Class : 11th
Subject : Maths
Chapter : 3
Chapter Name : Trigonometric Functions

Exercise 3.1

Q1 Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) $-47^{\circ}30'$ (iii) 240° (iv) 520°

Answer.
(i) 25°
We know that $180^{\circ}=\pi$ radian
$\therefore 25^{\circ}=\frac{\pi}{180}\times 25$ radian $=\frac{5\pi}{36}$ radian
(ii) $-47^{\circ}30'$
$-47^{\circ}30'=-47\frac{1}{2}$
$=\frac{-95}{2}$
Since $180^{\circ}=\pi$ radian
(iii) 240°
We know that $180^{\circ}=\pi$ radian
$\therefore 240^{\circ}=\frac{\pi}{180}\times 240$ radian $=\frac{4}{3}\pi$ radian
(iv) 520°
We know that $180^{\circ}=\pi$ radian
$\therefore 520^{\circ}=\frac{\pi}{180}\times 520$ radian $=\frac{26\pi}{9}$ radian

Page : 54 , Block Name : Exercise 3.1

Q2 Find the degree measures corresponding to the following radian measures (Use $\pi=\frac{22}{7}$).
(i) $\frac{11}{16}$ (ii) -4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Answer.
(i) $\frac{11}{16}$
We know that π radian $=180^{\circ}$

$$\therefore \frac{11}{16}\text{radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ degree } = \frac{45 \times 11}{\pi \times 4} \text{ deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree } = \frac{315}{8} \text{ degree}$$

$= 39\frac{3}{8}$ deg ree

$= 39^{\circ} + \frac{3\times 60}{8}$ min utes $\quad [1^{\circ} = 60^{\circ}]$

$= 39^{\circ} + 22' + \frac{1}{2}$ min utes

$= 39^{\circ}22'30'' \qquad [1' = 60'']$

$(ii) - 4$

We know that π radian $= 180^{\circ}$

-4 radian $= \frac{180}{\pi} \times (-4)$ deg ree $= \frac{180\times 7(-4)}{22}$ degree

$= \frac{-2520}{11}$ deg ree $= -229\frac{1}{11}$ degree

$= -229^{\circ} + \frac{1\times 60}{11}$ min utes $\quad [1^{\circ} = 60^{\circ}]$

$= -229^{\circ}5^{12}27^{n} \qquad [1' = 60'']$

(iii) $\frac{5\pi}{3}$

We know that π radlan $= 180^{\circ}$

$\therefore \frac{5\pi}{3}$ radian $= \frac{180}{\pi} \times \frac{5\pi}{3}$ degree $= 300^{\circ}$

(iv) $\frac{7\pi}{6}$

We know that π radian $= 180^{\circ}$

$\therefore \frac{7\pi}{6}$ radian $= \frac{180}{\pi} \times \frac{7\pi}{6} = 210^{\circ}$

Page : 55 , Block Name : Exercise 3.1

Q3 A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer. Number of revolutions made by the wheel in 1 minute = 360

$\therefore$ Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

Hence, In 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian, i.e., 12 π radian

Thus, In one second, the wheel turns and angle of 12 π radian.

Page : 55 , Block Name : Exercise 3.1

Q4 Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use $\pi = \frac{22}{7}$)

Answer. We know that in a circle of radius r unit, if an arc of length l unit subtends and angle θ radian at the centre, then

$\theta = \frac{1}{}$

$\theta = \frac{l}{r}$
Therefore, for r = 100 cm, l = 22 cm, we have

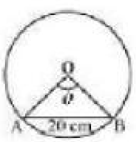
$$\theta = \frac{22}{100} \text{ radian } = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree } = \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$
$$= \frac{126}{10} \text{ deg ree } = 12\frac{3}{5} \text{ deg ree } = 12^{\circ}36^{\circ} \quad [1^{\circ} = 60']$$

Thus, the required angle is $12^{\circ}36'$.

Page : 55 , Block Name : Exercise 3.1

Q5 In a circle of diameter 40 cm, the length of a circle is 20 cm. Find the length of minor arc of the chord.

Answer. Diameter of the circle = 40 cm
Radius of the circle = $\frac{40}{2}\text{cm} = 20\text{cm}$
Let AB be a chord (length = 20 cm) of the circle.



$$\text{In } \triangle OAB, OA = OB = \text{ Radius of circle } = 20\text{cm}$$
$$\text{Also, } AB = 20\text{cm}$$
$$\text{Thus, } \triangle OAB \text{ is an equilateral triangle.}$$
$\therefore \theta = 60^{\circ} = \frac{\pi}{3}$
We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$
$\frac{\pi}{3} = \frac{\widetilde{AB}}{20} \Rightarrow \widetilde{AB} = \frac{20\pi}{3}\text{cm}$
Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}\text{cm}$.

Page : 55 , Block Name : Exercise 3.1

Q6 In the two circles , arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer. Let the radii of the two circles be r_1 and r_2 . Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1 , while let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_2.
Now, $60^{\circ} = \frac{\pi}{3}$ radian and $75^{\circ} = \frac{5\pi}{12}$
We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$ or $l = r\theta$
$\therefore l = \frac{r_1\pi}{3}$ and $l = \frac{r_2 5\pi}{12}$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$
$$\Rightarrow r_1 = \frac{r_2 5}{4}$$
$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$
Thus, the ratio of the radii is 5 : 4.

