Class : 11th
Subject : Maths
Chapter : 8
Chapter Name : Binomial Theorem

Exercise 8.1

Q1 Expand the expression $(1-2x)^5$

Answer.
$(1-2x)^5$
$={}^5C_0(1)^5-{}^5C_1(1)^4(2x)+{}^5C_2(1)^3(2x)^2-{}^5C_3(1)^2(2x)^3+{}^5C_4(1)^1(2x)^4-{}^5C_3(2x)^5$
$=1-5(2x)+10\left(4x^2\right)-10\left(8x^3\right)+5\left(16x^4\right)-\left(32x^2\right)$
$=1-10x+40x^2-80x^3+80x^4-32x^5$

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Q2 Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Answer.
$\left(\frac{2}{x}-\frac{x}{2}\right)^5={}^5C_0\left(\frac{2}{x}\right)^3-{}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right)+{}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$
$-{}^3C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3+{}^3C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4-{}^3C_5\left(\frac{x}{2}\right)^5$
$=\frac{32}{x^5}-5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right)+10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right)-10\left(\frac{4}{x^2}\right)+5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right)-\frac{x^5}{32}$
$=\frac{32}{x^5}-\frac{40}{x^3}+\frac{20}{x}-5x+\frac{5}{8}x^3-\frac{x^3}{32}$

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Q3 Expand the expression $(2x-3)^6$

Answer. By using Binomial theorem, the expression $(2x-3)^6$ can be expressed as
$(2x-3)^6={}^6C_0(2x)^6-{}^6C_1(2x)^5(3)+{}^6C_1(2x)^4(3)^2-{}^4C_3(2x)^3(3)^3$
$=64x^6-6\left(32x^5\right)(3)+15\left(16x^4\right)(9)-20\left(8x^3\right)(27)$
$+15\left(4x^2\right)(81)-6(2x)(243)+729$
$=64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729$

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Q4 Expand the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Answer. By using Binomial Theorem, the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ can be expand as

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^3C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^3C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2$$

$$= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^3}$$

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Q5 Expand the expression $\left(x+\frac{1}{x}\right)^6$

Answer.

$$\left(\mathrm{x}+\frac{1}{\mathrm{x}}\right)^6 = {}^6C_0(\mathrm{x})^6 + {}^6C_1(\mathrm{x})'\left(\frac{1}{\mathrm{x}}\right) + {}^6C_2(\mathrm{x})^4\left(\frac{1}{\mathrm{x}}\right)^2$$

$$+{}^6C_3(\mathrm{x})^3\left(\frac{1}{\mathrm{x}}\right)^3 + {}^6C_4(\mathrm{x})^2\left(\frac{1}{\mathrm{x}}\right)^4 + {}^6C_3(\mathrm{x})\left(\frac{1}{\mathrm{x}}\right)^5 + {}^6C_6\left(\frac{1}{\mathrm{x}}\right)^6$$

$$= \mathrm{x}^4 + 6(\mathrm{x})^3\left(\frac{1}{\mathrm{x}}\right) + 15(\mathrm{x})^4\left(\frac{1}{\mathrm{x}^2}\right) + 20(\mathrm{x})^3\left(\frac{1}{\mathrm{x}^3}\right) + 15(\mathrm{x})^2\left(\frac{1}{\mathrm{x}^4}\right) + 6(\mathrm{x})\left(\frac{1}{\mathrm{x}^5}\right) + \frac{1}{\mathrm{x}^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

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Q6 Using binomial theorem, evaluate $(96)^3$

Answer.

96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, $96 = 100 - 4$

$$\therefore (96)^3 = (100-4)^3$$
$$= {}^3C_\alpha(100)^3 - {}^3C_1(100)^2(4) + {}^3C_z(100)(4)^2 - {}^3C_3(4)^3$$
$$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$$
$$= 1000000 - 120000 + 4800 - 64$$
$$= 884736$$

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Q7 Using binomial theorem, evaluate $(102)^5$

Answer.

