Class : 11th
Subject : Maths
Chapter : 4
Chapter Name : Principle of Mathematical Induction

Exercise 4.1

Q1 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1 + 3 + 3^2 + \ldots + 3^{a-1} = \frac{(3^n - 1)}{2}$

Answer.
Let the given statement be $P(n)$, i.e.,
$P(n) : 1 + 3 + 3^2 + \ldots + 3^{n-1} = \frac{(3^n - 1)}{2}$, which is true
For $n = 1$, we have
$P(1) : 1 = \frac{(3' - 1)}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$
Let $P(k)$ be true for some positive integer k, i.e.,
$1 + 3 + 3^2 + \ldots + 3^{k-1} = \frac{\left(3^k - 1\right)}{2}$
We shall now prove that $P(k+1)$ is true.
Consider
$1 + 3 + 3^2 + \ldots + 3^{k-1} + 3^{(k+1)-1}$
$= \left(1 + 3 + 3^2 + \ldots + 3^{k-1}\right) + 3^k$
$= \frac{(3^t - 1)}{2} + 3^k$ [using (i)]
$= \frac{(3^t - 1) + 2.3^k}{2}$
$= \frac{(1+2)3^k - 1}{2}$
$= \frac{3.3^k - 1}{2}$
$= \frac{3^{k+1} - 1}{2}$
Thus, $P(k+1)$ is true whenever $P(k)$ is true.
Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e.,

Q2 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$

Answer.
Let the given statement be $P(n)$, i.e.,

$$1^3+2^3+3^3+\ldots+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

For $n=1$, we have
For $n=1$, we have

$$P(1): 1^3=1=\left(\frac{1(1+1)}{}\right)^2=\left(\frac{1.2}{2}\right)^2=1^2=1$$

, which is true.
Let $P(k)$ be true for some positive integer k, l.e.,

$$1^3+2^3+3^3+\ldots\ldots+k^3=\left(\frac{k(k+1)}{2}\right)^2$$

We shall now prove that $P(k+1)$ is true.
Consider

$$1^3+2^3+3^3+\ldots+k^3+(k+1)^3$$
$$=\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3 \quad [\text{ Using } (i)]$$
$$=\frac{k^2(k+1)^2}{4}+(k+1)^3$$
$$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$$
$$=\frac{(k+1)^2\left\{k^2+4(k+1)\right\}}{4}$$
$$=\frac{(k+1)^2\left\{k^2+4k+4\right\}}{4}$$
$$=\frac{(k+1)^2(k+2)^2}{4}$$
$$=\frac{(k+1)^2(k+1+1)^2}{4}$$
$$=\left(1^3+2^3+3^3+\ldots.+k^3\right)+(k+1)^3=\left(\frac{(k+1)(k+1+1)}{2}\right)^2$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.
Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e.,

Q3 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots n)}=\frac{2n}{(n+1)}$$

Answer.

Let the given statement be $P(n)$, i.e.,

$\mathrm{P(n)}: 1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots n}=\frac{2n}{n+1}$

For $n=1$, we have

$P(1): 1=\frac{2.1}{1+1}=\frac{2}{2}=1$

Let $P(k)$ be true for some positive integer k, i.e.,

$1+\frac{1}{1+2}+\ldots+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}=\frac{2k}{k+1}$............(i)

We shall now prove that $\mathrm{P}(k+1)$ is true.

Consider

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}+\frac{1}{1+2+2+k+(k+1)}$

$=\left(1+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+.k}\right)+\frac{1}{1+2+3+\ldots+k+(k+1)}$

$=\frac{2k}{k+1}+\frac{1}{1+2+3+\ldots+k+(k+1)}$ [Using (i)]

$=\frac{2k}{k+1}+\frac{1}{\left(\frac{(k+1)(k+1+1)}{2}\right)}\left[1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$

$=\frac{2k}{(k+1)}+\frac{2}{(k+1)(k+2)}$

$=\frac{2}{(k+1)}\left(k+\frac{1}{k+2}\right)$

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Q4 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1.2.3+2.3.4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

Answer.