Page : 55 , Block Name : Exercise 3.1

Q7 Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.
(i) 10 cm
(ii) 15 cm
(iii) 21 cm

Answer. We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.
It is given that r = 75 cm
(i) Here, l = 10 cm
$\theta = \frac{10}{75}$ radian $= \frac{2}{15}$ radian
(ii) Here, l = 15 cm
$\theta = \frac{15}{75}$ radian $= \frac{1}{5}$ radian
(iii) Here, l = 21 cm
$\theta = \frac{21}{75}$ radian $= \frac{7}{25}$ radian

Page : 55 , Block Name : Exercise 3.1

Exercise 3.2

Q1 Find the values of other five trigonometric functions in $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Answer.
$$\cos x = -\frac{1}{2}$$
$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$
$$\sin^2 x + \cos^2 x = 1$$
$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$
$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since x lies in the 3rd quadrant, the value of sin x will be negative.

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

Page : 63 , Block Name : Exercise 3.2

Q2 Find the values of other five trigonometric functions in $\sin x = \frac{3}{5}$, x lies in second quadrant.

Answer.

$\sin x = \frac{3}{5}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$

$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$

$\Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

Since x lies in the 2nd quadrant, the value of cos x will be negative

$\therefore \cos x = -\frac{4}{5}$

$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\cos x} = \frac{\left(-\frac{4}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$.

$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$

Page : 63 , Block Name : Exercise 3.2

Q3 Find the values of other five trigonometric functions in $\cot x = \frac{3}{4}$, x lies in third quadrant.

Answer.

$\cot x = \frac{3}{4}$

$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$

$1 + \tan^2 x = \sec^2 x$

$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$

$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$

$\Rightarrow \frac{25}{9} = \sec^2 x$

$\Rightarrow \sec x = \pm \frac{5}{3}$

Since, x lies in the 3rd quadrant, the value of sec x will be negative.

$\therefore \sec x = -\frac{5}{3}$

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$

$\tan x = \frac{\sin x}{\cos x}$

$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$

$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$.

$cosecx = \frac{1}{\sin x} = -\frac{5}{4}$

Page : 63 , Block Name : Exercise 3.2

Q4 Find the values of other five trigonometric functions in $\sec x = \frac{13}{5}$, lies in fourth quadrant.

Answer.

$\sec x = \frac{13}{5}$

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$

$\Rightarrow \sin^2 x = 1 - 25 \quad 144$

$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$

$\Rightarrow \sin x = \pm\frac{12}{13}$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$\therefore \sin x = -\frac{12}{13}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{12}{5}$

Page : 63 , Block Name : Exercise 3.2

Q5 Find the values of other five trigonometric functions in $\tan x = -\frac{5}{12}$, x lies in second quadrant.

Answer.

$\tan x = -\frac{5}{12}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$

$1 + \tan^2 x = \sec^2 x$

$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$

$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$

$\Rightarrow \frac{169}{144} = \sec^2 x$

$\Rightarrow \sec x = \pm\frac{13}{12}$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

$\therefore \sec x = -\frac{13}{12}$

$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$

$\tan x = \frac{\sin x}{\cos x}$

$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$

$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$

$cosecx = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$

Page : 63 , Block Name : Exercise 3.2

Q6 Find the values of the trigonometric functions in $\sin 765^{\circ}$.

Answer. It is known that the values of sin x repeat after an interval of 2π or 360^deg.

$\therefore \sin 765^{\circ}=\sin(2\times 360^{\circ}+45^{\circ})=\sin 45^{\circ}=\frac{1}{\sqrt{2}}$

Page : 63 , Block Name : Exercise 3.2

Q7 Find the values of the trigonometric functions in $cosec\,(-1410^{\circ})$.

Answer. It is known that the values of cosec x repeat after an interval of 2π or 360^o.

$$\begin{aligned}\therefore \text{ cosec } (-1410^{\circ}) &= cosec\,(-1410^{\circ}+4\times 360^{\circ})\\ &= cosec\,(-1410^{\circ}+1440^{\circ})\\ &= cosec 30^{\circ}=2\end{aligned}$$

Page : 63 , Block Name : Exercise 3.2

Q8 Find the values of the trigonometric functions in $\tan \frac{19\pi}{3}$.

Answer. It is known that the values of tan x repeat after an interval of π or 180° .

$\therefore \tan \frac{19\pi}{3}=\tan 6\frac{1}{3}\pi=\tan\left(6\pi+\frac{\pi}{3}\right)=\tan \frac{\pi}{3}=\tan 60^{\circ}=\sqrt{3}.$

Page : 63 , Block Name : Exercise 3.2

Q9 Find the values of the trigonometric functions in $\sin\left(-\frac{11\pi}{3}\right)$.

Answer. It is known that the values of sin x repeat after an interval of π or 360° .

