-e^{\frac{\pi}{2}}\times 1\right]$$
$$=e^{\frac{\pi}{2}}$$

Miscellaneous Exercise Class 11 Maths Question 26:

$$\int_{0}^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^{4}x+\sin^{4}x}dx$$

Solution:

Let $I=\int_{0}^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^{4}x+\sin^{4}x}dx$

$$\Rightarrow I=\int_{0}^{\frac{\pi}{4}}\frac{\frac{(\sin x\cos x)}{\cos^{4}x}}{\frac{(\cos^{4}x+\sin^{4}x)}{\cos^{4}x}}dx$$
$$\Rightarrow I=\int_{0}^{\frac{\pi}{4}}\frac{\tan x\sec^{2}x}{1+\tan^{4}x}dx$$
Let $\tan^{2}x=t \Rightarrow 2\tan x\sec^{2}x\,dx=dt$

When $x = 0$, $t = 0$ and when $x=\frac{\pi}{4}, t=1$

$$\therefore I=\frac{1}{2}\int_{0}^{1}\frac{dt}{1+t^{2}}$$
$$=\frac{1}{2}\left[\tan^{-1}t\right]_{0}^{1}$$
$$=\frac{1}{2}\left[\tan^{-1}1-\tan^{-1}0\right]$$
$$=\frac{1}{2}\left[\frac{\pi}{4}\right]$$
$$=\frac{\pi}{8}$$

Miscellaneous Exercise Class 11 Maths Question 27:

$$\int_{0}^{\frac{\pi}{2}}\frac{\cos^{2}x\,dx}{\cos^{2}x+4\sin^{2}x}$$

Solution:

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4\left(1-\cos^2 x\right)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4-4\cos^2 x}dx$$

$$\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}\frac{4-3\cos^2 x-4}{4-3\cos^2 x}dx$$

$$\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}\frac{4-3\cos^2 x}{4-3\cos^2 x}dx+\frac{1}{3}\int_0^{\frac{\pi}{2}}\frac{4}{4-3\cos^2 x}dx$$

$$\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}1\,dx+\frac{1}{3}\int_0^{\frac{\pi}{2}}\frac{4\sec^2 x}{4\sec^2 x-3}dx$$

$$\Rightarrow I=\frac{-1}{3}[x]_0^{\frac{\pi}{2}}+\frac{1}{3}\int_0^{\frac{\pi}{2}}\frac{4\sec^2 x}{4\left(1+\tan^2 x\right)-3}dx$$

$$\Rightarrow I=-\frac{\pi}{6}+\frac{2}{3}\int_0^{\frac{\pi}{2}}\frac{2\sec^2 x}{1+4\tan^2 x}dx \qquad ...(1)$$

$$\text{Consider, }\int_0^{\frac{\pi}{2}}\frac{2\sec^2 x}{1+4\tan^2 x}dx$$

Let $2\tan x=t \Rightarrow 2\sec^2 x\,dx=dt$

When $x=0, t=0$ and when $x=\frac{\pi}{2}, t=\infty$

$$\Rightarrow \int_0^{\frac{\pi}{2}}\frac{2\sec^2 x}{1+4\tan^2 x}dx=\int_0^{\infty}\frac{dt}{1+t^2}$$

$$=\left[\tan^{-1}t\right]_0^{\infty}$$

$$=\left[\tan^{-1}(\infty)-\tan^{-1}(0)\right]$$

$$=\frac{\pi}{2}$$

Therefore, from (1), we obtain

$$I=-\frac{\pi}{6}+\frac{2}{3}\left[\frac{\pi}{2}\right]=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

Miscellaneous Exercise Class 11 Maths Question 28:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$$

Solution:

$$\text{Let } I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{-(-\sin 2x)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{-(-1+1-2\sin x\cos x)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{1-\left(\sin^2 x+\cos^2 x-2\sin x\cos x\right)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)dx}{\sqrt{1-(\sin x-\cos x)^2}}$$