Let the given statement be $P(n)$, i.e.,

$P(n): 1.2.3+2.3.4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

For $n=1$, we have

$P(1): 1.2 \cdot 3=6=\frac{1(1+1)(1+2)(1+3)}{4}=\frac{1.23.4}{4}=6$, which is true

Let P(k) be true for some positive integer k, i.e.,

$1.2.3+2.3.4+\ldots+k(k+1)(k+2)=\frac{k(k+1)(k+2)(k+3)}{4}$ ....(i)

We shall now prove that $P(k+1)$ is true.

Consider

$1.2.3+2.3.4+\ldots+k(k+1)(k+2)+(k+1)(k+2)(k+3)$
$=\{1.2.3+2.3.4+\ldots+k(k+1)(k+2)\}+(k+1)(k+2)(k+3)$

$=\frac{k(k+1)(k+2)(k+3)}{4}+(k+1)(k+2)(k+3) \quad [\text{ Using }(i)]$
$=(k+1)(k+2)(k+3)\left(\frac{k}{4}+1\right)$
$=\frac{(k+1)(k+2)(k+3)(k+4)}{4}$
$=\frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, statement $P(n)$ is true for all natural numbers l.e., n .

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Q5 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.3+2.3^{2}+3.3^{3}+\ldots+n3^{n}=\frac{(2n-1)3^{n+1}+3}{4}$

Answer.

Let the given statement be $P(n)$, i.e.,

$1.3+2.3^{2}+3.3^{\circ}+\ldots+n3^{n}=\frac{(2n-1)3^{n+1}+3}{4}$
$P(n)$:

For $n=1$, we have

$P(1): 1.3=3 \quad =\frac{(2.1-1)3^{1+1}+3}{4}=\frac{3^{2}+3}{4}=\frac{12}{4}=3$

Let $P(k)$ be true for some positive integer k, i.e.,

$1.3+2.3^{2}+3.3^{3}+\ldots+k3^{4}=\frac{(2k-1)3^{k+1}+3}{4}$ ....(i)

We shall now prove that $P(k+1)$ is true.

Consider
$1.3+2.3^{2}+3.3^{3}+\ldots+k3^{k}+(k+1)3^{k+1}$
$=\left(1.3+2.3^{2}+3.3^{3}+\ldots+k.3^{k}\right)+(k+1)3^{k+1}$
$=\frac{(2k-1)3^{k+1}+3}{4}+(k+1)3^{k+1} \quad [\text{ Using }(\mathbf{i})]$
$=\frac{(2k-1)3^{3+1}+3+4(k+1)3^{k+1}}{4}$
$=\frac{3^{t+1}\{2k-1+4(k+1)\}+3}{4}$
$3^{k+1}\{6k+3\}+3$

$= \frac{\qquad}{4}$

$= \frac{3^{k+1} \cdot 3\{2k+1\}+3}{4}$

$= \frac{3^{(k+1)+1}\{2k+1\}+3}{4}$

$= \frac{\{2(k+1)-1\}3^{(k+1)+1}+3}{4}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers

i.e., n.

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Q6 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.2 + 2.3 + 3.4 + \ldots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$

Answer. Let the given statement be P(n), i.e.,
P(n) : $1.2 + 2.3 + 3.4 + \ldots + n \cdot (n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$
For n = 1, we have
P(1): $1.2 = 2 = \frac{1(1+1)(1+2)}{3} = \frac{1.2.3}{3} = 2$which is true.
Let P(k) be true for some positive integer k, i.e.,
$1.2 + 2.3 + 3.4 + \ldots. + k.(k+1) = \left[\frac{k(k+1)(k+2)}{3}\right]$ ....(i)
We shall now prove that P(k + 1) is true.
Consider
1.2 + 2.3 + 3.4 + ... + k.(k + 1) + (k + 1).(k + 2)
= [1.2 + 2.3 + 3.4 + ... + k.(k + 1)] + (k + 1).(k + 2)
$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$ [Using (i)]
$= (k+1)(k+2)\left(\frac{k}{3}+1\right)$
$= \frac{(k+1)(k+2)(k+3)}{3}$
$= \frac{(k+1)(k+1+1)(k+1+2)}{3}$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q7 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Answer. Let the given statement be P(n), i.e.,