$\therefore \sin\left(-\frac{11\pi}{3}\right)=\sin\left(-\frac{11\pi}{3}+2\times 2\pi\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

Page : 63 , Block Name : Exercise 3.2

Q10 Find the values of the trigonometric functions in $\cot\left(-\frac{15\pi}{4}\right)$.

Answer. It is known that the values of cot x repeat after an interval of π or 180° .

$\therefore \cot\left(-\frac{15\pi}{4}\right)=\cot\left(-\frac{15\pi}{4}+4\pi\right)=\cot \frac{\pi}{4}=1.$

Page : 63 , Block Name : Exercise 3.2

Exercise 3.3

Q1 Prove that: $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Answer.

$$\begin{aligned}&\text{L. H.S. }=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}\\&=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2\\&=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}\\&=\text{ R.H.S.}\end{aligned}$$

Page : 73 , Block Name : Exercise 3.3

Q2 Prove that: $2\sin^2\frac{\pi}{6}+cosec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Answer.

$$\begin{aligned}&\text{L.H.S. }=2\sin^2\frac{\pi}{6}+cosec^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}\\&=2\left(\frac{1}{2}\right)^2+\cos\text{ec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2\\&=2\times\frac{1}{4}+\left(-cosec\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)\\&=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}\\&=\text{ R.H.S.}\end{aligned}$$

Page : 73 , Block Name : Exercise 3.3

Q3 Prove that: $\cot^2\frac{\pi}{6}+cosec\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Answer.

$$\begin{aligned}&\text{L.H.S. }=\cot^2\frac{\pi}{6}+cosec\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}\\&=(\sqrt{3})^2+\cos\text{cc}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2\\&=3+\cos ec\frac{\pi}{6}+3\times\frac{1}{3}\\&=3+2+1=6\\&=RHS\end{aligned}$$

Page : 73 , Block Name : Exercise 3.3

Page : 73 , Block Name : Exercise 3.3

Q4 Prove that: $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Answer. $L.\text{H. S}=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$

$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$

$=2\left(\frac{1}{\sqrt{2}}\right)^2+1+8$

$=1+1+8$

$=10$

$=$ R.H.S

Page : 73 , Block Name : Exercise 3.3

Q5 Find the value of :

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Answer.

$(i)\sin 75^\circ=\sin(45^\circ+30^\circ)$

$=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$[\sin(x+y)=\sin x\cos y+\cos x\sin y]$

$=\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ=\tan(45^\circ-30^\circ)$

$=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}\quad\left[\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$

$=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}=\frac{\sqrt{3}-1}{\frac{\sqrt{3}}{\sqrt{3}}}$

$=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$

$=\frac{4-2\sqrt{3}}{3-1}=2-\sqrt{3}$

Page : 73 , Block Name : Exercise 3.3

Q6 Prove the following: $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Answer.

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$
$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$
$$=\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$
$$+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$
$$\left[\begin{array}{l}\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$
$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-\mathrm{x}\right)+\left(\frac{\pi}{4}-\mathrm{y}\right)\right\}\right]$$
$$=\cos\left[\frac{\pi}{2}-(\mathrm{x}+\mathrm{y})\right]$$
$$=\sin(\mathrm{x}+y)$$
$$=\mathrm{R.H.S}$$

Page : 73 , Block Name : Exercise 3.3

Q7 Prove the following: $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$

Answer. It is known that

$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ and $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\mathrm{R.H.S.}$$

Page : 73 , Block Name : Exercise 3.3

Q8 Prove the following: $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Answer.

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$
$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x]}$$

$$= \frac{-\cos^2 x}{-\cot^2 x}$$
$$= \cot^2 x$$
$$= R.H.S$$

Page : 73 , Block Name : Exercise 3.3

Q9 Prove the following: $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$

Answer. L.H.S $=\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$$= \sin x \cos x[\tan x + \cot x]$$
$$= \sin x \cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$
$$= (\sin x \cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x \cos x}\right]$$
$$= 1 = \mathrm{R.H.S.}$$

Page : 73 , Block Name : Exercise 3.3

Q10 Prove the following: $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Answer. L.H.S = $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

$$= \frac{1}{2}[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x]$$
$$= \frac{1}{2}\begin{bmatrix}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\ +\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{bmatrix}$$
$$\begin{bmatrix}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{bmatrix}$$

$$= \frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$$
$$= \cos(-x)=\cos x = R.HS$$

Page : 73 , Block Name : Exercise 3.3

Q11 Prove the following: $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Answer. It is known that

$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

$\therefore$ = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$= -2\sin\left\{\frac{\quad}{2}\right\} \cdot \sin\left\{\frac{\quad}{2}\right\}$

$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$

$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$

$= -2\sin\frac{\pi}{4}\sin x$

$= -2 \times \frac{1}{\sqrt{2}} \times \sin x$

$= -\sqrt{2}\sin x$

$= R.H.S$

Page : 73 , Block Name : Exercise 3.3

Q12 Prove the following: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Answer. It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore \text{L. H. S.} = \sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$= \left[2\sin\left(\frac{6\mathrm{x}+4\mathrm{x}}{2}\right)\cos\left(\frac{6\mathrm{x}-4\mathrm{x}}{2}\right)\right]\left[2\cos\left(\frac{6\mathrm{x}+4\mathrm{x}}{2}\right)\cdot\sin\left(\frac{6\mathrm{x}-4\mathrm{x}}{2}\right)\right]$

$= (2\sin 5x\cos x)(2\cos 5x\sin x)$

$= (2\sin 5x\cos 5x)(2\sin x\cos x)$

$= \sin 10x\sin 2x$

= R. H . S.

