Class : 11th
Subject : Maths
Chapter : 5
Chapter Name : Complex Number And Quadratic Equations

Exercise 5.1

Q1 Express each of the complex number in the form a + ib.
$(5i)\left(-\frac{3}{5}i\right)$

Answer.

$$\begin{aligned}(5i)\left(\frac{-3}{5}i\right) &= -5 \times \frac{3}{5} \times i \times i \\ &= -3i^2 \\ &= -3(-1) \\ &= 3\end{aligned}$$

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Q2 Express each of the complex number in the form a + ib.
$i^9 + i^{19}$

Answer.

$$\begin{aligned}& i^9 + i^{19} = i^{4x-1} + i^{4x+1} \\ & = \left(i^4\right)^2 \cdot j + \left(j^4\right)^4 \cdot i^3 \\ & = 1 \times i + | \times (-i) \\ & = i + (-i) \\ & = 0\end{aligned}$$

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Q3 Express each of the complex number in the form a + ib.
i^{-39}

Answer.

$$I^{-14} = f^{-1 \times 4-3} = \left(i^4\right)^{-9} \cdot l^{-3}$$

$$
\begin{aligned}
&= (1)^{-9} - F^{-3} \\
&= \frac{1}{p^3} = \frac{1}{-i} \\
&= \frac{-1}{i} \times \frac{j}{i} \\
&= \frac{-i}{i^2} = \frac{-i}{-1} = i
\end{aligned}
$$

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Q4 Express each of the complex number in the form a + ib.
$3(7+i7)+i(7+i7)$

Answer.

$$
\begin{aligned}
3(7+i7)+i(7+i7) &= 21+21i+7i+7i^2 \\
&= 21+28i+7\times(-1) \\
&= 14+28i
\end{aligned}
$$

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Q5 Express each of the complex number in the form a + ib.
$(1-i)-(-1+i6)$

Answer.

$$
\begin{aligned}
(1-i)-(-1+i6) &= 1-j+1-6i \\
&= 2-7i
\end{aligned}
$$

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Q6 Express each of the complex number in the form a + ib.
$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer.

$$
\begin{aligned}
&\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right) \\
&= \frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}l \\
&= \left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right) \\
&= \frac{-19}{5}+i\left(\frac{-21}{10}\right) \\
&= \frac{-19}{5}-\frac{21}{10}i
\end{aligned}
$$

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Q7 Express each of the complex number in the form a + ib.
$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Answer.
$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$
$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$
$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+\left(\frac{7}{3}+\frac{1}{3}-1\right)$
$=\frac{17}{3}+i\frac{5}{3}$

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Q8 Express each of the complex number in the form a + ib.
$(1-i)^4$

Answer.
$$\begin{aligned}(1-i)^t &= \left[(1-i)^2\right]^2\\ &= \left[1^2+i^2-2i\right]^2\\ &= [1-1-2i]^2\\ &= (-2i)^2\\ &= (-2i)\times(-2i)\\ &= 4i^2=-4\end{aligned}$$

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Q9 Express each of the complex number in the form a + ib.
$\left(\frac{1}{3}+3i\right)^3$

Answer.
$$\begin{aligned}\left(\frac{1}{3}+3i\right)^3 &= \left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)\\ &= \frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)\\ &= \frac{1}{27}+27(-i)+i+9i^2\\ &= \frac{1}{27}-27i+i-9\\ &= \left(\frac{1}{27}-9\right)+i(-27+1)\\ &= \frac{-242}{27}-26i\end{aligned}$$

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Q10 Express each of the complex number in the form a + ib.
$\left(-2-\frac{1}{3}i\right)^3$

Answer.

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^2+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

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Q11 Find the multiplicative inverse of each of the complex numbers given
$4-3i$

Answer.

Let $z=4-3i$

Then, $\bar{z}=4+3i$ and $|z|^2=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of $4-3i$ is given by

$z^{-1}=\frac{z}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$

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Q12 Find the multiplicative inverse of each of the complex numbers given
$\sqrt{5}+3i$

Answer.