Let $(\sin x-\cos x)=t \Rightarrow (\sin x+\cos x)dx=dt$

When $x=\frac{\pi}{6}, t=\left(\frac{1-\sqrt{3}}{2}\right)$ and when $x=\frac{\pi}{3}, t=\left(\frac{\sqrt{3}-1}{2}\right)$

$$I=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

$$\Rightarrow I=\int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

As $\frac{1}{\sqrt{1-(-t)^2}}=\frac{1}{\sqrt{1-t^2}}$, therefore, $\frac{1}{\sqrt{1-t^2}}$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a}f(x)\,dx=2\int_0^a f(x)\,dx$

$$\Rightarrow I=2\int_0^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

$$=\left[2\sin^{-1}t\right]_0^{\frac{\sqrt{3}-1}{2}}$$

$$=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

Miscellaneous Exercise Class 11 Maths Question 29:

$$\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

Solution:

$$\text{Let } I=\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$I=\int_0^1\frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)}\times\frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)}dx$$

$$=\int_0^1\frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$$

$$=\int_0^1\sqrt{1+x}\,dx+\int_0^1\sqrt{x}\,dx$$

$$=\left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1+\left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$=\frac{2}{3}\left[(2)^{\frac{3}{2}}-1\right]+\frac{2}{3}[1]$$

$$=\frac{2}{3}(2)^{\frac{3}{2}}$$

$$=\frac{2\cdot 2\sqrt{2}}{3}$$

$$=\frac{4\sqrt{2}}{3}$$

Miscellaneous Exercise Class 11 Maths Question 30:

$$\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$$

Let $I=\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

Also, let $\sin x-\cos x=t \Rightarrow (\cos x+\sin x)dx=dt$

When $x=0,\ t=-1$ and when $x=\frac{\pi}{4}, t=0$

$\Rightarrow (\sin x-\cos x)^2=t^2$

$\Rightarrow \sin^2 x+\cos^2 x-2\sin x\cos x=t^2$

$\Rightarrow 1-\sin 2x=t^2$

$\Rightarrow \sin 2x=1-t^2$

$$\therefore I=\int_{-1}^{0}\frac{dt}{9+16(1-t^2)}$$

$$=\int_{-1}^{0}\frac{dt}{9+16-16t^2}$$

$$=\int_{-1}^{0}\frac{dt}{25-16t^2}=\int_{-1}^{0}\frac{dt}{(5)^2-(4t)^2}$$

$$=\frac{1}{4}\left[\frac{1}{2(5)}\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$$

$$=\frac{1}{40}\left[\log(1)-\log\left|\frac{1}{9}\right|\right]$$

$$=\frac{1}{40}\log 9$$

Miscellaneous Exercise Class 11 Maths Question 31:

$$\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx=\int_0^{\frac{\pi}{2}}2\sin x\cos x\tan^{-1}(\sin x)dx$

Also, let $\sin x=t \Rightarrow \cos x\,dx=dt$

When $x=0, t=0$ and when $x=\frac{\pi}{2}, t=1$

$$\Rightarrow I=2\int_0^1 t\tan^{-1}(t)dt \qquad \ldots(1)$$

Consider $\int t\cdot\tan^{-1}t\,dt=\tan^{-1}t\cdot\int t\,dt-\int\left\{\frac{d}{dt}(\tan^{-1}t)\int t\,dt\right\}dt$

$$=\tan^{-1}t\cdot\frac{t^2}{2}-\int\frac{1}{1+t^2}\cdot\frac{t^2}{2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int\frac{t^2+1-1}{1+t^2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int 1\,dt+\frac{1}{2}\int\frac{1}{1+t^2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\cdot t+\frac{1}{2}\tan^{-1}t$$

$$\Rightarrow \int_0^1 t\cdot\tan^{-1}t\,dt=\left[\frac{t^2\cdot\tan^{-1}t}{2}-\frac{t}{2}+\frac{1}{2}\tan^{-1}t\right]_0^1$$

$$=\frac{1}{2}\left[\frac{\pi}{4}-1+\frac{\pi}{4}\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}-1\right]=\frac{\pi}{4}-\frac{1}{2}$$