Page : 73 , Block Name : Exercise 3.3

Q13 Prove the following: $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Answer. It is known that

$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore \text{L. H} \cdot \text{S,} = \cos^2 2x - \cos^2 6\mathrm{x}$

$= (\cos 2x + \cos 6x)(\cos 2x - 6x)$

$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$

$= [2\cos 4x\cos(-2x)][-2\sin 4x\sin(-2x)]$

$= [2\cos 4x\cos 2x][-2\sin 4x(-\sin 2x)]$

$= (2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$= \sin 8x\sin 4x$

$= R.H.S$

$= R.H.S.$

Page : 73 , Block Name : Exercise 3.3

Q14 Prove the following: $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Answer.

$$\text{LH. S.} = \sin 2x + 2\sin 4x + \sin 6x$$
$$= [\sin 2x + \sin 6x] + 2\sin 4x$$
$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$
$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$= 2\sin 4x\cos(-2x) + 2\sin 4x$$
$$= 2\sin 4x\cos 2x + 2\sin 4x$$
$$= 2\sin 4x(\cos 2x + 1)$$
$$= 2\sin 4x\left(2\cos^2 x - 1 + 1\right)$$
$$= 2\sin 4x\left(2\cos^2 x\right)$$
$$= 4\cos^2 x\sin 4x$$
$$= R.H.S$$

Page : 73 , Block Name : Exercise 3.3

Q15 Prove the following: cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Answer.

$$\text{L. H.S } = \cot 4x(\sin 5x + \sin 3x)$$
$$= \frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$
$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x\cos x]$$
$$= 2\cos 4x\cos x$$
$$R.4.5. = \cot x(\sin 5x - \sin 3x)$$
$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$
$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$= \frac{\cos x}{\sin x}[2\cos 4x \sin x]$
$= 2\cos 4x \cdot \cos x$
$L \cdot H \cdot S. = R.H.S$

Page : 73 , Block Name : Exercise 3.3

Q16 Prove the following: $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x} = -\frac{\sin 2x}{\cos 10x}$

Answer. It is known that
$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
$\therefore L.H.S = \frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$
$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\cdot\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\cdot\sin\left(\frac{17x-3x}{2}\right)}$
$= \frac{-2\sin 7x \cdot \sin 2x}{2\cos 10x \cdot \sin 7x}$
$= -\frac{\sin 2x}{\cos 10x}$
$= R.H.S$

Page : 73 , Block Name : Exercise 3.3

Q17 Prove the following: $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x} = \tan 4x$

Answer. It is known that
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
$\therefore L \cdot H \cdot S. = \frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$
$2\sin\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)$
$2\cos\left(\frac{5x+3x}{2}\right)\cdot\cos\left(\frac{5x-3x}{2}\right)$
$= \frac{2\sin 4x \cdot \cos x}{2\cos 4x \cdot \cos x}$
$= \frac{\sin 4x}{\cos 4x}$
= tan 4x = R.H.S

Page : 73 , Block Name : Exercise 3.3

Q18 Prove the following: $\frac{\sin x-\sin y}{\cos x+\cos y} = \tan\frac{x-y}{2}$

Answer. It is known that

$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore = = \frac{\sin x-\sin y}{\cos x+\cos y}$

$$=\frac{2\cos\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}$$

$$\sin\left(\frac{x-y}{2}\right)$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)=R.H.S$$

Page : 73 , Block Name : Exercise 3.3

Q19 Prove the following: $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Answer. It is known that

$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore L.H.S.=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$=\frac{\sin 2x}{\cos 2x}$$

$$=\tan 2x$$

$$=R.HS$$

Page : 73 , Block Name : Exercise 3.3

Q20 Prove the following: $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Answer. It is known that

$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos^2 A-\sin^2 A=\cos 2A$

$\therefore L\cdot H\cdot S=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$$2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$
$$= -2 \times (-\sin x)$$
$$= 2\sin x = R.H.S$$

Page : 73 , Block Name : Exercise 3.3

Q21 Prove the following: $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x} = \cot 3x$

Answer. L.H.S $= \frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$
$$2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x$$
$$= \frac{2\sin(4x+2x}{2})\cos\left(\frac{4x-2x}{2}\right) + \sin 3x$$
$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$
$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$
$$= \cot 3x = R.H.S$$

Page : 73 , Block Name : Exercise 3.3

Q22 Prove the following: $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Answer.

$$\text{L. H.S.} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$
$$= \cot x \cot 2x - \cot 3x(\cot 2x + \cot x)$$
$$= \cot x \cot 2x - \cot(2x + x)(\cot 2x + \cot x)$$
$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$
$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$
$$= \cot x \cot 2x - (\cot 2x \cot x - 1)$$
$$= 1 = R.H.S$$

Page : 74 , Block Name : Exercise 3.3

Q23 Prove the following: $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Answer. It is known that

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$\therefore$ L H.S. $= \tan 4x = \tan 2(2x)$

$= \frac{2\tan 2x}{1-\tan^2(2x)}$

$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$

$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{1-\frac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}]}$

$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$

$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$

$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = RHS$

Page : 74 , Block Name : Exercise 3.3

Q24 Prove the following: $\cos 4x = 1-8\sin^2 x\cos^2 x$

Answer.