P(n) : $1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

For $n = 1$, we have

$P(1) : 1.3 = 3 = \frac{1(4.1^2+6.1-1)}{3} = \frac{4+6-1}{3} = \frac{9}{3} = 3$, which is true

Let P(k) be true for sorne positive integer k, i.e.,

$1.3 + 3.5 + 5.7 + \ldots . + (2k-1)(2k+1) = \frac{k(4k^2+6k-1)}{3}$ ....(i)

We shall now prove that $P(k+1)$ is true.

Consider

$(1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1) + \{2(k+1)-1\}\{2(k+1)+1\}$

$= \frac{k(4k^2+6k-1)}{3} + (2k+2-1)(2k+2+1)$ [Using (i)]

$= \frac{k(4k^2+6k-1)}{3} + (2k+1)(2k+3)$

$= \frac{k(4k^2+6k-1)}{3} + (4k^2+8k+3)$

$= \frac{k(4k^2+6k-1)+3(4k^2+8k+3)}{3}$

$= \frac{4k^3+6k^2-k+12k^2+24k+9}{3}$

$= \frac{4k^3+18k^2+93k+9}{3}$

$= \frac{4k^3+14k^2+9k+4k^2+14k+9}{3}$

$= \frac{k(4k^2+14k+9)+1(4k^2+14k+9)}{3}$

$= \frac{(k+1)(4k^2+14k+9)}{3}$

$= \frac{(k+1)\{4k^2+8k+4+6k+6-1\}}{3}$

$= \frac{(k+1)\{4(k^2+2k+1)+6(k+1)-1\}}{3}$

$= \frac{(k+1)\{4(k+1)^2+6(k+1)-1\}}{3}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q8 Prove the following by using the principle of mathematical induction for all n ∈ N:

$1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^\circ = (n-1)2^{n+1} + 2$

Answer. Let the given statement be P(n), i.e.,

$P(n) : 1.2 + 2.2^2 + 3.2^2 + \ldots + n.2^n = (n-1)2^{n+1} + 2$

For n = 1, we have

For n = 1, we have
$P(1): 1.2 = 2 = (1-1)2^{1+1} + 2 = 0 + 2 = 2,$
Let P(k) be true for some positive integer k, i.e.,
$1.2 + 2.2^2 + 3.2^2 + \ldots + \mathrm{k}.2^{\mathrm{k}} = (\mathrm{k}-1)2^{\mathrm{k}+1} + 2 \ldots$..(i)
We shall now prove that P(k + 1) is true.
Consider
$\left\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\right\} + (k+1) \cdot 2^{k+1}$
$= (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$
$= 2^{k+1}\{(k-1) + (k+1)\} + 2$
$= 2^{k+1} \cdot 2k + 2$
$= k.2^{(k+1)+1} + 2$
$= \{(k+1) - 1\}2^{(k+1)+1} + 2$
Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q9 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Answer. Let the given statement be P(n), i.e.,
P(n): $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$
For n = 1, we have
P(1): $\frac{1}{2} = 1 - \frac{1}{2^1} = \frac{1}{2}$, which is true
Let P(k) be true for some positive integer k, i.e.,
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$
We shall now prove that P(k + 1) is true.
Consider
$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^k}\right) + \frac{1}{2^{k+1}}$ [using(i))]
$= \left(1 - \frac{1}{2^k}\right) + \frac{1}{2^{k+1}}$
$= 1 - \frac{1}{2^k} + \frac{1}{2 \cdot 2^k}$
$= 1 - \frac{1}{2^k}\left(1 - \frac{1}{2}\right)$
$= 1 - \frac{1}{2^k}\left(\frac{1}{2}\right)$
$= 1 - \frac{1}{2^{k+1}}$
Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q10 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$