Let $z=\sqrt{5}+3i$

Then, $\bar{z}=\sqrt{5}-3i$ and $|z|^2=(\sqrt{5})^2+3^2=5+9=14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

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Q13 Find the multiplicative inverse of each of the complex numbers given
$-i$

Answer.
Let $z=-i$
Then, $\bar{z}=i$ and $|z|^2=1^2=1$
Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

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Q14 Express the following expression in the form of a + ib

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$=\frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i}$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

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Exercise 5.2

Q1 Find the modulus and the arguments of each of the complex numbers
$z=-1-i\sqrt{3}$

Answer.
$z=-1-i\sqrt{3}$
Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$
On squaring and adding, we obtain
$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})^2$
$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+3$
$\Rightarrow$ Modulus $=2$

$\therefore 2\cos\theta=-1$ and $2\sin\theta=-\sqrt{3}$
$\Rightarrow \cos\theta=\frac{-1}{2}$ and $\sin\theta=\frac{-\sqrt{3}}{2}$
since both the values of $\sin\theta$ and $\cos\theta$ are negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant,
Argument $=-\left(\pi-\frac{\pi}{3}\right)=\frac{-2\pi}{3}$
Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Page : 108 , Block Name : Exercise 5.2

Q2 Find the modulus and the arguments of each of the complex numbers $z=-\sqrt{3}+i$

Answer.
$z=-\sqrt{3}+i$
Let $r\cos\theta=-\sqrt{3}$ and $r\sin\theta=1$
On squaring and adding, we obtain
$r^2\cos^2\theta+r^2\sin^2\theta=(-\sqrt{3})^2+1^2$
$\Rightarrow r^2=3+1=4$
$\Rightarrow r=\sqrt{4}=2$
$\therefore$ Modulus $=2$
$\therefore 2\cos\theta=-\sqrt{3}$ and $2\sin\theta=1$
$\Rightarrow \cos\theta=\frac{-\sqrt{3}}{2}$ and $\sin\theta=\frac{1}{2}$
$\therefore \theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$
Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Page : 108 , Block Name : Exercise 5.2

Q3 Convert each of the complex numbers given in the polar form:
$1-i$

Answer.
$1-i$
Let $r\cos\theta=1$ and $r\sin\theta=-1$
On squaring and adding, we obtain
$r^2\cos^2\theta+r^2\sin^2\theta=1^2+(-1)^2$
$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$
$\Rightarrow r^2=2$
$\Rightarrow r=\sqrt{2}$
$\therefore \sqrt{2}\cos\theta=1$ and $\sqrt{2}\sin\theta=-1$
$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$ and $\sin\theta=-\frac{1}{\sqrt{2}}$

$\therefore 1-i=r\cos\theta+r\sin\theta=\sqrt{2}\cos\left(-\frac{\pi}{4}\right)+i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$ This is the required polar form.

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Q4 Convert each of the complex numbers given in the polar form:
$-1+i$

Answer.
$-1+i$
Let $r\cos\theta=-1$ and $r\sin\theta=1$
On squaring and adding, we obtain
$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+1^2$
$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$
$\Rightarrow r=\sqrt{2}$ [Conventionally, $r>0$]
$\therefore \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$
$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}}$ and $\sin\theta=\frac{1}{\sqrt{2}}$
$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$ [As θ lies in the II quadrant]
It can be written,
$\therefore -1+i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{3\pi}{4}+i\sqrt{2}\sin\frac{3\pi}{4}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
This is the required polar form.

Page : 108 , Block Name : Exercise 5.2

Q5 Convert each of the complex numbers given in the polar form:

$-1-i$

Answer.

$-1-i$

Let $r\cos\theta=-1$ and $r\sin\theta=-1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+(-1)^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$

$\Rightarrow r^2=2$

$\Rightarrow r=\sqrt{2}$

$\therefore \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=-1$

$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}}$ and $\sin\theta=-\frac{1}{\sqrt{2}}$

$\therefore \theta=-\left(\pi-\frac{\pi}{4}\right)=-\frac{3\pi}{4}$ [As θ lies in the III quadrant]

$\therefore -1-i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{-3\pi}{4}+i\sqrt{2}\sin\frac{-3\pi}{4}=\sqrt{2}\left(\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4}\right)$ This is the required polar form.

