

NCERT Solutions for Class 12 Maths Chapter 13 Probability Ex 13.1

Ex 13.1 Class 12 Maths Question 1

Given that E and F are events such that

$P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$

find $P(E|F)$ and $P(F|E)$.

Solution:

Given: $P(E) = 0.6$, $P(F) = 0.3$, $P(E \cap F) = 0.2$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

Ex 13.1 Class 12 Maths Question 2

Compute $P(A|B)$ if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

Solution:

Given: $P(B) = 0.5$, $P(A \cap B) = 0.32$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.50} = \frac{32}{50} = 0.64$$

Ex 13.1 Class 12 Maths Question 3.

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find

(i) $P(A \cap B)$

(ii) $P(A/B)$

(iii) $P(A \cup B)$

Solution:

$$(i) P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.4 = \frac{P(A \cap B)}{0.8}$$

$$\therefore P(A \cap B) = 0.4 \times 0.8 = 0.32$$

$$(ii) P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

$$(iii) P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ = 0.8 + 0.5 - 0.32 = 1.30 - 0.32 = 0.98$$

Ex 13.1 Class 12 Maths Question 4.

Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$.

Solution:

Given:

$$2P(A) = P(B) = \frac{5}{13} \text{ and } P(A|B) = \frac{2}{5}$$

$$\therefore P(A) = \frac{5}{26}, P(B) = \frac{5}{13}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$= \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$$

$$\text{Now } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$$

Ex 13.1 Class 12 Maths Question 5.

If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$,

Find

(i) $P(A \cap B)$

(ii) $P(A|B)$

(iii) $P(B|A)$

Solution:

Given:

$$P(A) = \frac{6}{11}, P(B) = \frac{5}{11} \text{ and } P(A \cup B) = \frac{7}{11},$$

$$(i) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$$

$$(ii) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$$

$$(iii) P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6} = \frac{2}{3}$$

Ex 13.1 Class 12 Maths Question 6.

Determine $P(E/F)$ in question 6 to 9:

A coin is tossed three times, where

(i) E: head on third toss F: heads on first two tosses.

(ii) E: at least two heads F: at most two heads

(iii) E: at most two tails F: at least one tail

Solution:

(i) E = Head occurs on third toss as {HHH, HTH, THH, TTH}

F : Heads on first two tosses = {HHH, HHT} $E \cap F = \{HHH\}$

$$P(E \cap F) = \frac{1}{8}, P(F) = \frac{1}{4}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{8} \times 4 = \frac{1}{2}$$

(ii) E : At least two heads

= {HHT, HTH, THH, HHH}

F : At most two heads

= {TTT, HTT, THT, HTT, HHT, HTH, THH}

$E \cap F = \{HHT, HTH, THH\}$

$$P(E \cap F) = \frac{3}{8}, P(F) = \frac{7}{8},$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3}{8} \div \frac{7}{8} = \frac{3}{7}$$

(iii) E : At most two tails

= {HHT, THT, TTH, HHT, HTH, THH, HHH}

F : { THH, HTH, HHT, TTH, THT, HTT, TTT}

$E \cap F = \{HTT, THT, TTH, THH, HTH, HHT\}$

$$P(E \cap F) = \frac{6}{8}, P(F) = \frac{7}{8},$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{6}{8} \div \frac{7}{8} = \frac{6}{7}$$

Ex 13.1 Class 12 Maths Question 7.

Two coins are tossed once

(i) E: tail appears on one coin F: one coin shows head

(ii) E: no tail appears F: no head appears.

Solution:

$S = \{HH, TH, HT, TT\}$ $n(S) = 4$

(i) E : tail appears on one coin

$E = \{TH, HT\}$, $P(E) = \frac{1}{2}$

F : one coin shows head,

$F = \{TH, HT\}$, $P(F) = \frac{1}{2}$

$E \cap F = \{TH, HT\}$, $n(E \cap F) = 2$

$P(E \cap F) = \frac{2}{4} = \frac{1}{2}$,

Thus $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$.

(ii) $E = \{HH\}$, $n(E) = 1$

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$

$F = \{TT\} \Rightarrow n(F) = 1$

$P(F) = \frac{n(F)}{n(S)} = \frac{1}{4}$,

$E \cap F = \emptyset$, $n(E \cap F) = 0$, $P(E \cap F) = 0$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{1/4} = 0$.

Ex 13.1 Class 12 Maths Question 8.

A die is thrown three times.

E : 4 appears on the third toss

F : 6 and 5 appears respectively on first two tosses.

Solution:

A die is thrown three times E : 4 appears on third toss = $\{(1,1,4), (1,2,4), (1,3,4), (1,4,4), (1,5,4), (1,6,4), (2,1,4), (2,2,4), (2,3,4), (2,4,4), (2,5,4), (2,6,4), (3,1,4), (3,2,4), (3,3,4), (3,4,4), (3,5,4), (3,6,4), (4,1,4), (4,2,4), (4,3,4), (4,4,4), (4,5,4), (4,6,4), (5,1,4), (5,2,4), (5,3,4), (5,4,4), (5,5,4), (5,6,4), (6,1,4), (6,2,4), (6,3,4), (6,4,4), (6,5,4), (6,6,4)\}$

These are 36 cases

F : 6 and 5 appears respectively on first two tosses = $\{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$

These are six cases $E \cap F = \{(6,5,4)\}$

$$P(E \cap F) = \frac{1}{216}$$

\therefore from three dice, number of the exhaustive cases

$$= 6 \times 6 \times 6 = 216$$

$$P(F) = \frac{6}{216},$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{216} \div \frac{6}{216} = \frac{1}{6}$$

Ex 13.1 Class 12 Maths Question 9.