Answer. Let the given statement be P(n), i.e.,
P(n): $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{(6n+4)}$
For n = 1, we have
$P(1)=\frac{1}{2.5}=\frac{1}{10}=\frac{1}{6.1+4}=\frac{1}{10}$, which is true.
Let P(k) be true for some positive integer k, i.e.,
$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots+\frac{1}{(3k-1)(3k+2)}=\frac{k}{6k+4}$ ....(i)
We shall now prove that P(k + 1) is true.
Consider

$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots\ldots+\frac{1}{(3k-1)(3k+2)}+\frac{1}{\{3(k+1)-1\}\{3(k+1)+2\}}$
$=\frac{k}{6k+4}+\frac{1}{(3k+3-1)(3k+3+2)}$ [Using (i)]
$=\frac{k}{6k+4}+\frac{1}{(3k+2)(3k+5)}$
$=\frac{k}{2(3k+2)}+\frac{1}{(3k+2)(3k+5)}$
$=\frac{1}{(3k+2)}\left(\frac{k}{2}+\frac{1}{3k+5}\right)$
$=\frac{1}{(3k+2)}\left(\frac{k(3k+5)+2}{2(3k+5)}\right)$
$=\frac{1}{(3k+2)}\left(\frac{3k^2+5k+2}{2(3k+5)}\right)$
$=\frac{1}{(3k+2)}\left(\frac{(3k+2)(k+1)}{2(3k+5)}\right)$
$=\frac{(k+1)}{6k+10}$
$=\frac{(k+1)}{6(k+1)+4}$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q11 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$

Answer. Let the given statement be P(n),i.e., P(n):

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

For n = 1, we have

$P(1): \frac{1}{1\cdot2\cdot3}=\frac{1\cdot(1+3)}{4(1+1)(1+2)}=\frac{1\cdot4}{4\cdot2\cdot3}=\frac{1}{1\cdot2\cdot3}$, which is true

Let P(k) be true for some positive integer k, i.e.,

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{k(k+1)(k+2)}=\frac{k(k+3)}{4(k+1)(k+2)} \quad \ldots\ldots\text{(i)}$$

We shall now prove that P(k + 1) is true.

Consider

$$\left[\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\ldots+\frac{1}{k(k+1)(k+2)}\right]+\frac{1}{(k+1)(k+2)(k+3)}$$

$$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)} \quad [\text{ Using (i)}]$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)}{4}+\frac{1}{k+3}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+3)^2+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k\left(k^2+6k+9\right)+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k^3+6k^2+9k+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k^3+2k^2+k+4k^2+8k+4}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k\left(k^2+2k+1\right)+4\left(k^2+2k+1\right)}{4(k+3)}\right\}$$

$$=\frac{1}{(k+1)(k+2)}\left\{\frac{k(k+1)^2+4(k+1)^2}{4(k+3)}\right\}$$

$$=\frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}$$

$$=\frac{(k+1)\{(k+1)+3\}}{4\{(k+1)+1\}\{(k+1)+2\}}$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q12 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$a+ar+ar^2+\ldots+ar^{n-1}=\frac{a(r^n-1)}{r-1}$$

Answer. Let the given statement be P(n), i.e.,

$$P(n): a+ar+ar^2+\ldots+ar^{n-1}=\frac{a(r^n-1)}{\ldots}$$

$P(n): a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n-1)}{r-1}$

For n = 1, we have

$P(1): a = \frac{a(r^1-1)}{(r-1)} = a$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$a + ar + ar^2 + \ldots\ldots\ldots + ar^{k-1} = \frac{a(r^k-1)}{r-1}$ ....(i)

We shall now prove that P(k + 1) is true.