Page : 108 , Block Name : Exercise 5.2

Q6 Convert each of the complex numbers given in the polar form:

-3

Answer.

Let $r\cos\theta=-3$ and $r\sin\theta=0$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-3)^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=9$

$\Rightarrow r^2=9$

$\Rightarrow r=\sqrt{9}=3$

$\therefore 3\cos\theta=-1$ and $3\sin\theta=0$

$\Rightarrow \theta=\pi$

$\therefore -3=r\cos\theta+ir\sin\theta=3\cos\pi+B\sin\pi=3(\cos\pi+i\sin\pi)$

This is the required polar form.

Page : 108 , Block Name : Exercise 5.2

Q7 Convert each of the complex numbers given in the polar form:

$\sqrt{3}+i$

Answer.

$\sqrt{3}+i$

Let $r\cos\theta=\sqrt{3}$ and $r\sin\theta=1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(\sqrt{3})^2+1^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=3+1$

$\Rightarrow r^2=4$

$\Rightarrow r=\sqrt{4}=2\quad$ [Conventionally, $r>0$]

$\therefore 2\cos\theta=\sqrt{3}$ and $2\sin\theta=1$

$\Rightarrow \cos\theta=\frac{\sqrt{3}}{2}$ and $\sin\theta=\frac{1}{2}$

$\therefore \theta=\frac{\pi}{6}$

$\therefore \sqrt{3}+i=r\cos\theta+ir\sin\theta=2\cos\frac{\pi}{6}+i2\sin\frac{\pi}{6}=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

Page : 108 , Block Name : Exercise 5.2

Q8 Convert each of the complex numbers given in the polar form:
i

Answer.

Let $r\cos\theta=0$ and $r\sin\theta=1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=0^z+1^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1$

$\Rightarrow r^2=1$

$\Rightarrow r=\sqrt{1}=1$

$\therefore \cos\theta=0$ and $\sin\theta=1$

$\therefore \theta=\frac{\pi}{2}$

$\therefore i=r\cos\theta+ir\sin\theta=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

This is the required polar form.

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Exercise 5.3

Q1 Solve each of the following equations:
$x^2+3=0$

Answer.

The given quadratic equation is $x^2+3=0$

On comparing the given equation with $ax^2+bx+c=0,$ we obtain

$a=1, b=0,$ and $c=3$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=0^2-4\times1\times3=-12$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{\mathbf{D}}}{2a}=\frac{\pm\sqrt{-12}}{2\times1}=\frac{\pm\sqrt{12}i}{2}$$
$$=\frac{\pm2\sqrt{3}i}{2}=\pm\sqrt{3}i$$

Page : 109 , Block Name : Exercise 5.3

Q2 Solve each of the following equations:
$2x^2+x+1=0$

Answer.

The given quadratic equation is $2x^2+x+1=0$

On comparing the given equation with $ax^2+bx+c=0,$ we obtain

$a=2, b=1,$ and $c=1$

Ther efore, the discriminant of the given equation is

$\mathrm{D}=b^2-4\mathrm{ac}=1^2-4\times2\times1=1-8=-7$

Therefore, the requlred solutlons are

$$\frac{-b\pm\sqrt{\mathrm{D}}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times2}=\frac{-1\pm\sqrt{7}i}{4}$$

Page : 109 , Block Name : Exercise 5.3

Q3 Solve each of the following equations:
$x^2+3x+9=0$

Answer.

The given quadratic equation is $x^2+3x+9=0$

On comparing the given equation with $ax^2+bx+c=0,$ we obtain

$a=1, b=3,$ and $c=9$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=3^2-4\times1\times9=9-36=-27$

Ther efore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-27}}{2(1)}=\frac{-3\pm3\sqrt{-3}}{2}=\frac{-3\pm3\sqrt{3}i}{2}$$