102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, $102 = 100 + 2$

$$\therefore (102)^5 = (100+2)^5$$
$$= {}^*C_0(100)^5 + {}'C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3$$
$$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$$

$= 1000000000 + 1000000000 + 40000000 + 80000 + 8000 + 32$
$= 11040808032$

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Q8 Using binomial theorem, evaluate $(101)^4$

Answer.
101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.
It can be written that, $101 = 100 + 1$

$$\begin{aligned}\therefore (101)^4 &= (100+1)^4 \\ &= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^2 + {}^4C_4(1)^4 \\ &= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4 \\ &= 100000000 + 400000 + 60000 + 400 + 1 \\ &= 104060401\end{aligned}$$

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Q9 Using binomial theorem, evaluate $(99)^5$

Answer.
99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.
It can be written that, $99 = 100 - 1$

$$\begin{aligned}.(99)^5 &= (100-1)^5 \\ &= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 \\ &\quad + {}^3C_4(100)(1)^4 - {}^5C_5(1)^5\end{aligned}$$

$= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1$
$= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$
$= 10010000500 - 500100001$
$= 9509900499$

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Q10 Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000

Answer.
By splitting 1.1 and then applying Binomial Theorem, the first few terms of $(1.1)^{10000}$ can be obtained as

$$\begin{aligned}(1.1)^{10000} &= (1+0.1)^{10000} \\ &= (1+0.1)^{10000}C_1(1.1) + \text{ Other positive terms} \\ &= 1 + 10000 \times 1.1 + \text{ Other positive terms} \\ &= 1 + 11000 + \text{ Other positive terms} \\ &> 1000\end{aligned}$$

$(1.1)^{10000} > 1000$

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Q11 Find $(a+b)^4-(a-b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$

Answer. Useing Bionomial Theorm, the wxpression $(a+b)^4$ and $(a-b)^4$, can be expanded as

$(a+b)^4={}^4C_0a^4+{}^+C_1a^3b+{}^+C_2a^2b^2+{}^4C_3ab^3+{}^4C_4b^4$

$(a-b)^4={}^4C_0a^4-{}^4C_1a^3b+{}^4C_2a^2b^2-{}^4C_3ab^3+{}^4C_4b^4$

$\therefore (a+b)^4-(a-b)^4={}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4b^4$

$-\left[{}^4C_0a^4-{}^4C_1a^3b+{}^4C_2a^2b^2-{}^4C_3ab^3+{}^{\cdot}C_4b^4\right]$

$=2\left({}^4C_1a^3b+{}^4C_3ab^3\right)=2\left(4a^3b+4ab^3\right)$

$=8ab\left(a^2+b^2\right)$

By putting $a=\sqrt{3}$ and $b=\sqrt{2}$, we obtain

$(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4=8(\sqrt{3})(\sqrt{2})\left\{(\sqrt{3})^2+(\sqrt{2})^2\right\}$

$=8(\sqrt{6})\{3+2\}=40\sqrt{6}$

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Q12 Find $(x+1)^6+(x-1)^6\cdot$ Hence or otherwise evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$

Answer.

Using Binomial Theorem, the expressions, $(x+1)^6$ and $(x-1)^6$, can be expanded as

$(x+1)^6={}^6C_0x^6+{}^6C_1x^3+{}^6C_2x^4+{}^6C_3x^3+{}^6C_4x^2+{}^6C_5x+{}^6C_6$

$(x-1)^6={}^6C_0x^6-{}^6C_1x^5+{}^6C_2x^4-{}^6C_3x^3+{}^6C_4x^2-{}^6C_5x+{}^6C_6$

$\therefore (x+1)^6+(x-1)^6=2\left[{}^6C_0x^6+{}^6C_2x^4+{}^6C_4x^2+{}^6C_6\right]$

$=2\left[x^6+15x^4+15x^2+1\right]$

By putting $x=\sqrt{2}$, we obtain

$(\sqrt{2}+1)^6+(\sqrt{2}-1)^6=2\left[(\sqrt{2})^\circ+15(\sqrt{2})^4+15(\sqrt{2})^2+1\right]$

$=2(8+15\times 4+15\times 2+1)$

$=2(8+60+30+1)$

$=2(99)=198$

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Q13 Show that $9^{n+1}-8n-9$ is divisible by 64, whenever n

Answer.

In order to show that $9^{n+1}-8n-9$ is divisible by 64, it has to be proved that,

$9^{n+1}-8n-9=64k$, where k is some natural number

By Binomial Theorem,

$(1+a)^m={}^mC_0+{}^mC_1a+{}^mC_2a^2+\ldots+{}^mC_ma^m$

For $a=8$ and $m=n+1,$ we obtain
$(1+8)^{n+1}={ }^{n+1} C_0+{ }^{n+1} C_1(8)+{ }^{n+1} C_2(8)^2+\ldots+{ }^{n+1} C_{a+1}(8)^{n+1}$
$\Rightarrow 9^{n+1}=1+(n+1)(8)+8^2\left[{ }^{n+1} C_2+{ }^{n+1} C_3 \times 8+\ldots+{ }^{n+1} C_{n+1}(8)^{n-1}\right]$
$\Rightarrow 9^{n+1}=9+8 n+64\left[{ }^{n+1} C_2+{ }^{11} C_3 \times 8+\ldots+{ }^{n+1} C_{0+1}(8)^{n-1}\right]$
$\Rightarrow 9^{n+1}-8 n-9=64 k,$ where $k={ }^{n+1} C_2+{ }^{n+1} C_3 \times 8+\ldots+{ }^{n+1} C_{n+1}(8)^{n-1}$ is a natural number
Thus, $9^{n+1}-8 n-9$ is divisible by 64, whenever n is a positive integer.