From equation (1), we obtain

$$I=2\left[\frac{\pi}{4}-\frac{1}{2}\right]=\frac{\pi}{2}-1$$

Miscellaneous Exercise Class 11 Maths Question 32:

$$\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$$

Solution:

$$Let\ I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx \qquad ...(1)$$

$$I=\int_0^{\pi}\left\{\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\right\}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\pi}\left\{\frac{-(\pi-x)\tan x}{-(\sec x+\tan x)}\right\}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\pi}\frac{\pi\tan x}{\sec x+\tan x}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{\sin x+1-1}{1+\sin x}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}1.dx-\pi\int_0^{\pi}\frac{1}{1+\sin x}dx$$

$$\Rightarrow 2I=\pi[x]_0^{\pi}-\pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I=\pi^2-\pi\int_0^{\pi}(\sec^2 x-\tan x\sec x)dx$$

$$\Rightarrow 2I=\pi^2-\pi[\tan x-\sec x]_0^{\pi}$$

$$\Rightarrow 2I=\pi^2-\pi[\tan\pi-\sec\pi-\tan 0+\sec 0]$$

$$\Rightarrow 2I=\pi^2-\pi[0-(-1)-0+1]$$

$$\Rightarrow 2I=\pi^2-2\pi$$

$$\Rightarrow 2I=\pi(\pi-2)$$

$$\Rightarrow I=\frac{\pi}{2}(\pi-2)$$

Miscellaneous Exercise Class 11 Maths Question 33:

$$\int_1^4[|x-1|+|x-2|+|x-3|]dx$$

Solution:

$$\text{Let } I=\int_1^4[|x-1|+|x-2|+|x-3|]dx$$

$$\Rightarrow I=\int_1^4|x-1|dx+\int_1^4|x-2|dx+\int_1^4|x-3|dx$$

$$I=I_1+I_2+I_3 \qquad ...(1)$$

where, $I_1=\int_1^4|x-1|dx$, $I_2=\int_1^4|x-2|dx$, and $I_3=\int_1^4|x-3|dx$

$$I_1=\int_1^4|x-1|dx$$

$(x-1)\geq 0$ for $1\leq x\leq 4$

$$\therefore I_1=\int_1^4(x-1)dx$$

$$\Rightarrow I_1=\left[\frac{x^2}{x}-x\right]_1^4$$

$$\Rightarrow I_1=\left[8-4-\frac{1}{2}+1\right]=\frac{9}{2} \qquad ...(2)$$

$I_2 = \int_1^4 |x-2|\,dx$

$x-2 \ge 0$ for $2 \le x \le 4$ and $x-2 \le 0$ for $1 \le x \le 2$

$\therefore I_2 = \int_1^2 (2-x)\,dx + \int_2^4 (x-2)\,dx$

$\Rightarrow I_2 = \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$

$\Rightarrow I_2 = \left[4-2-2+\frac{1}{2}\right] + [8-8-2+4]$

$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2}$ ...(3)

$I_3 = \int_1^4 |x-3|\,dx$

$x-3 \ge 0$ for $3 \le x \le 4$ and $x-3 \le 0$ for $1 \le x \le 3$

$\therefore I_3 = \int_1^3 (3-x)\,dx + \int_3^4 (x-3)\,dx$

$\Rightarrow I_3 = \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$

$\Rightarrow I_3 = \left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right] + \left[8 - 12 - \frac{9}{2} + 9\right]$

$\Rightarrow I_3 = [6-4] + \left[\frac{1}{2}\right] = \frac{5}{2}$ ...(4)

From equations (1), (2), (3), and (4), we obtain

$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$

Miscellaneous Exercise Class 11 Maths Question 34:

$\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

Solution:

Let $I = \int_1^3 \frac{dx}{x^2(x+1)}$

Also, let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$\Rightarrow 1 = Ax(x+1) + B(x+1) + C(x^2)$

$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$A + B = 0$

$B = 1$

On solving these equations, we obtain

$A = -1$, $C = 1$, and $B = 1$

$$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$$

$$\Rightarrow I = \int_1^3 \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$$

$$= \left[-\log x - \frac{1}{x} + \log(x+1) \right]_1^3$$

$$= \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x} \right]_1^3$$

$$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$$

$$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$$

$$= \log 2 - \log 3 + \frac{2}{3}$$

$$= \log\left(\frac{2}{3}\right) + \frac{2}{3}$$

Hence, the given result is proved.