Consider

$\{a + ar + ar^2 + \ldots\ldots + ar^{k-1}\} + ar^{(k+1)-1}$

$= \frac{a(r^k-1)}{r-1} + ar^k$ [Using(i)]

$$= \frac{a\left(r^k - 1\right) + ar^k(r-1)}{r-1}$$

$$= \frac{a\left(r^k - 1\right) + ar^{k+1} - ar^k}{r-1}$$

$= \frac{ar^k - a + ar^{k+1} - ar^k}{r-1}$

$= \frac{ar^{k+1} - a}{r-1}$

$= \frac{a(r^{k+1}-1)}{r-1}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q13 Prove the following by using the principle of mathematical induction for all n ∈ N:

$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$

Answer.Let the given statement be P(n), i.e.,

$P(n): \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2n+1)}{n^2}\right) = (n+1)^2$

For n = 1, we have

$P(1): \left(1+\frac{3}{1}\right) = 4 = (1+1)^2 = 2^2 = 4,$

Let P(k) be true for some positive integer k, i.e.,

$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2k+1)}{k^2}\right) = (k+1)^2$ ......(1)

We shall now prove that P(k + 1) is true.

Consider

$\left[\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{(2k+1)}{k^2}\right)\right]\left\{1+\frac{\{2(k+1)+1\}}{(k+1)^2}\right\}$

$= (k+1)^2\left(1+\frac{2(k+1)+1}{(k+1)^2}\right)$ [(Using (1)]

$= (k+1)^2 \left[\frac{(k+1)^2+2(k+1)+1}{(k+1)^2}\right]$
$= (k+1)^2 + 2(k+1) + 1$
$= \{(k+1)+1\}^2$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q14 Prove the following by using the principle of mathematical induction for all n ∈ N:
$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right) = (n+1)$

Answer. Let the given statement be P(n), i.e.,
P(n): $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right) = (n+1)$
For n = 1, we have
$\mathrm{P}(1) : \left(1+\frac{1}{1}\right) = 2 = (1+1)$, which is true
Let P(k) be true for some positive integer k, i.e.,
$\mathrm{P}(k) : \left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right) = (k+1)$ ....(1)
We shall now prove that P(k + 1) is true.
Consider
$\left[\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right)\right]\left(1+\frac{1}{k+1}\right)$

$= (k+1)\left(1+\frac{1}{k+1}\right)$ [Using(1)]
$= (k+1)\left(\frac{(k+1)+1}{(k+1)}\right)$
$= (k+1)+1$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q15 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Answer. Let the given statement be P(n), i.e.,
$P(n) = 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

For $n=1,$ we have

$P(1)=1^2=1=\frac{1(2.1-1)(2.1+1)}{3}=\frac{1.1.3}{3}=1,$ which is true.

Let P(k) be true for some positive integer k, i.e.,

$P(k)=1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{k(2k-1)(2k+1)}{3}$ ....(1)

We shall now prove that P(k + 1) is true.

Consider

$\{1^2+3^2+5^2+\ldots+(2k-1)^2\}+\{2(k+1)-1\}^2$

$=\frac{k(2k-1)(2k+1)}{3}+(2k+2-1)^2$ [Using (1)]

$=\frac{k(2k-1)(2k+1)}{3}+(2k+1)^2$

$=\frac{k(2k-1)(2k+1)+3(2k+1)^2}{3}$

$=\frac{(2k+1)\{k(2k-1)+3(2k+1)\}}{3}$

$=\frac{(2k+1)\{2k^2-k+6k+3\}}{3}$

$=\frac{(2k+1)\{2k^2+5k+3\}}{3}$

$=\frac{(2k+1)\{2k^2+2k+3k+3\}}{3}$

$=\frac{(2k+1)\{2k(k+1)+3(k+1)\}}{3}$

$=\frac{(2k+1)(k+1)(2k+3)}{3}$

$=\frac{(k+1)\{2(k+1)-1\}\{2(k+1)+1\}}{3}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q16 Prove the following by using the principle of mathematical induction for all n ∈ N:

$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

Answer. Let the given statement be P(n), i.e.,

$P(n):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

For $n=1,$ we have

$P(1)=\frac{1}{1.4}=\frac{1}{3.1+1}=\frac{1}{4}=\frac{1}{1.4},$ which is true.