L.H.S. $= \cos 4x$

$= \cos 2(2x)$

$= 1-2\sin^2 2x\ [\cos 2A = 1-2\sin^2 A]$

$= 1-2\left(2\sin x\cos^2 x\right)^2 [\sin 2A = 2\sin A\cos A]$

$= 1-8\sin^2 x\cos^2 x$

$=$ R.H.S.

Page : 74 , Block Name : Exercise 3.3

Q25 Prove the following: $\cos 6x = 32\cos^6 x-48\cos^4 x+18\cos^2 x-1$

Answer.

L. H.S. $= \cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x-3\cos 2x\ [\cos 3A = 4\cos^3 A-3\cos A]$

$= 4\left[\left(2\cos^2 x-1\right)^3-3\left(2\cos^2 x-1\right)\ [\cos 2x = 2\cos^2 x-1]\right.$

$= 4\left[\left(2\cos^2 x\right)^3-(1)^3-3\left(2\cos^2 x\right)^2+3\left(2\cos^2 x\right)\right]-6\cos^2 x+3$

$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$
$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$
$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$
$= \text{R. H. S}$

Page : 74 , Block Name : Exercise 3.3

Exercise 3.4

Q1 Find the principal and general solutions of the following equations:
$\tan x = \sqrt{3}$

Answer. $\tan x = \sqrt{3}$
It is known that $\tan \frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$
Therefore, the principal solutions are x = $\frac{\pi}{3}$ and $\frac{4\pi}{3}$
Now, tan x = $\tan \frac{\pi}{3}$
$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$
Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$.

Page : 78 , Block Name : Exercise 3.4

Q2 Find the principal and general solutions of the following equations: sec x = 2

Answer. Sec x = 2
It is known that $\sec \frac{\pi}{3} = 2$ and $\sec \frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec \frac{\pi}{3} = 2$
Therefore, the principal solutions are x = $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
Now $_7 \sec x = \sec \frac{\pi}{3}$
$\Rightarrow \cos x = \cos \frac{\pi}{3}$
$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$
Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Page : 78 , Block Name : Exercise 3.4

Q3 Find the principal and general solutions of the following equations: cot x = $-\sqrt{3}$

Answer. Cot x = $-\sqrt{3}$
It is known that $\cot \frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi-\frac{\pi}{6}\right)=-\cot\frac{\pi}{6}=-\sqrt{3}$ and $\cot\left(2\pi-\frac{\pi}{6}\right)=-\cot\frac{\pi}{6}=-\sqrt{3}$

i.e., $\cot\frac{5\pi}{6}=-\sqrt{3}$ and $\cot\frac{11\pi}{6}=-\sqrt{3}$

Therefore, the principal solutions are x = $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Now, $\cot x=\cot\frac{5\pi}{6}$

$\Rightarrow \tan x=\tan\frac{5\pi}{6} \quad \left[\cot x=\frac{1}{\tan x}\right]$

$\Rightarrow x=n\pi+\frac{5\pi}{6}$, where $n\in Z$

Therefore, the general solution is $x=n\pi+\frac{5\pi}{6}$, where $n\in Z$.

Page : 78 , Block Name : Exercise 3.4

Q4 Find the principal and general solutions of the following equations: cosec x = – 2

Answer. Cosec x = -2

It is known that

$cosec\frac{\pi}{6}=2$

$\therefore \cos cc\left(\pi+\frac{\pi}{6}\right)=-\cos cc\frac{\pi}{6}=-2$ and cosec $\left(2\pi-\frac{\pi}{6}\right)=-\cos cc\frac{\pi}{6}=-2$

$cosec\frac{7\pi}{6}=-2$ and $cosec\frac{11\pi}{6}=-2$

Therefore, the principal solutions are x = $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Now, cosec x = cos ec $\frac{7\pi}{6}$

$\Rightarrow \sin x=\sin\frac{7\pi}{6} \quad \left[cosec x=\frac{1}{\sin x}\right]$

$\Rightarrow x=\mathbf{n}\pi+(-1)^n\frac{7\pi}{6}$, where $n\in Z$

Therefore, the general solution is $x=n\pi+(-1)^n\frac{7\pi}{6}$, where $n\in Z$.

Page : 78 , Block Name : Exercise 3.4

Q5 Find the general solution for each of the following equations: cos 4 x = cos 2 x

Answer.