Page : 109 , Block Name : Exercise 5.3

Q4 Solve each of the following equations:
$-x^2 + x - 2 = 0$

Answer.
The given quadratic equation is $-x^2 + x - 2 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we obtain
$a = -1, b = 1$, and $c = -2$
Therefore, the discriminant of the given equation is
$D = b^2 - 4ac = 1^2 - 4 \times (-1) \times (-2) = 1 - 8 = -7$
Therefore, the required solutions are
$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times (-1)} = \frac{-1 \pm \sqrt{7} i}{-2}$

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Q5 Solve each of the following equations:
$x^2 + 3x + 5 = 0$

Answer.
The given quadratic equation is $x^2 + 3x + 5 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we obtain
$a = 1, b = 3$, and $c = 5$
Therefore, the discriminant of the glven equation is
$D = b^2 - 4ac = 3^2 - 4 \times 1 \times 5 = 9 - 20 = -11$
Therefore, the requlred solutions are
$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11} i}{2}$

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Q6 Solve each of the following equations:
$x^2 - x + 2 = 0$

Answer.
The given quadratic equation is $x^2 - x + 2 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we obtain
$a = 1, b = -1$, and $c = 2$
Therefore, the discriminant of the given equation is

Therefore, the discriminant of the given equation is
$D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7$

Therefore, the required solutions are

$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-1)\pm\sqrt{-7}}{2\times1}=\frac{1\pm\sqrt{7}i}{2}$

Page : 109 , Block Name : Exercise 5.3

Q7 Solve each of the following equations:
$\sqrt{2}x^2+x+\sqrt{2}=0$

Answer.

The given quadratic equation is $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing the given equation with $ax^2+bx+c=0,$ we obtain
$a=\sqrt{2}, b=1,$ and $c=\sqrt{2}$

Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$

Therefore, the required solutions are

$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}$

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Q8 Solve each of the following equations:
$\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Answer.

The given quadratic equation is $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

On comparing the given equation with $ax^2+bx+c=0,$ we obtain
$a=\sqrt{3}, b=-\sqrt{2},$ and $c=3\sqrt{3}$

Therefore, the discriminant of the given equation is
$\mathrm{D}=b^2-4ac=(-\sqrt{2})^2-4(\sqrt{3})(3\sqrt{3})=2-36=-34$

Therefore, the required solutions are

$\frac{-b\pm\sqrt{\mathrm{D}}}{2a}=\frac{-(-\sqrt{2})\pm\sqrt{-34}}{2\times\sqrt{3}}=\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}} \quad [\sqrt{-1}=i]$

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Q9 Solve each of the following equations:
$x^2+x+\frac{1}{\sqrt{2}}=0$

Answer.

The given quadratic equation is

This equation can also be written as $\sqrt{2}x^2+\sqrt{2}x+1=0$

On comparing this equation with $ax^2+bx+c=0,$ we obtain

$a=\sqrt{2}, b=\sqrt{2},$ and $c=1$

$\therefore$ Discriminant $(D)=b^2-4ac=(\sqrt{2})^2-4\times(\sqrt{2})\times 1=2-4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2(1-2\sqrt{2})}}{2\sqrt{2}}=\frac{-1\pm(\sqrt{2\sqrt{2}-1})i}{2}$$

$$=\left(\frac{-\sqrt{2}\pm\sqrt{2}(\sqrt{2\sqrt{2}-1})}{2\sqrt{2}}\right)$$

Page : 109 , Block Name : Exercise 5.3

Q10 Solve each of the following equations:

$x^2+\frac{x}{\sqrt{2}}+1=0$

Answer.

The given quadratic equation is

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing this equation with $ax^2+bx+c=0,$ we obtain

$a=\sqrt{2}, b=1,$ and $c=\sqrt{2}$

$\therefore$ Discriminant $(D)=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}\quad[\sqrt{-1}=i]$$

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Miscellaneous Exercise

Q1 Evaluate

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

Answer.