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Q14 Prove that $\sum_{r=0}^n 3^{rn} \mathrm{C},=4^n$

Answer. By Binomial Theorem,
$\sum_{i=0}^n C_t a^{n-r} b^r=(a+b)^n$

By putting $b=3$ and $a=1$ in the above equation, we obtain
$\sum_{r=0}^n C_t(1)^{n-1}(3)^r=(1+3)^n$
$\Rightarrow \sum_{t=0}^n 3^n C_r=4^n$
Hence, proved.

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Exercise 8.2

Q1 Find the coefficient of $x^5 \ln (x+3)^8$

Answer.
It is known that $(r+1)^{\text {th }}$ term, $\left(T_{r+1}\right),$ in the binomial expansion of $(a+b)^n$ is given by
$T_{r+1}={ }^n C_r a^{n-t} b^t$
Assuming that x^5 occurs in the $(r+1)^{\text {th }}$ term of the expansion $(x+3)^8,$ we obtain
$T_{r+1}={ }^8 C_r(x)^{3-t}(3)^r$
Comparing the indices of x in x^5 and in $T_{r+1},$ we obtain
$r=3$
Thus, the coefficient of x^5 is ${ }^3 \mathrm{C}_3(3)^3=\frac{8!}{3!5!} \times 3^3=\frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 5!} \cdot 3^3=1512$

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Q2 Find the coefficient of $a^5 b^7 \ln (a-2 b)^{12}$

Answer.
It is known that $(r+1)^{\text {th }}$ term, $\left(T_{r+1}\right),$ in the binomial expansion of $(a+b)^n$ is given by
$T_{r+1}={ }^n C_r a^{n-1} b^r$

a^5b^7 occurs in the $(r+1)^{th}$ term of the expansion $(a-2b)^{12}$,
$T_{r+1} = {}^{12}C_r(a)^{12-r}(-2b)' = {}^{12}C_r(-2)'(a)^{12-T}(b)^r$
Thus the coefficient of a^5b^7 is
${}^{12}C_7(-2)^7 = -\frac{12!}{715!} \cdot 2^7 = -\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8.7!}{5 \cdot 4 \cdot 3 \cdot 2.7!} \cdot 2^7 = -(792)(128) = -101376$

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Q3 Write the general term in the expansion of $\left(x^2-y\right)^6$

Answer.
It is known that the general term T_{r+1} {which is the $(r+1)^{\text{th}}$ term } in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_ra^{n-\tau}b^r$

Thus, the general term in the expansion of $\left(x^2-y^6\right)$ is
$T_{r+1} = {}^6C_1\left(x^2\right)^{6-\tau}(-y)^t = (-1)^rC_r \cdot x^{12-2r}y^r$

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Q4 Write the general term in the expansion of $\left(x^2-yx\right)^{12}, x \neq 0$

Answer.
It is known that the general term T_{r+1} $\{$ which is the $(r+1)^{\text{th}}$ term $\}$ in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_ra^{n-\tau}b^r$
Thus the general term of expansion of $\left(x^2-yx\right)^{12}$ is
$\mathrm{T_{r+1}} = {}^{12}\mathrm{C_r}\left(\mathrm{x}^2\right)^{12-\mathrm{t}}(-\mathrm{yx})^{\mathrm{r}} = (-1)\mathrm{r}^{12}\mathrm{C_r} \cdot \mathrm{x}^{2+2\mathrm{r}} \cdot \mathrm{y}^{\mathrm{r}} \cdot \mathrm{xr} = (-1)^{\mathrm{r}}\mathrm{C_r} \cdot \mathrm{x}^{24-\mathrm{r}} \cdot \mathrm{y}^{\mathrm{r}}$

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Q5 Find the 4^{th} term in the expansion of $(x-2y)^{12}$

Answer.
It is known that $(r+1)^{\text{th}}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_1a^{n-1}b'$
Thus, the 4^{th} term in the expansion of $(x-2y)^{12}$ is
$T_4 = T_{7+1} = {}^{12}C_3(x)^{12-3}(-2y)^3 = (-1)^3 \cdot \frac{12!}{3!9!} \cdot x^0 \cdot (2)^3 \cdot y^4 = -\frac{12 \cdot 11 \cdot 10}{3 \cdot 2} \cdot (2)^3x^4y^3 = -1760x^9y^3$