$\cos 4x=\cos 2x$

$\Rightarrow \cos 4x-\cos 2x=0$

$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)=0$

$\left[\because \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x\sin x=0$

$\Rightarrow \sin 3x=0 \quad$ or $\quad \sin x=0$

$\therefore 3x=n\pi$

$\Rightarrow x=\frac{n\pi}{3}$ or $x=n\pi$, where $n\in Z$

Page : 78 , Block Name : Exercise 3.4

Q6 Find the general solution for each of the following equations: cos 3x + cos x – cos 2x = 0

Answer.

$\cos 3x+\cos x-\cos 2x=0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-\cos 2x=0 \quad \left[\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x\cos x-\cos 2x=0$

$\Rightarrow \cos 2x(2\cos x-1)=0$

$\Rightarrow \cos 2x=0$ or $2\cos x-1=0$

$\Rightarrow \cos 2x=0$ or $\cos x=\frac{1}{2}$

$\therefore 2\mathrm{x}=(2\mathrm{n}+1)\frac{\pi}{2}$ or $\cos\mathrm{x}=\cos\frac{\pi}{3}$, where $\mathrm{n}\in Z$

$\Rightarrow \mathrm{x}=(2\mathrm{n}+1)\frac{\pi}{4}$ or $\mathrm{x}=2\mathrm{n}\pi\pm\frac{\pi}{3}$, where $\mathrm{n}\in Z$

Page : 78 , Block Name : Exercise 3.4

Q7 Find the general solution for each of the following equations: sin 2x + cos x = 0

Answer.

$\sin 2x+\cos x=0$

$\Rightarrow 2\sin x\cos x+\cos x=0$

$\Rightarrow \cos x(2\sin x+1)=0$

$\Rightarrow \cos x=0$ or $2\sin x+1=0$

Now, $\cos x=0\Rightarrow \cos x=(2n+1)\frac{\pi}{2}$, where $n\in Z$

$2\sin x+1=0$

$\Rightarrow \sin x=\frac{-1}{2}=-\sin\frac{\pi}{6}=\sin\left(\pi+\frac{\pi}{6}\right)=\sin\left(\pi+\frac{\pi}{6}\right)=\sin\frac{7\pi}{6}$

$\Rightarrow \mathrm{x}=\mathrm{n}\pi+(-1)^{\mathrm{n}}\frac{7\pi}{6}$, where $\mathrm{n}\in Z$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi+(-1)^n\frac{7\pi}{6}, n\in Z.$

Page : 78 , Block Name : Exercise 3.4

Q8 Find the general solution for each of the following equations: $\sec^2 2x=1-\tan 2x$

Answer.

$\sec^2 2x=1-\tan 2x$

$\Rightarrow 1+\tan^2 2x=1-\tan 2x$

$\Rightarrow \tan^2 2x+\tan 2x=0$

$$\Rightarrow \tan 2x(\tan 2x+1)=0$$
$$\Rightarrow \tan 2x=0 \quad \text{or} \quad \tan 2x+1=0$$

$$\text{Now, } \tan 2x=0$$
$$\Rightarrow \tan 2x=\tan 0$$
$$\Rightarrow 2x=n\pi+0, \text{ where } n\in Z$$
$$\Rightarrow x=\frac{n\pi}{2}, \text{ where } n\in Z$$
$$\tan 2x+1=0$$
$$\Rightarrow \tan 2x=-1=-\tan\frac{\pi}{4}=\tan\left(\pi-\frac{\pi}{4}\right)=\tan\frac{3\pi}{4}$$
$$\Rightarrow 2x=n\pi+\frac{3\pi}{4}, \text{ where } n\in Z$$
$$\Rightarrow x=\frac{n\pi}{2}+\frac{3\pi}{8}, \text{ where } n\in Z$$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2}+\frac{3\pi}{8}, n\in Z$.

Page : 78 , Block Name : Exercise 3.4

Q9 Find the general solution for each of the following equations: sin x + sin 3x + sin 5x = 0

Answer. sin x + sin 3x + sin 5x = 0

$$(\sin x+\sin 5x)+\sin 3x=0$$
$$\Rightarrow \left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right]+\sin 3x=0 \quad \left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$\Rightarrow 2\sin 3x\cos(-2x)+\sin 3x=0$$
$$\Rightarrow 2\sin 3x\cos 2x+\sin 3x=0$$
$$\Rightarrow \sin 3x(2\cos 2x+1)=0$$
$$\Rightarrow \sin 3x=0 \quad \text{or} \quad 2\cos 2x+1=0$$
$$\text{Now, } \sin 3x=0 \Rightarrow 3x=n\pi, \text{ where } n\in Z$$
$$\text{i.c., } x=\frac{n\pi}{3}, \text{ where } n\in Z$$
$$2\cos 2x+1=0$$
$$\Rightarrow \cos 2x=\frac{-1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)$$
$$\Rightarrow \cos 2x=\cos\frac{2\pi}{3}$$
$$\Rightarrow 2x=2n\pi\pm\frac{2\pi}{3}, \text{ where } n\in Z$$
$$\Rightarrow x=n\pi\pm\frac{\pi}{3}, \text{ where } n\in Z$$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi\pm\frac{\pi}{3}, n\in Z$

Page : 78 , Block Name : Exercise 3.4

Miscellaneous Exercise

Q1 Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Answer.