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

$$= \left[i^{4x+42} + \frac{1}{i^{4x+1}}\right]^3$$

$$= \left[\left(i^4\right)^4 - i^2 + \frac{1}{\left(i^4\right)^6 \cdot i}\right]^3$$

$$= \left[t^2 + \frac{1}{i}\right]^3$$

$$= \left[-1 + \frac{1}{i} \times \frac{i}{i}\right]^3$$

$$= \left[-1 + \frac{j}{i^2}\right]^3$$

$$= \left[-1 + \frac{j}{j^2}\right]^3$$

$$= [-1 - i]^3$$

$$= (-1)^3[1 + i]^3$$

$$= -\left[1 + i^5 + 3i + 3i^2\right]$$

$$= [1 - i + 3i - 3]$$

$$= -[-2 + 2i]$$

$$= 2 - 2i$$

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Q2

For any two complex numbers z_1 and z_2 , prove that

$\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$

Answer.

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1(x_2 + iy_2) + iy_1(x_2 + ty_2)$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2$$

$$= (x_1x_2 - y_1y_2) + i(x_2y_2 + y_1x_2)$$

$$\Rightarrow \operatorname{Re}(z_1 z_2) = x_1x_2 - y_1y_2$$

$$\Rightarrow \operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{lm} z_1 \operatorname{Im} z_2$$

Hence, proved.

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Q3 Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to standard form

Answer.

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right]=\left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$=\left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right]=\frac{33+31i}{28-10i}=\frac{33+31i}{2(14-5i)}$$

$$=\frac{(33+31i)}{2(14-5i)}\times\frac{(14+5i)}{(14+5i)}\quad[\text{ On multiplying numerator and denominator by }(14+5i)]$$

$$=\frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]}=\frac{307+599i}{2\left(196-25i^2\right)}$$

$$=\frac{307+599i}{2(221)}=\frac{307+599i}{442}=\frac{307}{442}+\frac{599i}{442}$$

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Q4 if $x-iy=\sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$

Answer.

$$\mathrm{x}-\mathrm{iy}=\sqrt{\frac{\mathrm{a}-\mathrm{ib}}{\mathrm{c}-\mathrm{id}}}$$

$$=\sqrt{\frac{\mathrm{a}-\mathrm{ib}}{\mathrm{c}-\mathrm{id}}\times\frac{\mathrm{c}+\mathrm{id}}{\mathrm{c}+\mathrm{id}}}[\text{On multiplying numerator and denomin ator by }(\mathrm{c}+\mathrm{id})]$$

$$=\sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

$$\therefore(x-iy)^2=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2-y^2-2ixy=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\therefore(x-iy)^2=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2-y^2-2ixy=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$x^2-y^2=\frac{ac+bd}{c^2+d^2},-2xy=\frac{ad-bc}{c^2+d^2}$$

$$\left(x^2+y^2\right)^2=\left(x^2-y^2\right)^2+4x^2y^2\qquad[\text{Using (i)}]$$

$$=\left(\frac{ac+bd}{c^2+d^2}\right)^2+\left(\frac{ad-bc}{c^2+d^2}\right)^2$$

$$=\frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{\left(c^2+d^2\right)^2}$$

$$=\frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{\left(c^2+d^2\right)^2}$$

$$= \frac{a^2\left(c^2+d^2\right)+b^2\left(c^2+d^2\right)}{\left(c^2+d^2\right)^2}$$
$= \frac{\left(c^2+d^2\right)\left(a^2+b^2\right)}{\left(c^2+d^2\right)^2}$
$= \frac{a^2+b^2}{c^2+d^2}$

Hence, proved.

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Q5

Convert the following in the polar form:

$(i)\, \frac{1+7i}{(2-i)^2}$, (ii) $\frac{1+3i}{1-2i}$

Answer.

(i) Here $z=\frac{1+7i}{(2-i)^2}$

$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$

$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$

$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$

$= -1+i$

$= -1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\left(\cos^2\theta+\sin^2\theta\right) = 1+1$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right) = 2$

$\Rightarrow r^2 = 2$ $\left[\cos^2\theta+\sin^2\theta = 1\right]$

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r>0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\therefore$: $\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

: $z = r\cos\theta + ir\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

(ii) Here $z=\frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$= \dfrac{1+2i+3i-6}{1+4}$

$= \dfrac{-5+5i}{5} = -1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 2$

$\Rightarrow r^2 = 2 \qquad \left[\cos^2\theta + \sin^2\theta = 1\right]$

$\Rightarrow r = \sqrt{2} \qquad [\text{ Conventionally, } r > 0]$

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore z = r\cos\theta + ir\sin\theta$

$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

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Q6 Solve the equation $3x^2 - 4x + \frac{20}{3} = 0$

Answer.