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Q6 Find the 13^{th} term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$

Answer.
$(r+1)^{\text{th}}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$

$$\mathrm{T_{13}} = \mathrm{T_{12+1}} = {}^{18}\mathrm{C_{12}}(9\mathrm{x})^{18-12}\left(-\frac{1}{3\sqrt{\mathrm{x}}}\right)^{12}$$

$$=(-1)^{12}\frac{18!}{12!6!}(9)^6(x)^6\left(\frac{1}{3}\right)^{12}\left(\frac{1}{\sqrt{x}}\right)^{12}$$
$$=\frac{18\cdot17\cdot16\cdot15\cdot14\cdot13\cdot12!}{12!6\cdot5\cdot4\cdot3\cdot2}\cdot x^6\cdot\left(\frac{1}{x^6}\right)\cdot3^{12}\left(\frac{1}{3^{12}}\right)$$
$$=18564$$

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Q7 Find the middle terms in the expansions of $\left(3-\frac{x^3}{6}\right)^7$

Answer. It is known that in the expansion of $(a+b)^n$, if n if n is odd then there are two middle terms, namely $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{t}$ Term

$$T_4=T_{3+1}={}^7C_3(3)^{1-3}\left(-\frac{x^3}{6}\right)^3=(-1)^3\frac{7!}{3!4!}\cdot3^4\cdot\frac{x^9}{6^3}$$
$$=-\frac{7\cdot6\cdot5.4!}{3\cdot2.4!}\cdot3^4\cdot\frac{1}{2^3\cdot3^3}\cdot x^9=-\frac{105}{8}x^9$$
$$T_5=T_{4,1}={}^7C_4(3)^{7-4}\left(-\frac{x^3}{6}\right)^4=(-1)^4\frac{7!}{4!3!}(3)^3\cdot\frac{x^{12}}{6^4}$$
$$=\frac{7\cdot6\cdot5\cdot4!}{4!.3\cdot2}\cdot\frac{3^3}{2^4\cdot3^4}\cdot x^{12}=\frac{35}{48}x^{12}$$
$\left(3-\frac{x^3}{6}\right)^7$ are $-\frac{105}{8}x^9$ and $\frac{35}{48}x^{12}$

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Q8 Find the middle terms in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$

Answer.

It is known that in the expansion $(a+b)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.

$$T_6=T_{5+1}={}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5=\frac{10!}{5!5!}\cdot\frac{x^5}{3^5}\cdot9^5\cdot y^5$$
$$=\frac{10.9\cdot8\cdot7.6.5!}{5\cdot4\cdot3\cdot2.5!}\cdot\frac{1}{3^5}.^3\cdot x^5y^5$$
$$=252\times3^5\cdot x^5\cdot y^5=61236x^5y^5$$

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Q9 In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n

Answer.

It is known that $(r+1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by $T_{r+1}={}^nC_ra^{n-r}b^r$

Assuming that a^m occurs in the $(r+1)^{\text{th}}$ term of the expansion $(1+a)^{m+n}$, we obtain
$T_{r+1} = {}^{m+n}C_r(1)^{m+n-r}(a)^r = {}^{m+n}C_r a^r$
Therefore, the coefficient of a^m is
$$\mathrm{m+nC_m} = \frac{(m+n)!}{m!(m+n-m)!} = \frac{(m+n)!}{m!n!}$$
Assuming that a^n occurs in the $(k+1)^{\text{th}}$ term of the expansion $(1+a)^{m+n}$, we obtain
$T_{k+1} = {}^{m+n}C_k(1)^{m+n-k}(a)^k = {}^{m+n}C_k(a)^k$
Therefore, the coefficient of a^n is
$$\mathrm{m+nC_n} = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!}$$
Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1+a)^{m+n}$ are equal.