Mother, father and son line up at random for a family picture:

E : son on one end, F : father in middle

Solution:

Mother (m), Father (f) and son (s) line up at random E : son on one end: {(s, m, f), (s, f, m), (f, m, s), (m, f, s)}

F: Father in middle: {(m, f, s), (s, f, m), (s, f, m)}

$E \cap F = \{(m, f, s), (s, f, m)\}$

$$P(E \cap F) = \frac{2}{3!} = \frac{2}{6} = \frac{1}{3}, \quad P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{3} \div \frac{1}{3} = 1$$

Ex 13.1 Class 12 Maths Question 10.

A Mack and a red die are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Solution:

(a) $n(S) = 6 \times 6 = 36$

Let A represent obtaining a sum greater than 9 and B represents black die resulted in a 5.

$$A = \{46, 64, 55, 36, 63, 45, 54, 65, 56, 66\}$$

$$n(A) = 10 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{10}{216}$$

$$B = \{51, 52, 53, 54, 55, 56\} \Rightarrow n(B) = 6$$

$$P(B) = \frac{6}{216},$$

$$A \cap B = \{55, 56\} \Rightarrow n(A \cap B) = 2$$

$$P(A \cap B) = \frac{2}{216},$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{216}}{\frac{6}{216}} = \frac{2}{6} = \frac{1}{3}.$$

(b) Let A denotes the sum is 8

$$\therefore A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

B = Red die results in a number less than 4
either first or second die is red.

$$B = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

$$A \cap B = \{(2, 6), (3, 5)\}$$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}, \quad P(B) = \frac{18}{36} = \frac{1}{2}$$

$$\text{Hence } P(A|B) = \frac{P(AB)}{P(B)} = \frac{1}{9}.$$

Ex 13.1 Class 12 Maths Question 11

A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$, Find

(i) $P(E|F)$ and $P(F|E)$

(ii) $P(E|G)$ and $P(G|E)$

(iii) $P((E \cup F) | G)$ and $P(E \cap F | G)$

Solution:

$$(i) E = \{1, 3, 5\}, F = \{2, 3\}, E \cap F = \{3\}$$

$$P(E) = \frac{3}{6}, P(F) = \frac{2}{6}, P(E \cap F) = \frac{1}{6},$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{6} \div \frac{2}{6} = \frac{1}{2}$$

$$P(FE) = \frac{P(E \cap F)}{P(E)} = \frac{1}{6} \div \frac{3}{6} = \frac{1}{3}$$

$$(ii) E = \{1, 3, 5\}, G = \{2, 3, 4, 5\}, E \cap G = \{3, 5\}$$

$$P(E) = \frac{3}{6}, P(G) = \frac{4}{6}, P(E \cap G) = \frac{2}{6}$$

$$P(E/G) = \frac{P(E \cap G)}{P(G)} = \frac{2}{6} \div \frac{4}{6} = \frac{2}{4} = \frac{1}{2}$$

$$P(G/E) = \frac{P(E \cap G)}{P(E)} = \frac{2}{6} \div \frac{3}{6} = \frac{2}{3}$$

$$(iii) E = \{1, 3, 5\}, F = \{2, 3\}, G = \{2, 3, 4, 5\}$$

$$E \cap G = \{3, 5\}, F \cap G = \{2, 3\},$$

$$(E \cap F) \cap G = \{3\}$$

$$P(E \cap G) = \frac{2}{6}, P(F \cap G) = \frac{2}{6}, P[(E \cap F) \cap G] = \frac{1}{6}$$

$$\text{Now } P(E \cup F/G)$$

$$= P(E/G) + P(F/G) - P[(E \cap F)/G]$$

$$= \frac{P(E \cap G)}{P(G)} + \frac{P(F \cap G)}{P(G)} - \frac{P[(E \cap F) \cap G]}{P(G)}$$

$$= \left(\frac{2}{6} \div \frac{4}{6} \right) + \left(\frac{2}{6} \div \frac{4}{6} \right) - \left(\frac{1}{6} \div \frac{4}{6} \right)$$

$$= \frac{2}{4} + \frac{2}{4} - \frac{1}{4} = \frac{3}{4}$$

$$P(E \cap F/G) = \frac{P[(E \cap F) \cap G]}{P(G)} = \frac{1}{6} \div \frac{4}{6} = \frac{1}{6}$$

Ex 13.1 Class 12 Maths Question 12.

Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(i) the youngest is a girl,

(ii) at least one is girl?

Solution:

Let first and second girls are denoted by G1 and G2 and Boys by B1 and B2.

Sample space

$$S = \{(G1G2), (G1B2), (G2B1), (B1B2)\}$$

Let A = Both the children are girls = {G1G2}

B = youngest child is a girls = {G1G2, B1G2}

C = at least one is a girl = {G1B2, G1G2, B1G2}

$$A \cap B = \{G1G2\},$$

$$A \cap C = \{G1G2\}$$

$$P(A \cap B) = \frac{1}{4}, P(A \cap C) = \frac{1}{4}$$

$$P(B) = \frac{2}{4}, P(C) = \frac{3}{4}$$

$$(i) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \div \frac{2}{4} = \frac{1}{2}$$

$$(ii) P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$$

Ex 13.1 Class 12 Maths Question 13.