The given quadratic equation is

This equation can also be written as $9x^2 - 12x + 20 = 0$

On comparing this equation with $ax^2 + bx + c = 0,$ we obtain

$a = 9, b = -12,$ and $c = 20$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Therefore, the required solutions are

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Therefore, the required solutions are

$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-12)\pm\sqrt{-576}}{2\times9} = \frac{12\pm\sqrt{576}i}{18}$

$= \frac{12\pm24i}{18} = \frac{6(2\pm4i)}{18} = \frac{2\pm4i}{3} = \frac{2}{3} \pm \frac{4}{3}i$

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Q7 Solve the equation $x^2 - 2x + \frac{3}{2} = 0$

Answer.

$x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -4,$ and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Therefore, the required solutions are

$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}i}{4} \quad [\sqrt{-1} = i]$

$= \frac{2 \pm \sqrt{2}i}{2} = 1 \pm \frac{\sqrt{2}}{2}i$

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Q8 Solve the equation $27x^2 - 10x + 1 = 0$

Answer.

The given quadratic equation is $27x^2 - 10x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 27, b = -10,$ and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}i}{54}$

$= \frac{5 \pm \sqrt{2}i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i$

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Q9 Solve the equation $21x^2 - 28x + 10 = 0$

Answer.

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 21, b = -28,$ and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$\frac{-b \pm \sqrt{\mathrm{D}}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}i}{42}$

$= \frac{28 \pm 2\sqrt{14}i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i$

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Q10 if $z_1 = 2 - i, z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$

Answer.

$z_1 = 2 - i, z_2 = 1 + i$

$\therefore \left|\frac{z_1+z_2+1}{z_1-z_2+1}\right| = \frac{|(2-i)+(1+i)+1}{(2-i)-(1+i)+1}$

$= \frac{4}{2-2i} = \frac{4}{2(1-i)}|$

$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{l^2-i^2}\right|$

$= \left|\frac{2(1+i)}{1+1}\right| \qquad [i^2 = -1]$

$$= \left|\frac{2(1+i)}{2}\right|$$

$= |1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

$\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$ is $\sqrt{2}$

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Q11 if $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Answer.

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$
$$= \frac{x^2+i^2+2xi}{2x^2+1}$$
$$= \frac{x^2-1+i2x}{2x^2+1}$$
$$= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$
$$\therefore a^2 + b^2 = \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2$$
$$= \frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$$
$$= \frac{x^4+1+2x^2}{(2x^2+1)^2}$$
$$= \frac{(x^z+1)^2}{(2x^2+1)^2}$$

$\therefore a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Hence, proved.

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Q12

Let $z_1 = 2 - i, z_2 = -2 + i$. Find

(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\text{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

Answer.

$z_1 = 2 - i, z_2 = -2 + i$

$$Z_1 I_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$$

$\overline{Z}_1 = 2 + i$

$\therefore \frac{Z_1 Z_2}{\overline{Z}_1} = \frac{-3+4i}{2+i}$

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+l^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+1li}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = \frac{-2}{5}$

$\frac{1}{z_1 \bar{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$\text{lm}\left(\frac{1}{z_1 \bar{z}_1}\right) = 0$

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Q13 Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Answer.

$z = \frac{1+2i}{1-3i}$,

$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$

$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$

Let $z = r\cos\theta + ir\sin\theta$

i.e., $r\cos\theta = \frac{-1}{2}$ and $r\sin\theta = \frac{1}{2}$

i.e., $r\cos\theta = \frac{-1}{2}$ and $r\sin\theta = \frac{1}{2}$

On squaring and adding, we obtain

$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\Rightarrow r = \frac{1}{\sqrt{2}}$ [Conventionally, $r > 0$]

$\therefore \frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2}$ and $\frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in the II quadrant]

Therefore, the modulus and argument of the given complex number are $\sqrt{2}$ and $\frac{3\pi}{4}$

Therefore, the modulus and argument of the given complex number are $\sqrt{2}$ and $\frac{\pi}{4}$ respectively.