Question 35:

$$\int_0^1 xe^x dx = 1$$

Solution:

Let $I = \int_0^1 xe^x \ dx$

Integrating by parts, we obtain

$$I = x\int_0^1 e^x dx - \int_0^1 \left\{ \left(\frac{d}{dx}(x) \right) \int e^x dx \right\} dx$$

$$= \left[xe^x \right]_0^1 - \int_0^1 e^x dx$$

$$= \left[xe^x \right]_0^1 - \left[e^x \right]_0^1$$

$$= e - e + 1$$

$$= 1$$

Hence, the given result is proved.

Miscellaneous Exercise Class 11 Maths Question 36:

$$\int_{-1}^1 x^{17} \cos^4 x dx = 0$$

Solution:

Let $I = \int_{-1}^1 x^{17} \cos^4 x dx$

Also, let $f(x) = x^{17} \cos^4 x$

$$\Rightarrow f(-x) = (-x)^{17} \cos^4(-x) = -x^{17} \cos^4 x = -f(x)$$

Therefore, $f(x)$ is an odd function.

It is known that if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$$\therefore I = \int_{-1}^1 x^{17} \cos^4 x \, dx = 0$$

Hence, the given result is proved.

Miscellaneous Exercise Class 11 Maths Question 37:

$$\int_0^{\frac{\pi}{2}} \sin^3 x \, dx = \frac{2}{3}$$

Solution:

Let $I=\int_0^{\frac{\pi}{2}}\sin^3 x\,dx$

$I=\int_0^{\frac{\pi}{2}}\sin^2 x\cdot\sin x\,dx$

$=\int_0^{\frac{\pi}{2}}\left(1-\cos^2 x\right)\sin x\,dx$

$=\int_0^{\frac{\pi}{2}}\sin x\,dx-\int_0^{\frac{\pi}{2}}\cos^2 x\cdot\sin x\,dx$

$=\left[-\cos x\right]_0^{\frac{\pi}{2}}+\left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$

$=1+\frac{1}{3}[-1]=1-\frac{1}{3}=\frac{2}{3}$

Hence, the given result is proved.

Miscellaneous Exercise Class 11 Maths Question 38:

$\int_0^{\frac{\pi}{4}}2\tan^3 x dx=1-\log 2$

Solution:

Let $I=\int_0^{\frac{\pi}{4}}2\tan^3 x\,dx$

$I=2\int_0^{\frac{\pi}{4}}\tan^2 x\tan x\,dx=2\int_0^{\frac{\pi}{4}}\left(\sec^2 x-1\right)\tan x\,dx$

$=2\int_0^{\frac{\pi}{4}}\sec^2 x\tan x\,dx-2\int_0^{\frac{\pi}{4}}\tan x\,dx$

$=2\left[\frac{\tan^2 x}{2}\right]_0^{\frac{\pi}{4}}+2\left[\log\cos x\right]_0^{\frac{\pi}{4}}$

$=1+2\left[\log\cos\frac{\pi}{4}-\log\cos 0\right]$

$=1+2\left[\log\frac{1}{\sqrt{2}}-\log 1\right]$

$=1-\log 2-\log 1=1-\log 2$

Hence, the given result is proved.

Miscellaneous Exercise Class 11 Maths Question 39:

$\int_0^1\sin^{-1} x\,dx=\frac{\pi}{2}-1$

Solution:

Let $I=\int_0^1\sin^{-1} x\,dx$

$\Rightarrow I=\int_0^1\sin^{-1} x\cdot 1\cdot dx$

Integrating by parts, we obtain

$I=\left[\sin^{-1} x\cdot x\right]_0^1-\int_0^1\frac{1}{\sqrt{1-x^2}}\cdot x\,dx$

$=\left[x\sin^{-1} x\right]_0^1+\frac{1}{2}\int_0^1\frac{(-2x)}{\sqrt{1-x^2}}dx$

Let $1-x^2=t\Rightarrow -2x\,dx=dt$

When $x=0,\ t=1$ and when $x=1,\ t=0$

$I=\left[x\sin^{-1} x\right]_0^1+\frac{1}{2}\int_1^0\frac{dt}{\sqrt{t}}$

$=\left[x\sin^{-1} x\right]_0^1+\frac{1}{2}\left[2\sqrt{t}\right]_1^0$

$=\sin^{-1}(1)+\left[-\sqrt{1}\right]$

$=\frac{\pi}{2}-1$

Hence, the given result is proved.