$$\begin{aligned}
&\text{L.H.S.}\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+\cos\tfrac{5\pi}{13}+\cos\tfrac{5\pi}{13}\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)\ \left[\cos x+\cos y=2\cos\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right)\right]\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\tfrac{4\pi}{13}\cos\left(\tfrac{-\pi}{13}\right)\\
&=2\cos\tfrac{\pi}{13}\cos\tfrac{9\pi}{13}+2\cos\tfrac{4\pi}{13}\cos\tfrac{\pi}{13}\\
&=2\cos\tfrac{\pi}{13}\left[\cos\tfrac{9\pi}{13}+\cos\tfrac{4\pi}{13}\right]\\
&=2\cos\tfrac{\pi}{13}\left[2\cos\left(\tfrac{9\pi}{13}+\tfrac{4\pi}{13}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]\\
&=2\cos\tfrac{\pi}{13}\left[2\cos\tfrac{\pi}{2}\cos\tfrac{5\pi}{26}\right]\\
&=2\cos\tfrac{\pi}{13}\times 2\times 0\times\cos\tfrac{5\pi}{26}\\
&=0=\text{R. H. S}
\end{aligned}$$

Page : 81 , Block Name : Miscellaneous Exercise

Q2 Prove that: $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$

Answer.

$$\begin{aligned}
&\text{L.H.S.}\\
&=(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x\\
&=\sin 3x\sin x+\sin^2 x+\cos 3x\cos x-\cos^2 x\\
&=\cos 3x\cos x+\sin 3x\sin x-\left(\cos^2 x-\sin^2 x\right)\\
&=\cos(3x-x)-\cos 2x\quad[\cos(A-B)=\cos A\cos B+\sin A\sin B]\\
&=\cos 2x-\cos 2x\\
&=0\\
&=R.H.S
\end{aligned}$$

Page : 81 , Block Name : Miscellaneous Exercise

Q3 Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Answer.

$$\begin{aligned}
\text{L.H.S. } &= (\cos x+\cos y)^2+(\sin x-\sin y)^2 \\
&= \cos^2 x+\cos^2 y+2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y \\
&= \left(\cos^2 x+\sin^2 x\right)+\left(\cos^2 y+\sin^2 y\right)+2(\cos x\cos y-\sin x\sin y) \\
&= 1+1+2\cos(x+y) \qquad [\cos(A+B)=(\cos A\cos B-\sin A\sin B)] \\
&= 2+2\cos(x+y)
\end{aligned}$$

$$\begin{aligned}
&= 2[1+\cos(\mathrm{x}+\mathrm{y})] \\
&= 2\left[1+2\cos^2\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)-1\right] \qquad \left[\cos 2\mathrm{A}=2\cos^2\mathrm{A}-1\right] \\
&= 4\cos^2\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)=\mathrm{R.H.S}
\end{aligned}$$

Page : 82 , Block Name : Miscellaneous Exercise

Q4 Prove that: $(\cos x-\cos y)^2+(\sin x-\sin y)^2=4\sin^2\frac{x-y}{2}$

Answer. $\text{L.H.S}=(\cos x-\cos y)^2+(\sin x-\sin y)^2$

$$\begin{aligned}
&= \cos^2 x+\cos^2 y-2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y \\
&= \left(\cos^2 x+\sin^2 x\right)+\left(\cos^2 y+\sin^2 y\right)-2[\cos x\cos y+\sin x\sin y] \\
&= 1+1-2[\cos(x-y)] \qquad [\cos(A-B)=\cos A\cos B+\sin A\sin B] \\
&= 2[1-\cos(x-y)] \\
&= 2\left[1-\left\{1-2\sin^2\left(\tfrac{x-y}{2}\right)\right\}\right] \qquad \left[\cos 2A=1-2\sin^2 A\right] \\
&= 4\sin^2\left(\tfrac{x-y}{2}\right)=R.H.S
\end{aligned}$$

Page : 82 , Block Name : Miscellaneous Exercise

Q5 Prove that: sin x + sin 3x + sin 5x + sin 7x = 4 cos x cos 2x sin 4x

Answer. It is known that $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$$\begin{aligned}
\text{L.H.S. } &= \sin x+\sin 3x+\sin 5x+\sin 7x \\
&= (\sin x+\sin 5x)+(\sin 3x+\sin 7x) \\
&= 2\sin\left(\tfrac{x+5x}{2}\right)\cdot\cos\left(\tfrac{x-5x}{2}\right)+2\sin\left(\tfrac{3x+7x}{2}\right)\cos\left(\tfrac{3x-7x}{2}\right)
\end{aligned}$$

$= 2\sin 3x\cos(-2x) + 2\sin 5x\cos(-2x)$
$= 2\sin 3x\cos 2x + 2\sin 5x\cos 2x$
$= 2\cos 2x[\sin 3x + \sin 5x]$
$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cdot\cos\left(\frac{3x-5x}{2}\right)\right]$
$= 2\cos 2x[2\sin 4x\cdot\cos(-x)]$
$= 4\cos 2x\sin 4x\cos x = \text{ R.H.S.}$