Let P(k) be true for some positive integer k, i.e.,

$P(k)=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$ ....(1)

We shall now prove that P(k + 1) is true.

Consider

Consider

$$\left\{\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}\right\}+\frac{1}{\{3(k+1)-2\}\{3(k+1)+1\}}$$

$$=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)} \text{ [Using (1)]}$$

$$=\frac{1}{(3k+1)}\left\{k+\frac{1}{(3k+4)}\right\}$$

$$=\frac{1}{(3k+1)}\left\{\frac{k(3k+4)+1}{(3k+4)}\right\}$$

$$=\frac{1}{(3k+1)}\left\{\frac{3k^2+4k+1}{(3k+4)}\right\}$$

$$=\frac{1}{(3k+1)}\left\{\frac{3k^2+3k+k+1}{(3k+4)}\right\}$$

$$=\frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$$

$$=\frac{(k+1)}{3(k+1)+1}$$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q17 Prove the following by using the principle of mathematical induction for all n ∈ N:

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

Answer. Let the given statement be P(n), i.e.,

$$P(n):\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

For n = 1, we have

$P(1):\frac{1}{3.5}=\frac{1}{3(2.1+3)}=\frac{1}{3.5}$, which is true.

Let P(k) be true for some positive integer k, i.e.,

$P(k):\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}=\frac{k}{3(2k+3)}$ ......(1)

We shall now prove that P(k + 1) is true.

Consider

$$\left[\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}\right]+\frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$$

$$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)}\ [\text{U sing}(1)]$$

$$=\frac{1}{(2k+3)}\left[\frac{k}{3}+\frac{1}{(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{3(2k+5)}\right]$$

$$=\frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{3(2k+5)}\right]$$

$= \frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{3(2k+5)}\right]$

$= \frac{(k+1)(2k+3)}{3(2k+3)(2k+5)}$

$= \frac{(k+1)}{3\{2(k+1)+3\}}$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q18 Prove the following by using the principle of mathematical induction for all n ∈ N:
$1 + 2 + 3 + \ldots + n < \frac{1}{8}(2n + 1)^2$

Answer. Let the given statement be P(n), i.e.,
$P(n) : 1 + 2 + 3 + \ldots + n < \frac{1}{8}(2n + 1)^2$
It can be noted that P(n) is true for n = 1 since $1 < \frac{1}{8}(2.1 + 1)^2 = \frac{9}{8}$
Let P(k) be true for some positive integer k, i.e.,
$1 + 2 + \ldots + k < \frac{1}{8}(2k + 1)^2$ ......(1)
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$(1 + 2 + \ldots + k) + (k + 1) < \frac{1}{g}(2k + 1)^2 + (k + 1)$ [Using (1)]
$< \frac{1}{8}\left\{(2k + 1)^2 + 8(k + 1)\right\}$
$< \frac{1}{8}\left\{4k^2 + 4k + 1 + 8k + 8\right\}$
$< \frac{1}{8}\left\{4k^2 + 12k + 9\right\}$
$< \frac{1}{8}(2k + 3)^2$
$< \frac{1}{8}\{2(k + 1) + 1\}^2$
Hence, $(1 + 2 + 3 + \ldots + k) + (k + 1) < \frac{1}{8}(2k + 1)^2 + (k + 1)$
Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q19 Prove the following by using the principle of mathematical induction for all n ∈ N:
$n(n + 1)(n + 5)$ is a multiple of 3

Answer. Let the given statement be P(n), i.e.,
P(n): n (n + 1) (n + 5), which is a multiple of 3.