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Q10
The coefficients of the $(r-1)^{\text{th}}$, r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

Answer.
It is known that $(k+1)^{\text{th}}$ term, (T_{k+1}), in the binomial expansion of $(a+b)^n$ is given by
$T_{k+1} = {}^nC_k a^{n-k} b^k$
$(r-1)^{\text{th}}$ term in the expansion of $(x+1)^n$ is $T_{r-1} = {}^nC_{r-2}(x)^{n-(r-2)}(1)^{(r-2)} = {}^nC_{r-2}x^{n-r+2}$
r^{th} term in the expansion of $(x+1)^n$ is $T_r = {}^nC_{r-1}(x)^{n-(r-1)}(1)^{(r-1)} = {}^nC_{r-1}x^{n-r+1}$
Therefore, the coefficients of the $(r-1)^{\text{th}}$, r^{th}, and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are ${}^nC_{r-2}, {}^nC_{r-1}$, and ${}^nC_r$ respectively.
since these coefficients are in the ratio } 1 : 3 : 5, we obtain
$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3}$ and $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$
$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!}$$
$$= \frac{(r-1)(r-2)!(n-r+1)!}{n!}$$
$$= \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!}$$
$$= \frac{r-1}{n-r+2}$$
$\therefore \frac{r-1}{n-r+2} = \frac{1}{3}$
$\Rightarrow 3r - 3 = n - r + 2$
$\Rightarrow n - 4r + 5 = 0$
$\frac{{}^nC_{r-1}=}{{}^nC_r} = \frac{n!}{(r-1)!(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!}$
$= \frac{r}{n-r+1}$
$\therefore \frac{r}{n-r+1} = \frac{3}{5}$
$\Rightarrow 5r = 3n - 3r + 3$

$\Rightarrow 3\mathrm{n}-8\mathrm{r}+3=0$

Multiplying (1) by 3 and subtracting it from (2), we obtain

Putting the value of r in (1), we obtain

$n-12+5=0$

$\Rightarrow n=7$

Thus, $n=7$ and $r=3$

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Q11

Prove that the coefficient of x^{n} in the expansion of $(1+x)^{2}$ istwice the coefficient of x^{n} in the expansion of $(1+x)^{2n-1}$.

Answer.

It is known that $(r+1)^{\text{th}}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^{n}$ is given by

$T_{r+1}={}^{n}C_{r}a^{n-t}b^{r}$

Assuming that x^{n} occurs in the $(r+1)^{\text{th}}$ term of the expansion of $(1+x)^{2n}$, we obtain

$T_{r+1}={}^{2n}C_{r}(1)^{2n-r}(x)^{r}={}^{2n}C_{r}(x)^{r}$

Therefore, the coefficient of x^{n} in the expansion of $(1+x)^{2n}$ is

$^{2n}C_{n}=\frac{(2n)!}{n!(2n-n)!}=\frac{(2n)!}{n!n!}=\frac{(2n)!}{(n!)^{2}}$

Assuming that x^{n} occurs in the $(k+1)^{\text{th}}$ term of the expansion $(1+x)^{2n-1}$, we obtain

$T_{k+1}={}^{2n-1}C_{k}(1)^{2n-1-k}(x)^{k}={}^{2n-1}C_{k}(x)^{k}$

x^{n} in the expansion of $(1+x)^{2n-1}$

$$2n-1=\frac{(2n-1)!}{n!(2n-1-n)!}=\frac{(2n-1)!}{n!(n-1)!}$$

$$=\frac{2n\cdot(2n-1)!}{2n.\,n!(n-1)!}=\frac{(2n)!}{2.\,n!n!}=\frac{1}{2}\left[\frac{(2n)!}{(n!)^{2}}\right]$$

From (1) and (2), it is observed that

$\frac{1}{2}\left({}^{2n}\mathrm{C}_{\mathrm{n}}\right)={}^{2n-1}\mathrm{C}_{\mathrm{n}}$

$\Rightarrow{}^{2n}\mathrm{C}_{\mathrm{n}}=2\left({}^{2n-1}\mathrm{C}_{\mathrm{n}}\right)$

Therefore, the coefficient of x^{n} in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^{n} in the expansion of $(1+x)^{2n-1}$.

Hence, proved.

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Q12

Find a positive value of m for which the coefficient of x^{2} in the expansion $(1+x)^{n}$ is 6.

Answer.

It is known that $(r+1)^{\text{th}}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^{n}$ is given by