Miscellaneous Exercise Class 11 Maths Question 40:

Evaluate $\int_0^1 e^{2-3x}dx$ as a limit of a sum.

Solution:

Let $I = \int_0^1 e^{2-3x} dx$

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+...+f(a+(n-1)h)\right]$$

Where, $h = \frac{b-a}{n}$

Here, $a = 0, b = 1,$ and $f(x) = e^{2-3x}$

$\Rightarrow h = \frac{1-0}{n} = \frac{1}{n}$

$$\therefore \int_0^1 e^{2-3x}dx = (1-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(0+h)+...+f(0+(n-1)h)\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2 + e^{2-3h} + ...e^{2-3(n-1)h}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{1+e^{-3h}+e^{-6h}+e^{-9h}+...e^{-3(n-1)h}\right\}\right]$$

$$= \lim_{h\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-3h}\right)^n}{1-\left(e^{-3h}\right)}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-e^{-\frac{3}{n}\times n}}{1-e^{-\frac{3}{n}}}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2\left(1-e^{-3}\right)}{1-e^{-\frac{3}{n}}}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{e^{-\frac{3}{n}}-1}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\left(-\frac{1}{3}\right)\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}\lim_{n\to\infty}\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}(1) \qquad \left[\lim_{n\to\infty}\frac{x}{e^x-1}\right]$$

$$= \frac{-e^{-1}+e^2}{3}$$

$$= \frac{1}{3}\left(e^2-\frac{1}{e}\right)$$

Miscellaneous Exercise Class 11 Maths Question 41:

$\int \frac{dx}{e^x+e^{-x}}$ **is equal to**

Solution:

A. $\tan^{-1}\left(e^x\right)+C$

B. $\tan^{-1}\left(e^{-x}\right)+C$

C. $\log\left(e^x-e^{-x}\right)+C$

D. $\log\left(e^x+e^{-x}\right)+C$

Let $I = \int \frac{dx}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x}+1} dx$

Also, let $e^x = t \Rightarrow e^x dx = dt$

$\therefore I = \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + C$

$= \tan^{-1}(e^x) + C$

Hence, the correct answer is A.

Miscellaneous Exercise Class 11 Maths Question 42:

$\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ **is equal to**

Solution:

A. $\frac{-1}{\sin x + \cos x} + C$

B. $\log|\sin x + \cos x| + C$

C. $\log|\sin x - \cos x| + C$

D. $\frac{1}{(\sin x + \cos x)^2}$

Let $I = \frac{\cos 2x}{(\cos x + \sin x)^2}$

$I = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$

$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$

$= \int \frac{\cos x - \sin x}{\cos + \sin x} dx$

Let $\cos x + \sin x = t \Rightarrow (\cos x - \sin x) dx = dt$

$\therefore I = \int \frac{dt}{t}$

$= \log|t| + C$

$= \log|\cos x + \sin x| + C$

Hence, the correct answer is B.

Miscellaneous Exercise Class 11 Maths Question 43:

If $f(a+b-x) = f(x)$**, then** $\int_a^b x f(x) dx$ **is equal to**

Solution:

A. $\frac{a+b}{2} \int_a^b f(b-x) dx$

B. $\frac{a+b}{2} \int_a^b f(b+x) dx$

C. $\frac{b-a}{2} \int_a^b f(x) dx$

D. $\frac{a+b}{2} \int_a^b f(x) dx$

$$\text{Let } I=\int_a^b x f(x)\,dx \qquad \ldots(1)$$

$$I=\int_a^b (a+b-x) f(a+b-x)\,dx \qquad \left(\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx\right)$$

$$\Rightarrow I=\int_a^b (a+b-x) f(x)\,dx$$

$$\Rightarrow I=(a+b)\int_a^b f(x)\,dx \quad -I \qquad [\text{Using}(1)]$$

$$\Rightarrow I+I=(a+b)\int_a^b f(x)\,dx$$

$$\Rightarrow 2I=(a+b)\int_a^b f(x)\,dx$$

$$\Rightarrow I=\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$$

Hence, the correct answer is D.