An instructor has a question bank consisting of 300 easy True/ False questions, 200 difficult True/ False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Solution:

The given data may be tabulated as

	Easy	Difficult	Total
True/False	300	200	500
Multiple Choice	500	400	900
Total	800	600	1400

Let us denote E = Easy question, D = Difficult question,

T = true/ false question and

M = Multiple choice question number of easy multiple choice question = 500

Total numbers of question = 1400

$P(E \cap M)$ = Probability of selecting an easy and multiple choice question = $\frac{500}{1400}$

Total number of multiple choice question = 500 + 400 = 900

$P(M)$ = Probability of selecting one multiple choice question = $\frac{900}{1400}$

$$P(E/M) = \frac{P(E \cap M)}{P(M)} = \frac{500}{1400} \div \frac{900}{1400} = \frac{5}{9}$$

Ex 13.1 Class 12 Maths Question 14.

Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Solution:

$$S = \left[\begin{array}{cccccc} 11 & 21 & 31 & 41 & 51 & 61 \\ 12 & 22 & 32 & 42 & 52 & 62 \\ 13 & 23 & 33 & 43 & 53 & 63 \\ 14 & 24 & 34 & 44 & 54 & 64 \\ 15 & 25 & 35 & 45 & 55 & 65 \\ 16 & 26 & 36 & 46 & 56 & 66 \end{array} \right], n(S) = 36$$

Let A represents the event "the sum of numbers on the dice is 4"

and B represents the event "the two numbers appearing on throwing two dice are different."

$$A = \{13, 22, 31\}, n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$$

$$B = \left\{ \begin{array}{cccccc} 12 & 21 & 32 & 41 & 51 & 61 \\ 13 & 23 & 33 & 43 & 53 & 63 \\ 14 & 24 & 34 & 44 & 54 & 64 \\ 15 & 25 & 35 & 45 & 55 & 65 \\ 16 & 26 & 36 & 46 & 56 & 66 \end{array} \right\}$$

$$n(B) = 30 \Rightarrow P(B) = \frac{30}{36}$$

$$A \cap B = \{13, 31\} \Rightarrow n(A \cap B) = 2$$

$$P(A \cap B) = 2/36$$

Hence required probability

$$= P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{2}{30} = \frac{1}{15}$$

Ex 13.1 Class 12 Maths Question 15.

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail' given that 'at least one die shows a 3'.

Solution:

Let there be n throws in which a multiple of 3 occurs every time.

Probability of getting a multiple of 3 (i.e. 3 or 6)

$$\text{in one throw} = \frac{2}{6} = \frac{1}{3}$$

\therefore Probability of getting a multiple of 3 in n

$$\text{throws} = \left(\frac{1}{3}\right)^n$$

$$\therefore \text{Probability of getting a 6 in } n \text{ throws} = \left(\frac{1}{6}\right)^n$$

\Rightarrow Probability of getting at least a 3 in n throws

$$= \left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n$$

Let a multiple of 3 does not occur in $(n+1)^{\text{th}}$ throw.

\therefore Probability of getting 1, 2, 3, 4, 5 (not a multiple of 3) in $(n+1)^{\text{th}}$ throw $= \frac{4}{6} = \frac{2}{3}$

In the next throw a coin is tossed and tail occurs.

$$\therefore \text{Probability of getting a tail} = \frac{1}{2}$$

\Rightarrow Probability of getting at least a 3 and a tail in the end in $(n+2)^{\text{th}}$ throw.

$$= \left[\left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n \right] \times \frac{2}{3} \times \frac{1}{2}$$

As $n \rightarrow \infty$; the probability of getting at least a 3 till tail is obtained.

$$= \sum_{n=1}^{\infty} \left[\left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n \right] \times \frac{2}{3} \times \frac{1}{2}$$

$$= \left[\frac{\frac{1}{3}}{1 - \frac{1}{3}} - \frac{\frac{1}{6}}{1 - \frac{1}{6}} \right] \times \frac{1}{3} = \left(\frac{1}{3} \times \frac{3}{2} - \frac{1}{6} \times \frac{6}{5} \right) \times \frac{1}{3}$$

$$= \left(\frac{1}{2} - \frac{1}{5} \right) \times \frac{1}{3} = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10}$$

Ex 13.1 Class 12 Maths Question 16.

If $P(A) = \frac{1}{2}$, $P(B) = 0$ then $P(A|B)$ is

- (a) 0
- (b) $\frac{1}{2}$
- (c) not defined
- (d) 1

Solution:

$$P(A) = P(B) = 0$$

$$\therefore P(A \cap B) = 0$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{0}$$

Thus option C is correct

Ex 13.1 Class 12 Maths Question 17.

If A and B are events such that $P(A|B) = P(B|A)$ then

- (a) $A \subset B$ but $A \neq B$
- (b) $A = B$
- (c) $A \cap B = \emptyset$
- (d) $P(A) = P(B)$

Solution:

$$P(A|B) = P(B|A)$$

Thus option (d) is correct.

$$\frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(A) = P(B)$$

NCERT Solutions for Class 12 Maths Chapter 13 Probability Ex 13.2

Question 1

If, $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$ find $P(A \cap B)$ if A and B are independent events.