$T_{r+1}={}^{n}C_{r}a^{n-t}b^{r}$

$T_{r+1} = {}^nC_r a^{n-r} b^r$
Assuming that x^2 occurs in the $(r+1)^{\text{th}}$ term of the expansion $(1+x)^m$, we obtain
$T_{r+1} = {}^m C_r (1)^{m-r} (x)^r = {}^m C_r (x)^r$
Comparing the indices of x in x^2 and in T_{r+1}, we obtain
$r = 2$
Therefore, the coefficient of x^2 is ${}^m C_2$
It is given that the coefficient of x^2 in the expansion $(1+x)^m$ is 6
$\therefore {}^{\mathrm{m}} \mathrm{C}_2 = 6$
$\Rightarrow \frac{\mathrm{m}!}{2!(\mathrm{m}-2)!} = 6$
$\Rightarrow \frac{\mathrm{m}(\mathrm{m}-1)(\mathrm{m}-2)!}{2 \times (\mathrm{m}-2)!} = 6$
$\Rightarrow \mathrm{m}(\mathrm{m}-1) = 12$
$\Rightarrow \mathrm{m}^2 - \mathrm{m} - 12 = 0$
$\Rightarrow \mathrm{m}^2 - 4\mathrm{m} + 3\mathrm{m} - 12 = 0$
$\Rightarrow \mathrm{m}(\mathrm{m}-4) + 3(\mathrm{m}-4) = 0$
$\Rightarrow (m-4)(m+3) = 0$
$\Rightarrow (m-4) = 0$ or $(m+3) = 0$
$\Rightarrow m = 4$ or $m = -3$
Thus, the positive value of m, for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6, is 4 .

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Miscellaneous Exercise

Q1
Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are $729, 7290$ and 30375, respectively.

Answer.
It is known that $(r+1)^{\text{th}}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by
$T_{r+1} = {}^n C_r a^{n-t} b^r$
The first three terms of the expansion are given as $729, 7290$, and 30375 respectively.
Therefore, we obtain
$T_1 = {}^n C_0 a^{n-0} b^0 = a^n = 729$
$T_2 = {}^n C_1 a^{n-2} b^1 = n a^{n-1} b = 7290$
$T_3 = {}^n C_2 a^{n-2} b^2 = \frac{n(n-1)}{2} a^{n-2} b^2 = 30375 \qquad \ldots(3)$
Dividing (2) by (1), we obtain
$\mathrm{na}^{\mathrm{n}-1} \mathrm{ba}^{\mathrm{n}} = \frac{7290}{729}$
$\Rightarrow \mathrm{nba} = 10$
Dividing (3) by (2), we obtain
$n(n-1) a^{n-2} b^2 2 n a^{n-1} b = \frac{30375}{7290}$

$\Rightarrow (n-1)b2a = \frac{30375}{7290}$

$\Rightarrow (n-1)ba = \frac{30375\times 2}{7290} = \frac{25}{3}$

$\Rightarrow \mathrm{nba} - \frac{\mathrm{b}}{\mathrm{a}} = \frac{25}{3}$

$\Rightarrow 10 - \mathrm{ba} = \frac{25}{3}$

$\Rightarrow \mathrm{ba} = 10 - \frac{25}{3} = \frac{5}{3}$

From (4) and (5), we obtain

$\mathrm{n} \cdot \frac{5}{3} = 10$

$\Rightarrow \mathrm{n} = 6$

Substituting $n = 6$ in equation (1), we obtain

$a^6 = 729$

$\Rightarrow a = \sqrt[6]{729} = 3$

From (5), we obtain

$\frac{b}{3} = \frac{5}{3} \Rightarrow b = 5$

Thus, $a = 3, b = 5,$ and $n = 6$

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Q2 Find a if the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$

Answer.

It is known that $(r+1)^{\text{th}}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by

$T_{r+1} = {}^n C_r a^{n-T} b^r$

Assuming that x^2 occurs in the $(r+1)^{\text{th}}$ term in the expansion of $(3+ax)^9$, we obtain

$T_{r+1} = {}^9 C_r (3)^{9-t} (ax)^r = {}^9 C_r (3)^{9-r} a' x^r$

Thus, the coefficient of x^2 is

${}^9C_2(3)^{9-2}a^2 = \frac{9!}{2!7!}(3)^7 a^2 = 36(3)^7 a^2$

Assuming that x^3 occurs in the $(k+1)^{\text{th}}$ term in the expansion of $(3+ax)^9$, we obtain

$T_{k+1} = {}^9 C_k (3)^{9-k} (ax)^k = {}^9 C_k (3)^{9-k} a^k x^k$

Thus, the coefficient of x^3 is

${}^9\mathrm{C}_3(3)^{9-3}\mathrm{a}^3 = \frac{9!}{3!6!}(3)^6\mathrm{a}^3 = 84(3)^6\mathrm{a}^3$

$84(3)^6\mathrm{a}^3 = 36(3)^7\mathrm{a}^2$

$\Rightarrow 84\mathrm{a} = 36 \times 3$

$\Rightarrow \mathrm{a} = \frac{36\times 3}{84} = \frac{104}{84}$

$\Rightarrow \mathrm{a} = \frac{9}{7}$

Thus, the required value of a is $\frac{9}{7}$

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Q3 Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$

Answer.

$$(1+2x)^6 = {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$
$$= 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6$$
$$= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$$
$$(1-x)^7 = {}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7$$
$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$
$$\therefore (1+2x)^6(1-x)^7$$
$$= \left(1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6\right)\left(1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7\right)$$

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Q4

If a and b are distinct integers, prove that $a-b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

Answer.