Miscellaneous Exercise Class 11 Maths Question 44:

The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ **is**

Solution:

A. 1

B. 0

C. - 1

D. $\frac{\pi}{4}$

$$\text{Let } I=\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$$

$$\Rightarrow I=\int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$$

$$\Rightarrow I=\int_0^1\left[\tan^{-1}x-\tan^{-1}(1-x)\right]dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(1-1+x)\right]dx$$

$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx$$

$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^1\left(\tan^{-1}x+\tan^{-1}(1-x)-\tan^{-1}(1-x)-\tan^{-1}x\right)dx$$

$$\Rightarrow 2I=0$$

$$\Rightarrow I=0$$

Hence, the correct answer is B.

Integration Formulas

1. Integration is the inverse process of differentiation. In the differential calculus, we are given a function and we have to find the derivative or differential of this function, but in the integral calculus, we are to find a function whose differential is given. Thus, integration is a process which is the inverse of differentiation.

   Then, $\int f(x)dx = F(x) + C$, these integrals are called indefinite integrals or general integrals. C is an arbitrary constant by varying which one gets different anti-derivatives of the given function.

   NOTE Derivative of a function is unique but a function can have infinite anti-derivatives or integrals.

2. **Properties of Indefinite Integral**

   (i) $\int [f(x) + g(x)]\,dx = \int f(x)dx + \int g(x)dx$

   (ii) For any real number k, $\int k\,f(x)dx = k\int f(x)dx$.

   (iii) In general, if $f_1, f_2, \ldots, f_n$ are functions and $k_1, k_2, \ldots, k_n$ are real numbers, then

   $$\int [k_1\, f_1(x) + k_2\, f_2(x) + \ldots + k_n f_n(x)]\,dx = k_1 \int f_1(x)dx + k_2 \int f_2(x)dx + \ldots + k_n \int f_n(x)dx$$

3. **Basic Formulae**

   (i) $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C, n \neq -1$

   (ii) $\int e^{ax}\, dx = \dfrac{e^{ax}}{a} + C$

   (iii) $\int a^x\, dx = \dfrac{a^x}{\log a} + C$

   (iv) $\int \sin x\, dx = -\cos x + C$

   (v) $\int \cos x\, dx = \sin x + C$

   (vi) $\int \tan x\, dx = -\log|\cos x| + C = \log|\sec x| + C$

   (vii) $\int \cot x\, dx = \log|\sin x| + C = -\log|\operatorname{cosec} x| + C$

   (viii) $\int \sec x\, dx = \log|\sec x + \tan x| + C = \log\left|\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right| + C$

   (ix) $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C = \log\left|\tan\dfrac{x}{2}\right| + C$

   (x) $\int \sec x \tan x\, dx = \sec x + C$

   (xi) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$

   (xii) $\int \sec^2 x\, dx = \tan x + C$

   (xiii) $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$

   (xiv) $\int \dfrac{1}{\sqrt{1 - x^2}}\, dx = \sin^{-1} x + C$

   (xv) $\int \dfrac{-1}{\sqrt{1 - x^2}}\, dx = \cos^{-1} x + C$

   (xvi) $\int \dfrac{1}{1 + x^2}\, dx = \tan^{-1} x + C$

   (xvii) $\int \dfrac{-1}{1 + x^2}\, dx = \cot^{-1} x + C$

   (xviii) $\int \dfrac{1}{x\sqrt{x^2 - 1}}\, dx = \sec^{-1} x + C$

   (xix) $\int \dfrac{-1}{x\sqrt{x^2 - 1}}\, dx = \operatorname{cosec}^{-1} x + C$

   (xx) $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \dfrac{x}{a} + C$

   (xxi) $\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$

   (xxii) $\int \dfrac{dx}{\sqrt{x^2 + a^2}} = \log|x + \sqrt{x^2 + a^2}| + C$

   (xxiii) $\int \dfrac{1}{a^2 + x^2}\, dx = \dfrac{1}{a} \tan^{-1}\left(\dfrac{x}{a}\right) + C$