Page : 82 , Block Name : Miscellaneous Exercise

Q6 Prove that: $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

Answer. It is known that
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\text{L.H.S} = \frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$
$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)-\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$
$= [2\sin 6x\cdot\cos x] + [2\sin 6x\cdot\cos 3x]$
$= [2\cos 6x\cdot\cos x] + [2\cos 6x\cdot\cos 3x]$
$= \frac{2\sin 6x[\cos x+\cos 3x]}{2\cos 6x[\cos x+\cos 3x]}$
$= \tan 6x$
$= \text{ R.H.S.}$

Page : 82 , Block Name : Miscellaneous Exercise

Q7 Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Answer. $\text{L.H.S} = \sin 3x + \sin 2x - \sin x$
$= \sin 3x + (\sin 2x - \sin x)$
$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \quad \left[\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$
$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$
$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$
$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \quad [\sin 2A = 2\sin A\cdot\cos B]$
$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$
$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right] \left[\left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]\right.$

$= 2\cos\left(\frac{3x}{2}\right) \cdot 2\sin x \cos\left(\frac{x}{2}\right)$

$= 4\sin x \cos\left(\frac{x}{2}\right) \cos\left(\frac{3x}{2}\right) = R.H.S$

Page : 82 , Block Name : Miscellaneous Exercise

Q8 Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in $\tan' x = -\frac{4}{3}, x$ in quadrant II

Answer. Here, x is in quadrant II.

$\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.

It is given that $\tan x = -\frac{4}{3}$

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

As x is in quadrant II, cos x is negative.

$\cos x = \frac{-3}{5}$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow \frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow 2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2\frac{x}{2} = \frac{2}{5}$

$\Rightarrow \cos^2\frac{x}{2} = \frac{1}{5}$

$\Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ $\qquad \because \cos\frac{x}{2}$ is positive $]$

$\therefore \cos\frac{x}{2} = \frac{\sqrt{5}}{5}$

$\sin^2\frac{x}{2} + \cos^2\frac{x}{2} = 1$

$\Rightarrow \sin^2\frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$

$\Rightarrow \sin^2\frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$

$\Rightarrow \sin\frac{x}{2} = \frac{2}{\sqrt{5}}$ $\left[\because \sin\frac{x}{2} \text{ is positive}\right]$

$\therefore \sin\frac{x}{2} = \frac{2\sqrt{5}}{5}$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)}=2$$

Thus, the respective values of $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2} are \frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}$, and 2

Page : 82 , Block Name : Miscellaneous Exercise

Q9 Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in $\cos x=-\frac{1}{3}, x$ in quadrant III.

Answer. Here, x is in quadrant III.

i.e., $\pi<\mathrm{x}<\frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2}<\frac{\mathrm{x}}{2}<\frac{3\pi}{4}$

Therefore,

$\cos\frac{x}{2} and \tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x=-\frac{1}{3}$.

$\cos x=1-2\sin^2\frac{x}{2}$

$\Rightarrow \sin^2\frac{x}{2}=\frac{1-\cos x}{2}$

$\Rightarrow \sin^2\frac{x}{2}=\frac{1-\left(-\frac{1}{3}\right)}{2}=\frac{\left(1+\frac{1}{3}\right)}{2}=\frac{3}{2}=\frac{2}{3}$

$\Rightarrow \sin\frac{\mathrm{x}}{2}=\frac{\sqrt{2}}{\sqrt{3}} \quad \left[\because \sin\frac{\mathrm{x}}{2} \text{ is positive}\right]$

$\therefore \sin\frac{x}{2}=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

$\cos x=2\cos^2\frac{x}{2}-1$

$\Rightarrow \cos^2\frac{x}{2}=\frac{1+\cos x}{2}=\frac{1+\left(-\frac{1}{3}\right)}{2}=\frac{\left(\frac{3-1}{3}\right)}{2}=\frac{\left(\frac{2}{3}\right)}{2}=\frac{1}{3}$

$\Rightarrow \cos\frac{x}{2}=-\frac{1}{\sqrt{3}}$

$\therefore \cos\frac{x}{2}=-\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{-\sqrt{3}}{3}$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)}=-\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

Page : 82 , Block Name : Miscellaneous Exercise

Q10 Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in $\sin x=\frac{1}{4}, x$ is in quadrant II.

Answer. Here, x is in quadrant II.

i.e., $\frac{\pi}{2}<x<\pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore,

$\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$

$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$ [$\cos x$ is negative in quadrant II]

$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$

$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} \quad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$

$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$

$= \sqrt{\frac{8+2\sqrt{15}}{16}}$

$= \frac{\sqrt{8+2\sqrt{15}}}{4}$

$\cos^2 \frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$

$\Rightarrow \cos \frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} \quad \left[\because \cos \frac{x}{2} \text{ is positive}\right]$

$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$

$= \sqrt{\frac{8-2\sqrt{15}}{16}}$

$= \frac{\sqrt{8-2\sqrt{15}}}{4}$

$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right.}{\left.\frac{\sqrt{8-2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$

$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$

$= \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$

Thus, the respective values of $\sin \frac{x}{2}, \cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4 + \sqrt{15}$.