In order to prove that $(a-b)$ is a factor of $\left(a^n - b^n\right)$, it has to be proved that $a^n - b^n = k(a-b)$, where k is some natural number

It can be written that, $a = a - b + b$

$$\therefore a^n = (a-b+b)^n = [(a-b)+b]^n$$
$$= {}^nC_0(a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + {}^nC_nb^n$$
$$= (a-b)^n + {}^nC_1(a-b)^{n-1}b + \ldots + {}^nC_{n-1}(a-b)b^{n-1} + b^n$$
$$\Rightarrow a^n - b^n = (a-b)\left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$$
$$\Rightarrow a'' - b^n = k(a-b)$$

where, $k = \left[(a-b)^{n-1} + {}^nC_1(a-b)^{n-2}b + \ldots + {}^nC_{n-1}b^{n-1}\right]$ is a natural number

This shows that $(a-b)$ is a factor of $\left(a^n - b^n\right)$, where n is a positive integer.

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Q5 Evaluate

$(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

Answer.

Firstly, the expression $(a+b)^6 - (a-b)^6$ is simplified by using Binomial Theorem.

This can be done as

$$(a+b)^6 = {}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^3 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5a^1b^5 + {}^6C_6b^6$$
$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$
$$(a-b)^6 = {}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5a^1b^5 + {}^6C_6b^6$$
$$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$(\sqrt{3}+\sqrt{2})^6-(\sqrt{3}-\sqrt{2})^6 = 2\left[6(\sqrt{3})^5(\sqrt{2})+20(\sqrt{3})^3(\sqrt{2})^3+6(\sqrt{3})(\sqrt{2})^5\right]$$
$$= 2[54\sqrt{6}+120\sqrt{6}+24\sqrt{6}]$$
$$= 2\times 198\sqrt{6}$$
$$= 396\sqrt{6}$$

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Q6 Find the value of
$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4$

Answer.
Firstly, the expression $(x+y)^4+(x-y)^4$ is simplified by using Binomial Theorem.
This can be done as
$$(x+y)^4 = {}^4C_0x^4+{}^4C_1x^3y+{}^4C_2x^2y^2+{}^+C_3xy^3+{}^4C_4y^4$$
$$= x^4+4x^3y+6x^2y^2+4xy^3+y^4$$
$$(x-y)^4 = {}^4C_0x^4-{}^4C_1x^3y+{}^4C_2x^2y^2-{}^4C_3xy^3+{}^4C_4y^4$$
$$= x^4-4x^3y+6x^2y^2-4xy^3+y^4$$
$$\therefore (x+y)^4+(x-y)^4 = 2\left(x^4+6x^2y^2+y^4\right)$$
Putting $x=a^2$ and $y=\sqrt{a^2-1}$, we obtain
$$\left(a^2+\sqrt{a^2-1}\right)^4+\left(a^2-\sqrt{a^2-1}\right)^4 = 2\left[\left(a^2\right)^4+6\left(a^2\right)^2\left(\sqrt{a^2-1}\right)^2+\left(\sqrt{a^2-1}\right)^4\right]$$
$$= 2\left[a^8+6a^4\left(a^2-1\right)+\left(a^2-1\right)^2\right]$$
$$= 2\left[a^8+6a^6-6a^4+a^4-2a^2+1\right]$$
$$= 2\left[a^8+6a^6-5a^4-2a^2+1\right]$$
$$= 2a^8+12a^6-10a^4-4a^2+2$$

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Q7 Find an approximation of $(0.99)^5$ using the first three terms of its expansion

Answer.

$$0.99 = 1-0.01$$
$$\therefore (0.99)^5 = (1-0.01)^5$$
$$= {}^5C_0(1)^5-{}^5C_1(1)^4(0.01)+{}^5C_2(1)^3(0.01)^2$$
$$= 1-5(0.01)+10(0.01)^2$$
$$= 1-0.05+0.001$$
$$= 0.951$$
$$= 0.951$$

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Q8

Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

Answer.