   (xxiv) $\int \dfrac{1}{a^2 - x^2}\, dx = \dfrac{1}{2a} \log\left|\dfrac{a + x}{a - x}\right| + C$

   (xxv) $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a} \log\left|\dfrac{x - a}{x + a}\right| + C$

(xxvi) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left| x + \sqrt{x^2 - a^2} \right| + C$

(xxvii) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

(xxviii) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left| x + \sqrt{x^2 + a^2} \right| + C$

(xxix) $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C, n \neq -1$

(xxx) $\int e^x[f(x) + f'(x)]\, dx = f(x)\, e^x + C$

4. **Integration using Trigonometric Identities**

When the integrand involves some trigonometric functions, we use the following identities to find the integral:

(i) $2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$ (ii) $2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

(iii) $2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$ (iv) $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

(v) $2\sin A \cdot \cos A = \sin 2A$ (vi) $\cos^2 A - \sin^2 A = \cos 2A$

(vii) $\sin^2 A = \left(\frac{1-\cos 2A}{2}\right)$ (viii) $\sin^2 A + \cos^2 A = 1$

(ix) $\sin^3 A = \frac{3\sin A - \sin 3A}{4}$ (x) $\cos^3 A = \frac{3\cos A + \cos 3A}{4}$

5. **Integration by Substitutions**

Substitution method is used, when a suitable substitution of variable leads to simplification of integral.

If $I = \int f(x)dx$, then by putting $x = g(z)$, we get

$$I = \int f[g(z)]g'(z)dz$$

NOTE Try to substitute the variable whose derivative is present in original integral and final integral must be written in terms of the original variable of integration.

6. **Integration by Parts**

For given functions $f(x)$ and $g(x)$, we have

$$\int [\underset{\mathrm{I}}{f(x)} \cdot \underset{\mathrm{II}}{g(x)}]dx = f(x) \cdot \int g(x)\, dx - \int \{f'(x) \cdot \int g(x)\, dx\} dx$$

Here, we can choose first function according to its position in ILATE, where

I = Inverse trigonometric function L = Logarithmic function

A = Algebraic function T = Trigonometric function

E = Exponential function

[the function which comes first in ILATE should taken as first junction and other as second function]

NOTE

(i) Keep in mind, ILATE is not a rule as all questions of integration by parts cannot be done by above method.

(ii) It is worth mentioning that integration by parts is not applicable to product of functions in all cases. For instance, the method does not work for $\int \sqrt{x} \sin x \, dx$. The reason is that there does not exist any function whose derivative is $\sqrt{x} \sin x$.

(iii) Observe that while finding the integral of the second function, we did not add any constant of integration.

7. **Integration by Partial Fractions**

A rational function is ratio of two polynomials of the form $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials in x and $q(x) \neq 0$. If degree of $p(x) >$ degree of $q(x)$, then we may divide $p(x)$ by $q(x)$ so that $\frac{p(x)}{q(x)} = t(x) + \frac{p_1(x)}{q(x)}$, where $t(x)$ is a polynomial in x which can be integrated easily and degree of $p_1(x)$ is less than the degree of $q(x)$. $\frac{p_1(x)}{q(x)}$ can be integrated by expressing $\frac{p_1(x)}{q(x)}$ as the sum of partial fractions of the following type:

(i) $\frac{p(x)+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}, a \neq b$

(ii) $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$

(iii) $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

(iv) $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$

(v) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+c}{x^2+bx+c}$, where x^2+bx+c cannot be factorised further.

8. Integrals of the types $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ can be transformed into standard form by expressing $ax^2+bx+c = a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] = a\left[\left(x+\frac{b}{2a}\right)^2 + \left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right]$.

9. Integrals of the types $\int \frac{px+q}{ax^2+bx+c}dx$ or $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$ can be transformed into standard form by expressing $px+q = A\frac{d}{dx}(ax^2+bx+c)+B = A(2ax+b)+B$, where A and B are determined by comparing coefficients on both sides.