It can be noted that P(n) is true for n = 1 since 1 (1 + 1) (1 + 5) = 12, which is a multiple of 3.
Let P(k) be true for some positive integer k, i.e.,
k (k + 1) (k + 5) is a multiple of 3.
∴k (k + 1) (k + 5) = 3m, where m ∈ N ... (1)
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$$(k+1)\{(k+1)+1\}\{(k+1)+5\}$$
$$=(k+1)(k+2)\{(k+5)+1\}$$
$$=(k+1)(k+2)(k+5)+(k+1)(k+2)$$
$$=\{k(k+1)(k+5)+2(k+1)(k+5)\}$$
$$=3m+(k+1)\{2(k+5)+(k+2)\}$$
$$=3m+(k+1)\{2k+10+k+2\}$$
$$=3m+(k+1)\{2k+12)$$
$$=3m+3(k+1)(k+4)$$
$$=3\{m+(k+1)(k+4)\}=3\times q,\text{ where } q=\{m+(k+1)(k+4)\}\text{ is some natural number}$$

Therefore, $(k+1)\{(k+1)+1\}\{(k+1)+5\}$ is a multiple of 3.

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q20 Prove the following by using the principle of mathematical induction for all n ∈ N:
$10^{1n-1}+1$ is divisible by 11

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P(n)}: 10^{2\mathrm{n}-1}+1$ is divisible by 11.
It can be observed that $\mathrm{P(n)}$ is true for $\mathrm{n}=1$ since $\mathrm{P}(1)=10^{2.1-1}+1=11$, which is divisible by 11.
Let P(k) be true for some positive integer k, i.e.,
$10^{2\mathrm{k}-1}+1$ is divisible by 11.
$\therefore 10^{2\mathrm{k}-1}+1=11\mathrm{m}$, where $\mathrm{m}\in\mathrm{N}\ldots(1)$
We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider

$$10^{2(k+1)-1}+1$$
$$=10^{2k+2-1}+1$$
$$=10^{2k+1}+1$$
$$=10^2\left(10^{2k-1}+1-1\right)+1$$

$= 10^2 \left(10^{2k-1} + 1 - 1\right) + 1$

$= 10^2 \left(10^{2k-1} + 1\right) - 10^2 + 1$

$= 10^2 \cdot 11m - 100 + 1$ [Using (1)]

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q21 Prove the following by using the principle of mathematical induction for all n ∈ N:
$x^{2x} - y^{3n}$ is divisible by $x + y$

Answer. Let the given statement be P(n), i.e.,
$P(n) : x^{2n} - y^{2n}$ is divisible by $x + y$.

It can be observed that $P(n)$ is true for $n = 1$.

This is so because $x^2 \times 1 - y^2 \times 1 = x^2 - y^2 = (x + y)(x - y)$ is divisible by $(x + y)$.

Let P(k) be true for some positive integer k, i.e.,
$x^{2k} - y^{2k}$ is divisible by $x + y$.

$\therefore x^{2k} - y^{2k} = m(x + y),$ where $m \in N \ldots (1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$x^{2(k+1)} - y^{2(k+1)}$

$= x^{2k} \cdot x^2 - y^{2k} \cdot y^2$

$= x^2 \left(x^{2k} - y^{2k} + y^{2k}\right) - y^{2k} \cdot y^2$

$= x^2 \left\{m(x + y) + y^{2k}\right\} - y^{2k} \cdot y^2$ [Using (1)]

$= m(x + y)x^2 + y^{2k} \cdot x^2 - y^{2k} \cdot y^2$

$= m(x + y)x^2 + y^{2k} \left(x^2 - y^2\right)$

$= m(x + y)x^2 + y^{2k}(x + y)(x - y)$

$= (x + y) \left\{mx^2 + y^{2k}(x - y)\right\}$, which is a factor of $(x + y)$

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q22 Prove the following by using the principle of mathematical induction for all n ∈ N:
$3^{2n+2} - 8n - 9$ is divisible by 8

Answer. Let the given statement be P(n), i.e.

Answer. Let the given statement be P(n), i.e.,

$P(n): 3^{2n+2} - 8n - 9$ is divisible by 8

It can be observed that $P(n)$ is true for $n = 1$ since $3^{2\times1+2} - 8\times1 - 9 = 64$, which is divisible by 8 .