Solution:

A and B are independent if $P(A \cap B)$

$$= P(A) \times P(B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$$

Question 2.

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Solution:

Number of exhaustive cases = 52

Number of black cards = 26

One black card may be drawn in 26 ways

\therefore Probability of getting a black card,

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

After drawing one card, number of cards left

= 51

After drawing a black card number of black cards

left = 25

\therefore Probability of getting both the black cards,

$$P(A)P(B/A) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Question 3.

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Solution:

$S = \{12 \text{ good oranges}, 3 \text{ bad oranges}\}$,

$n(S) = 15$

$$P(\text{a box is approved}) = \frac{C(12,3)}{C(15,3)} = \frac{12 \times 11 \times 10}{15 \times 14 \times 13} = \frac{44}{91}$$

Question 4.

A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not

Solution:

When a coin is thrown, head or tail will occur

Probability of getting head $P(A) = \frac{1}{2}$

When a die is tossed 1, 2, 3, 4, 5, 6 one of them will appear

\therefore Probability of getting 3 = $P(B) = \frac{1}{6}$

When a die and coin is tossed, total number of cases are

H1, H2, H3, H4, H5, H6

T1, T2, T3, T4, T5, T6

Head and 3 will occur only in 1 way

\therefore Probability of getting head and 3 = $\frac{1}{12}$

$$\text{i.e., } P(A \cap B) = \frac{1}{12}, P(A) \times P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$\therefore P(A \cap B) = P(A) \times P(B)$$

\Rightarrow Events A and B are independent.

Question 5.

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even', and B be the event, 'the number is red'. Are A and B independent?

Solution:

Even numbers on die are 2,4,6

∴ Probability of getting even number

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

There are two colours of the die – red and green.

Probability of getting red colour, $P(B) = \frac{1}{2}$

Even number in red colour is 2

∴ Probability of getting red colour and even number

$$= P(A \cap B) = \frac{1}{6}$$

$$P(A)P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} \neq P(A \cap B) = \frac{1}{6}$$

⇒ A and B are not independent.

Question 6.

Let E and F be the events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

Solution:

$$P(E) = \frac{3}{5}, P(F) = \frac{3}{10},$$

$$\therefore P(E) \times P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$$

$$P(E \cap F) \neq P(E) \times P(F)$$

∴ The event A and B are not independent.

Question 7.

Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are

(i) mutually exclusive

(ii) independent.

Sol:

Sol. Let $P(A \cap B) = x$, Now $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$, $P(B) = p$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore \frac{3}{5} = \frac{1}{2} + p - x$$

$$\text{or } p - x = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10} \quad \dots(i)$$

(i) When events A and B are mutually exclusive

$$x=0, p=1/10$$

(ii) When events A and B are independent

$$P(A \cap B) = P(A) \times P(B)$$

$$x = \frac{1}{2} \times p \quad \dots(ii)$$

Also $p - x = \frac{1}{10}$ from (i), subtracting value of

$$x = \frac{p}{2} \text{ in } p - x = \frac{1}{10}, \text{ we get}$$

$$p - \frac{p}{2} = \frac{1}{10} \Rightarrow \frac{p}{2} = \frac{1}{10} \Rightarrow p = \frac{1}{5}$$

Question 8.

Let A and B independent events $P(A) = 0.3$ and $P(B) = 0.4$. Find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A | B)$

(iv) $P(B | A)$

Solution:

$$P(A) = 0.3,$$

$$P(B) = 0.4$$

A and B are independent events

$$(i) \therefore P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12.$$

$$(ii) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.4 - 0.3 \times 0.4 = 0.7 - 0.12 = 0.58.$$

$$(iii) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = 0.3$$

$$(iv) P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) \cdot P(B)}{P(A)} = 0.4$$

Question 9.

If A and B are two events, such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, and $P(A \cap B) = \frac{1}{8}$. Find $P(\text{not } A \text{ and not } B)$

Solution:

Event not A and not B = $\bar{A} \cap \bar{B}$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

$$\text{But } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

$$\left[\because P(A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{8} \right]$$

$$\therefore P(\bar{A} \cap \bar{B}) = 1 - \frac{5}{8} = \frac{3}{8}$$

Question 10

Events A and B are such that

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether A and B are independent

Solution:

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

$$\Rightarrow \frac{1}{4} = 1 - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4},$$

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}, P(A \cap B) = \frac{3}{4}$$

$$\Rightarrow P(A \cap B) \neq P(A) \times P(B)$$

Hence A and B are not independent events.