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Q14 Find the real number x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$

Answer.

Let $z=(x-ty)(3+5i)$

$z=3x+5xi-3yi-5yi^2=3x+5xi-3yi+5y=(3x+5y)+i(5x-3y)$

$\therefore \bar{z}=(3x+5y)-i(5x-3y)$

It is given that, $\bar{z}=-6-24i$

$\therefore (3x+5y)-i(5x-3y)=-6-24i$

Equating real and imaginary parts, we obtain

$3x+5y=-6$

$5x-3y=24$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

Putting the value of x in equation (i), we obtain

$3(3)+5y=-6$

$\Rightarrow 5y=-6-15$

$\Rightarrow y=-3$

Thus, the values of x and y are 3 and -3 respectlvely.

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Q15 Find the modulas of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$

Answer.

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-j)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{l^2+1^2}$$

$$\therefore=\frac{4i}{2}=2i$$

$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

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Q16

$(x+iy)^3=u+iv,$ then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$

Answer.

$(x+iy)^3=u+iv$

$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$

$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i=u+iv$

$\Rightarrow x^3-4y^3+3x^2yi-3xy^2=u+iv$

$\Rightarrow \left(x^3-3xy^2\right)+i\left(3x^zy-y^3\right)=u+iv$

$u=x^3-3xy^2, v=3x^2y-y^3$

$$
\begin{aligned}
\therefore \frac{u}{x}+\frac{v}{y} &= \frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y} \\
&= \frac{x\left(x^2-3y^2\right)}{x}+\frac{y\left(3x^2-y^2\right)}{y} \\
&= x^2-3y^2+3x^2-y^2 \\
&= 4\left(x^2-4y^2\right) \\
&= 4\left(x^2-4y^2\right) \\
\therefore \frac{u}{x}+\frac{v}{y} &= 4\left(x^2-y^2\right)
\end{aligned}
$$

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Q17 if α and β are different complex numbers with $|\beta|=1,$ then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Answer.

Let $\mathrm{a}=\mathrm{a}+$ ib and $\beta=x+iy$

It is given that, $|\beta|=1$

$\therefore \sqrt{\mathrm{x}^2+\mathrm{y}^2}=1$

$\Rightarrow \mathrm{x}^2+\mathrm{y}^2=\mathrm{I}$

$$
\begin{aligned}
\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| &= \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right| \\
&= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right| \\
&= \frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\Big|
\end{aligned}
$$

$=\frac{|(x-a)+i(y-b)|}{(1-ax-by)+i(bx-ay)|}$

$=\frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$

$=\frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$

$=\frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$

$=\frac{\sqrt{1+a^2+b^2-2ax-2by}}{}$ $[U \sin g(1)]$

$= \frac{}{\sqrt{1+a^2+b^2-2ax-2by}}[0 \sin y(1)]$

$= 1$

$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

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Q18 Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Answer.

$|1-i|^x = 2^x$

$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$

$\Rightarrow (\sqrt{2})^x = 2^x$

$\Rightarrow 2^{\frac{x}{2}} = 2^x$

$\Rightarrow x = 2x$

$\Rightarrow 2x - x = 0$

$\Rightarrow x = 0$

Thus, 0 is the only integral solution of the given equation. Therefore, the nurnber of non-integral solutions of Y--,e given equation is 0.

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Q19

If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$, then show that

$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2+B^2$

Answer.

$(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\therefore (a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

On squaring both sides, we obtain

$\left(a^2+b^2\right)\left(c^2+c^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right) = A^2+B^2$

Hence, proved.

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Q20

$\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m

Answer.

$\left(\frac{1+i}{1-i}\right)^m$

$$\left(\frac{1+i}{1-i}\right) = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{m} = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{l^2+1^2}\right)^{\prime\prime} = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^{n} = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^{n} = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^{n} = 1$$

$$\Rightarrow i^{n} = 1$$

$\therefore m = 4k,$ where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is $4(= 4 \times 1)$.