Let P(k) be true for some positive integer k, i.e.,

$3^{2k+2} - 8k - 9$ is divisible by 8 .

$\therefore 3^{2k+2} - 8k - 9 = 8m;$ where $m \in N \ldots (1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$3^{2(k+1)+2} - 8(k+1) - 9$

$= 3^{2k+2} \cdot 3^2 - 8k - 8 - 9$

$= 3^2\left(3^{2k+2} - 8k - 9 + 8k + 9\right) - 8k - 17$

$= 3^2\left(3^{2k+2} - 8k - 9\right) + 3^2(8k+9) - 8k - 17$

$= 9.8m + 9(8k+9) - 8k - 17$

$= 9.8m + 9(8k+9) - 8k - 17$

$= 9.8m + 72k + 81 - 8k - 17$

$= 9.8m + 64k + 64$

$= 8(9m + 8k + 8)$

$= 8r,$ where $r = (9m + 8k + 8)$ is a natural number

Therefore, $3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8 .

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q23 Prove the following by using the principle of mathematical induction for all n ∈ N:

$41^n - 14^n$ is a multiple of 27

Answer. Let the given statement be P(n), i.e.,

$P(n): 41^n - 14^n$ is a multiple of 27

It can be observed that P(n) is true for n = 1 since $41^1 - 14^1 = 27$ which is a multiple of 27.

Let P(k) be true for some positive integer k, i.e.,

$41^k - 14^k$ is a multiple of 27

$\therefore 41^k - 14^k = 27m,$ where $m \in N \ldots$ (1)

We shall now prove that P(k + 1) is true whenever P(k) is true.

Consider

$41^{k+1}-14^{k+1}$
$=41^{k}\cdot 41-14^{k}\cdot 14$
$=41\left(41^{k}-14^{k}+14^{k}\right)-14^{k}\cdot 14$
$=41\left(41^{k}-14^{k}\right)+41.14^{k}-14^{k}\cdot 14$
$=41.27m+14^{k}(41-14)$
$=41.27m+27.14^{k}$
$=27\left(41m-14^{k}\right)$
$=27\times r,$ where $r=\left(41m-14^{k}\right)$ is a natural number

Therefore, $41^{k+1}-14^{k+1}$ is a multiple of 27

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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Q24 Prove the following by using the principle of mathematical induction for all n ∈ N:
$(2n+7)<(n+3)^{2}$

Answer. Let the given statement be P(n), i.e.,
$\mathrm{P(n)}:(2\mathrm{n}+7)<(\mathrm{n}+3)^{2}$

It can be observed that P(n) is true for $\mathrm{n}=1$ since $2.1+7=9<(1+3)^{2}$
$=16,$ which is true.

Let P(k) be true for some positive integer k, i.e.,
$(2k+7)<(k+3)^{2}\ldots(1)$

We shall now prove that P(k + 1) is true whenever P(k) is true.
Consider
$\{2(k+1)+7\}=(2k+7)+2$
$\therefore\{2(\mathrm{k}+1)+7\}=(2\mathrm{k}+7)+2<(\mathrm{k}+3)^{2}+2\quad[$ using $(1)]$
$2(\mathrm{k}+1)+7<\mathrm{k}^{2}+6\mathrm{k}+9+2$
$2(\mathrm{k}+1)+7<\mathrm{k}^{2}+6\mathrm{k}+11$

Now, $\mathrm{k}^{2}+6\mathrm{k}+11<\mathrm{k}^{2}+8\mathrm{k}+16$
$\therefore 2(\mathrm{k}+1)+7<(\mathrm{k}+4)^{2}$
$2(\mathrm{k}+1)+7<\{(\mathrm{k}+1)+3\}^{2}$

Thus, P(k + 1) is true whenever P(k) is true.
Hence, by the principle of mathematical induction, statement P(n) is true for all natural numbers i.e., n.

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