Question 11

Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find

(i) $P(A \text{ and } B)$

(ii) $P(A \text{ and not } B)$

(iii) $P(A \text{ or } B)$

(iv) $P(\text{neither } A \text{ nor } B)$

Solution:

(i) A and B are independent events

$$\therefore P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

$$= 0.3 \times 0.6 \quad [\because P(A) = 0.3, P(B) = 0.6]$$

$$\therefore P(A \text{ and } B) = 0.18$$

(ii) $P(A \text{ and } B) =$ and

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= 0.3 - 0.18 = 0.12$$

$$[\because P(A) = 0.3 \text{ and } P(A \cap B) = 0.18 \text{ from (i)}]$$

(iii) Here $P(A) = 0.3$, $P(B) = 0.6$ and

$$P(A \cap B) = 0.18$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.6 - 0.18 = 0.9 - 0.18 = 0.72$$

$$P(A \text{ or } B) = 0.72$$

(iv) $P(\text{neither } A \text{ nor } B)$

$$= P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$$

[when A and B are independent]

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \times P(\bar{B})$$

$$(1 - 0.3) \times (1 - 0.6)$$

$$= 0.7 \times 0.4 = 0.28$$

Question 12

A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

Let A represents an odd number.

$$A = \{1, 3, 5\}, n(A) = 3 \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = 3, P(\text{atleast one success}) = 1 - P(X = 0)$$

$$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8}.$$

Question 13

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) the first ball is "black and the second is red.

(iii) one of them is black and other is red.

Solution:

$$S = \{10 \text{ black balls}, 8 \text{ red balls}\}, n(S) = 18$$

Let drawing of a red ball be a success.

$$A = \{8 \text{ red balls}\}, n(A) = 8$$

$$P(A) = \frac{8}{18} = \frac{4}{9}, \Rightarrow P(\bar{A}) = 1 - \frac{4}{9} = \frac{5}{9} \quad n = 2$$

(i) $P(\text{both are red balls}) = P(A).$

$$P(A) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

(ii) $P(\text{first is black ball and second is red})$

$$= P(\bar{A}).P(A) = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$$

(iii) $P(\text{one of them is black and other is red})$

$$= P(\bar{A}).P(A) + P(A).P(\bar{A})$$

$$= \frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9} = \frac{40}{81}.$$

Question 14

Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

Solution:

Probability that A solves the problem = $\frac{1}{2}$

=> Probability that A does not solve the problem

$$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Probability that B solves the problem = $\frac{1}{3}$

=> Probability that B does not solve the problem

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

(i) Probability that problem is not solved

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) P(\bar{B}) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

=> Probability that problem is solved

$$P(A \cap B) = 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) Exactly one of them solves the problem

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) P(\bar{B}) + P(\bar{A}) P(B)$$

Since A and B are independent events So $\bar{A} \cap B$

and $A \cap \bar{B}$ are also independent

Now

$$P(A) = \frac{1}{2}, P(B) = \frac{2}{3}, P(\bar{A}) = \frac{1}{2}, P(\bar{B}) = \frac{1}{3}$$

Exactly one of them solves the problem

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Question 15

One card is drawn at random from a well-shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king or queen'

F: 'the card drawn is a queen or jack'.

Solution:

$$n(S) = 52$$

$$(i) E = \{13 \text{ spades}\}, P(E) = \frac{13}{52} = \frac{1}{4}$$

$$F = \{4 \text{ aces}\}, P(F) = \frac{4}{52} = \frac{1}{13}$$

$$E \cap F = \{\text{an ace of spade}\}, P(E \cap F) = \frac{1}{52}$$

$$\text{Also, } P(E) \cdot P(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

$$\therefore P(E \cap F) = P(E) \cdot P(F)$$

Hence, E and F are independent events.

$$(ii) E = \{26 \text{ black cards}\}, P(E) = \frac{26}{52} = \frac{1}{2}.$$

$$F = \{4 \text{ kings}\}, P(F) = \frac{4}{52} = \frac{1}{13}$$

$$E \cap F = \{2 \text{ black kings}\},$$

$$P(E \cap F) = \frac{2}{52} = \frac{1}{26}$$

$$\text{Now, } P(E) \cdot P(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

$$\therefore P(E \cap F) = P(E) \cdot P(F).$$

Hence E and F are independent events.

$$(iii) E = \{4 \text{ kings, 4 queens}\}, n(E) = 8$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

$$F = \{4 \text{ queens, 4 jacks}\}, P(F) = \frac{8}{52} = \frac{2}{13}$$

$$E \cap F = \{4 \text{ queens}\}$$

$$P(E \cap F) = \frac{4}{52} = \frac{1}{13}$$

$$\text{Now, } P(E) \cdot P(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$$

$$\text{Thus } P(E \cap F) \neq P(E) \cdot P(F)$$

Hence E and F are not independent events.

Question 16

In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers.

A student is selected at random

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspapers.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Solution:

(a) Let H and E represent the event that a student reads Hindi and English newspaper respectively

$$P(H) = 0.6, P(E) = 0.4, P(H \cap E) = 0.2$$

Probability that the student reads at least one paper

$$= P(H \cup E)$$

$$\text{Now } P(H) = 0.6, P(E) = 0.4, P(H \cap E) = 0.2$$

$$\therefore P(H \cup E) = 0.6 + 0.4 - 0.2 = 1 - 0.2 = 0.8$$

\therefore Probability that a student reads neither Hindi nor English newspaper

$$= 1 - P(H \cup E) = 1 - 0.8 = 0.2$$

(b) The probability that the student reads English newspaper if she reads Hindi

$$= P(E/H) = \frac{P(E \cap H)}{P(H)}$$

$$\text{Now } P(E \cap H) = 0.2, P(H) = 0.6$$

$$\therefore P(E/H) = \frac{0.2}{0.6} = \frac{1}{3}$$

(c) The probability that she reads Hindi newspaper if she reads English newspaper

$$= P(H/E) = \frac{P(H \cap E)}{P(E)} = 0.2, P(E) = 0.4$$

$$\therefore P(H/E) = \frac{P(H \cap E)}{P(E)} = \frac{0.2}{0.4} = \frac{1}{2}$$

Choose the correct answer in the following Question 17 and 18:

Question 17

The probability of obtaining an even prime number on each die when a pair of dice is rolled is

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{1}{12}$

(d) $\frac{1}{36}$

Solution:

$$(d) n(S) = 36$$

Let A represents an even prime number one each dice.

$$A = \{2\}, n(A) = 1 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Question 18

Two events A and B are said to be independent, if

(a) A and B are mutually exclusive

$$(b) P(A \cap B) = [1 - P(A)] [1 - P(B)]$$

$$(c) P(A) = P(B)$$

$$(d) P(A) + P(B) = 1$$

Solution:

$$(b) P(A \text{ and } B) = [1 - P(A)] [1 - P(B)] = P(A) \cdot P(B)$$

Thus option (b) is correct.

NCERT Solutions for Class 12 Maths Chapter 13 Probability Ex 13.3

Question 1.

An urn contain 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution:

Urn contain 5 red and 5 black balls.

(i) Let a red ball is drawn.

$$\text{probability of drawing a red ball} = \frac{5}{10} = \frac{1}{2}$$

Now two red balls are added to the urn.

=> The urn contains 7 red and 5 black balls.

$$\text{Probability of drawing a red ball} = \frac{7}{12}$$

(ii) Let a black ball is drawn at first attempt

$$\text{Probability of drawing a black ball} = \frac{5}{10} = \frac{1}{2}$$

Next two black balls are added to the urn

Now urn contains 5 red and 7 black balls

$$\text{Probability of getting a red ball} = \frac{5}{12}$$

=> Probability of drawing a second ball as red

$$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}$$

Question 2.

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution:

Let A be the event that ball drawn is red and let E₁ and E₂ be the events that the ball drawn is from the first bag and second bag

respectively. $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$.

$P(A|E_1)$ = Probability of drawing a red ball from bag

$$I = \frac{4}{8} = \frac{1}{2}$$

$P(A|E_2)$ = Probability of drawing a red ball from bag

$$II = \frac{2}{8} = \frac{1}{4}$$

Therefore by Bayes' theorem

$P(E_1|A)$ = Probability that the red ball drawn is from bag I

$$= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}} = \frac{2}{3}$$

Question 3.

Of the students In a college, it is known that 60% reside In hostel and 40% are day scholars (not residing In hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student Is chosen at random from the college and he has an A- grade what Is the probability that the student is a hostlier?

Solution:

Let E_1 , E_2 and A represents the following:

E_1 = students residing in the hostel,

E_2 day scholars (not residing in the hostel)

and A = students who attain grade A

$$\text{Now } P(E_1) = \frac{60}{100}, P(E_2) = \frac{40}{100}$$

$$P(A|E_1) = \frac{30}{100}, P(A|E_2) = \frac{20}{100}$$

Now by Bayes' theorem

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} \\ &= \frac{\frac{60}{100} \times \frac{30}{100}}{\frac{60}{100} \times \frac{30}{100} + \frac{40}{100} \times \frac{20}{100}} = \frac{9}{13} \end{aligned}$$

Question 4.

In answering a question on a multiple choice test, a student either knows the answer or 3 guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Solution:

Let the event E_1 = student knows the answer , E_2 = He gusses the answer

$$P(E_1) = \frac{3}{4}, P(E_2) = \frac{1}{4}$$

Let A is the event that answer is correct, if the student knows the answer

$$\Rightarrow \text{Answer is correct} \quad \therefore P(A/E_1) = 1$$

$$\text{If he guesses the answer} \quad \therefore P(A/E_2) = \frac{1}{4}$$

\therefore Probability that a student knows the answer given that answer is correct is,

$$\begin{aligned} P(E_1/A) &= \frac{P(E_1) P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)} \\ &= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{3}{4} \times \frac{16}{13} = \frac{12}{13} \end{aligned}$$

Question 5.

A laboratory blood test is 99% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Solution:

Let

E_1 = The person selected is suffering from certain disease,

E_2 = The person selected is not suffering from certain disease.

A = The doctor diagnoses correctly

$$\text{Now } P(E_1) = 0.1\% = \frac{1}{1000} = 0.001$$

$$P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000} = 0.999$$

Question 6.

There are three coins. One is a two headed coin, another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it was the two headed coin?

Solution:

Let E_1 , E_2 , E_3 and A denotes the following:

E_1 = a two headed coin, E_2 = a biased coin,

E_3 = an unbiased coin, A = A head is shown

$$\text{Now } P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$$

$$P(A|E_1) = 1, P(A|E_2) = \frac{75}{100} = \frac{3}{4}, P(A|E_3) = \frac{1}{2}$$

By Bayes' theorem

$$P(E_1|A) = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{4}{9}$$

Question 7.

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01, 0.03, 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Solution:

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. Total number of drivers

$$= 2000 + 4000 + 6000 = 12,000$$

Probability of selecting a scooter driver

$$= P(E_1) = \frac{2000}{12000} = \frac{1}{6}$$

Probability of selecting a car driver

$$= P(E_2) = \frac{4000}{12000} = \frac{1}{3}$$

Probability of selecting a truck driver

$$= P(E_3) = \frac{6000}{12000} = \frac{1}{2}$$

Let A be the event that insured person meet with an accident.