It is known that in the expansion $(a+b)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ term.

$${}^nC_4(\sqrt[4]{2})^{n-1}\left(\frac{1}{\sqrt[4]{3}}\right)^4={}^nC_4\frac{(\sqrt[4]{2})^n}{(\sqrt[4]{2})^4}\cdot\frac{1}{3}={}^nC_4\frac{(\sqrt[4]{2})^n}{2}\cdot\frac{1}{3}=\frac{n!}{6.4!(n-4)!}(\sqrt[4]{2})^n$$

$${}^nC_{n-4}(\sqrt[4]{2})^4\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}={}^nC_{n-1}\cdot 2\cdot\frac{(\sqrt[4]{3})^4}{(\sqrt[4]{3})^n}={}^nC_{n-1}\cdot 2\cdot\frac{3}{(\sqrt[4]{3})^n}=\frac{6n!}{(n-4)!4!}\cdot\frac{1}{(\sqrt[4]{3})^n}$$

$$\frac{n!}{6.4!(n-4)!}(\sqrt[4]{2})^n:\frac{6n!}{(n-4)!!4!}\cdot\frac{1}{(\sqrt[4]{3})^n}=\sqrt{6}:1$$

$$\Rightarrow\frac{(\sqrt[4]{2})^n}{6}:\frac{6}{(\sqrt[4]{3})^n}=\sqrt{6}:1$$

$$\Rightarrow\frac{(\sqrt[4]{2})^n}{6}\times\frac{(\sqrt[4]{3})^n}{6}=\sqrt{6}$$

$$\Rightarrow(\sqrt[4]{6})^n=36\sqrt{6}$$

$$\Rightarrow 6^{\frac{n}{4}}=6^{\frac{5}{2}}$$

$$\Rightarrow\frac{n}{4}=\frac{5}{2}$$

$$\Rightarrow n=4\times\frac{5}{2}=10$$

Thus, the value of n is 10.

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Q9 Expand using binomial theorem $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x\neq 0$

Answer.

It is known that $(r+1)^{\text{th}}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by $T_{r+1}={}^nC_ia^{n-1}b'$

Assuming that a^m occurs in the $(r+1)^{\text{th}}$ term of the expansion $(1+a)^{m+n}$, we obtain $T_{r+1}={}^{m+a}C_r(1)^{m+a-1}(a)^r={}^{m+n}C_ra^r$

Therefore, the coefficient of a^m is

$$m+a_m=\frac{(m+n)!}{m!(m+n-m)!}=\frac{(m+n)!}{m!n!}$$

Therefore, the coefficient of a^n is

$$m+nC_n=\frac{(m+n)!}{n!(m+n-n)!}=\frac{(m+n)!}{n!m!}$$

$${}^{m+n}C_n=\frac{(m+n)!}{n!(m+n-n)!}=\frac{(m+n)!}{n!m!}$$

Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1+a)^{m+n}$ are equal.

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Q10 Find the expression $\left(3x^2-2ax+3a^2\right)^3$ using binomial theorem

Answer.

It is known that $(k+1)^{\text{th}}$ term, (T_{k+1}), in the binomial expansion of $(a+b)^n$ is given b

$T_{k+1} = ^n C_1 a^{n-k} b^k$

Therefore, $(r-1)^{\text{th}}$ term in the expansion of $(x+1)^n$ is

$T_{r-1} = ^n C_{1-2}(x)^{n-(r-2)}(1)^{(r-2)} = ^n C_{r-2} x^{n-r+2}$

$\left[\left(3x^2 - 2ax\right) + 3a^2\right]^3$

$= ^3 C_0\left(3x^2 - 2ax\right)^3 + ^3 C_1\left(3x^2 - 2ax\right)^2\left(3a^2\right) + ^3 C_2\left(3x^2 - 2ax\right)\left(3a^2\right)^2 + ^3 C_3\left(3a^2\right)^3$

$= \left(3x^2 - 2ax\right)^3 + 3\left(9x^4 - 12ax^3 + 4a^2x^2\right)\left(3a^2\right) + 3\left(3x^2 - 2ax\right)\left(ga^4\right) + 27a^6$

$= \left(3x^2 - 2ax\right)^3 + 81a^2x^4 - 108a^3x^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6$

$= \left(3x^2 - 2ax\right)^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$

Again by using Binomial Theorem, we obtain

$\left(3x^2 - 2ax\right)^3$

$= ^3 C_0\left(3x^2\right)^3 - ^3 C_1\left(3x^2\right)^2(2ax) + ^3 C_2\left(3x^2\right)(2ax)^2 - ^3 C_3(2ax)^3$

$= 27x^6 - 3\left(9x^4\right)(2ax) + 3\left(3x^2\right)\left(4a^2x^2\right) - 8a^3x^3$

$= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3$

$\left(3x^2 - 2ax + 3a^2\right)^3$

$= 27x^6 - 54ax^5 + 36a^2x^4 - 8a^3x^3 + 81a^2x^4 - 108a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$

$= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$