Probability of accident of car drivers

$$= P(A|E_2) = 0.03$$

Probability of accident of truck drivers

$$= P(A|E_3) = 0.15$$

Probability of scooter driver who has met an accident that he is a scooter driver $P(E_1|A)$

$$= \frac{P(E_1) P(A|E_1)}{P(E_1) P(A|E_1) + P(E_2) P(A|E_2) + P(E_3) P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15} = \frac{1}{52}$$

Question 8

A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B.?

Solution:

E_1 and E_2 are the events the percentage of production of items by machine A and machine B respectively.

Let A denotes defective item.

Machine A's production of items = 60 %

Probability of production of items by machine

A, $P(E_1) = 60\% = 0.6$

Probability of production of items by machine

B, $P(E_2) = 40\% = 0.4$

Probability that machine A produced defective item

i.e. $P(A/E_1) = 0.02$, $P(A/E_2) = 0.01$

We have to find the probability of the defective item selected at random was from machine

$B = P(E_2/A)$

$$= \frac{P(E_2) P(A/E_2)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$= \frac{0.4 \times 0.01}{0.6 \times 0.02 + 0.4 \times 0.01} = \frac{0.04}{0.016} = \frac{4}{16} = \frac{1}{4}$$

Question 9.

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution:

Given: $P(G_1) = 0.6$, $P(G_2) = 0.4$

P represents the launching of new product $P(P|G_1) = 0.7$ and $P(P|G_2) = 0.3$

By Baye's theorem

$$P(G_2|P) = \frac{P(G_2) \times P(P/G_2)}{P(G_1) \times P(P/G_1) + P(G_2) \times P(P/G_2)}$$

$$= \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3} = \frac{2}{9}$$

Question 10.

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die?

Solution:

When a die is thrown there are 6 exhaustive cases.

If she gets 5 or 6 the probability of $E_1 = \frac{2}{6} = \frac{1}{3}$

i.e., $P(E_1) = \frac{1}{3}$

If she gets 1, 2, 3, 4 and probability of $E_2 = \frac{4}{6} = \frac{2}{3}$

When she gets 5 or 6, she throws a coin three times. The exhaustive cases are 8 viz.

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

One head may be obtained as {HHT, THT, TTH}

Probability of getting exactly one head = $\frac{3}{8}$

If A denotes the event of getting exactly one

head, then $P(A/E_1) = \frac{3}{8}$

When she gets 1, 2, 3, 4 she thrown a coin once.

The probability of getting one head = $\frac{1}{2}$

i.e. $P(A/E_2) = \frac{1}{2}$

Probability that she gets exactly one head when she threw 1, 2, 3 or 4 from die = $P(E_2/A)$

$\therefore P(E_2/A)$

$$= \frac{P(E_2) P(A/E_2)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8} + \frac{1}{3}} = \frac{8}{11}$$

Question 11.

A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the

time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Solution:

Let E_1 , E_2 , E_3 and A be the events defined as follows:

E_1 = the item is manufactured by the operator A

E_2 = the item is manufactured by the operator B

E_3 = the item is manufactured by the operator C

and A = the item is defective

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$$

$P(A | E_1)$ = Probability that the item drawn is

manufactured by operator A = $\frac{1}{100}$

$$\text{Similarly } P(A | E_2) = \frac{5}{100} \text{ and } P(A | E_3) = \frac{7}{100}$$

Now required probability = Probability that the item is manufactured by operator A given that the item drawn is defective = $P(E_1 | A)$

$$P(E_1 | A) = \frac{P(E_1)P(A | E_1)}{P(E_1)P(A | E_1) + P(E_2)P(A | E_2) + P(E_3)P(A | E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}} = \frac{5}{34}$$

Question 12.

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond?

Solution:

E_1 = Event that lost card is diamond,

E_2 = Event that lost card is not diamond.

There are 13 diamond cards, out of a pack of 52 cards

$$P(E_1) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$$

There are 39 cards which are not diamond.

$$P(E_2) = \frac{39}{52} = \frac{3}{4}$$

- (i) When one diamond card is lost, 12 diamond cards are left and in total 51 cards are left. Out of 12 cards 2 may be drawn in ${}^{12}C_2$ way.

\therefore Probability of getting 2 diamond cards when one diamond card is lost

$$P(A/E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12 \times 11}{51 \times 50}$$

Where A denotes the lost card

When diamond card is not lost, there are 13 diamond cards. The probability of drawing 2 diamond cards

$$= \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13 \times 12}{51 \times 50}$$

Probability that the lost card is diamond

$$= P(E_1/A) = \frac{P(E_1) P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times \frac{12 \times 11}{51 \times 50}}{\frac{1}{4} \times \frac{12 \times 11}{51 \times 50} + \frac{3}{4} \times \frac{13 \times 12}{51 \times 50}} = \frac{11}{50}$$

Question 13.

Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

- A. 4/5
- B. 1/2
- C. 1/5
- D. 2/5

Let A be the event that the man reports that head occurs in tossing a coin and let E₁ be the event that head occurs and E₂ be the event head does not occurs.

$$P(E_1) = \frac{1}{2} = P(E_2)$$

$P(A|E_1)$ = Probability that A reports that head occurs when head has actually occurs red on the coin

$$= \text{Probability that the man speaks truth} = \frac{4}{5}$$

$P(A|E_2)$ = Probability that A reports that head occurs when head has not occurred on the coin
= Probability that the man does not speak the truth

$$= 1 - \frac{4}{5} = \frac{1}{5}$$

By Baye's theorem

$P(E_1 | A)$ = Probability that the report of A that head occurs, is actually a head

$$= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{4}{5}$$

Thus option (a) is correct.

Question 14.

If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct:

- (a) $P(A|B) = \frac{P(B)}{P(A)}$
- (b) $P(A|B) < P(A)$
- (c) $P(A|B) \geq P(A)$
- (d) None of these

Solution: (c) $A \subset B \Rightarrow A \cap B = A$ and $P(B) \neq 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

$$P(B) \neq 0, \therefore \frac{P(A)}{P(B)} < 1$$

$$\Rightarrow P(A) < P(B) \therefore P(A|B) \geq P(A)$$

NCERT Solutions for Class 12 Maths Chapter 13 Probability Ex 13.4

Question 1.

State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

Solution:

$$P(0) + P(1) + P(2) = 0.4 + 0.4 + 0.2 = 1$$

It is a probability distribution.

(ii) $P(3) = -0.1$ which is not possible.

Thus it is not a probability distribution.

$$(iii) P(-1) + P(0) + P(1) = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$$

Thus it is not a probability distribution.

$$(iv) P(3) + P(2) + P(1) + P(0) + P(-1)$$

$$= 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$$

Hence it is not a probability distribution.

Question 2.

An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable?

Solution:

These two balls may be selected as RR, RB, BR, BB, where R represents red and B represents black ball, variable X has the value 0, 1, 2, i.e., there may be no black balls, may be one black ball, or both the balls are black. Yes, X is a random variable.

Question 3.

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

Solution:

For one coin, $S = \{H, T\}$

$n(S) = 2$, Let A represent Head

$$\therefore A = \{H\}, n(A) = 1$$

$$P(A) = \frac{1}{2}, P(\bar{A}) = \frac{1}{2}$$

$$n = 6, r = 0, 1, 2, 3, 4, 5, 6$$

$$P(X=0) = [P(\bar{X})]^6 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$P(X=1) = 6P(X) [P(\bar{X})]^5 = 6 \times \left(\frac{1}{2}\right)^6 = \frac{6}{64}$$

$$P(X=2) = 15[P(X)]^2 [P(\bar{X})]^4 = 15 \left(\frac{1}{2}\right)^6 = \frac{15}{64}$$

$$P(X=3) = 20[P(X)]^3 [P(\bar{X})]^3 = 20 \left(\frac{1}{2}\right)^6 = \frac{20}{64}$$

$$P(X=4) = 15[P(X)]^4 [P(\bar{X})]^2 = 15 \left(\frac{1}{2}\right)^6 = \frac{15}{64}$$

$$P(X=5) = 6[P(X)]^5 [P(\bar{X})] = 6 \left(\frac{1}{2}\right)^6 = \frac{6}{64}$$

$$P(X=6) = [P(X)]^6 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

Question 4.

Find the probability distribution of

(a) number of heads in two tosses of a coin.

(b) number of tails in the simultaneous tosses of three coins.

(c) number of heads in four tosses of a coin.

Solution:

(a) When two tosses of a coin are there sample space

= {TT, TH, HT, HH}

Zero success => No heads => Two tails (TT)

$$\therefore P(X=0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

One success => 1 head or 1 tail (TH, HT)

$$P(X=1) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Two success => Both heads (HH)

$$P(X=2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

\therefore Probability distribution

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(b) When three coins are tossed simultaneously sample space :

{ TTT, TTH, THT, HHT, THH, HTH, HHT, HHH }

Zero success = No tail = All heads {HHH}

$$P(0) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

1 success = 1 tail and 2 heads

= {THH, HTH, HHT}

$$P(1) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

2 success = 2 tail and 1 heads

= {TTH, THT, HHT}

$$P(2) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

3 success = All the tails = {TTT}

$$P(3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Probability Distribution is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(c) When a coin is tossed 4 times

Zero success = No head = All are tails

$$= \{TTTT\} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

1 success = 1 head 3 tails

= {HTTT, THTT, TTHT, TTTH}

$$P(1) = {}^4C_1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{16} = \frac{1}{4}$$

Two success = 2 heads and 2 tails

= {HTTT, HTHT, THTT, TTHT, TTTH}

$$P(2) = {}^4C_2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{6}{16} = \frac{3}{8}$$

3 success = 3 heads and 1 tail

= {HHHT, HHTH, HTTH, THHH}

$$P(3) = {}^4C_3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{4}{16} = \frac{1}{4}$$

4 success = All the heads = {HHHH}

$$P(4) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

Thus, the probability distribution is

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Question 5.

Find the probability distribution of the number of successes in two tosses of a die where a success is defined as

- (i) number greater than 4
- (ii) six appears on at least one die

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

(i) Let A be the set of favorable events.

$$A = \{5, 6\}, n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}, P(\bar{A}) = \frac{2}{3}$$

Question 6.

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution:

There are 30 bulbs which include 6 defective bulbs

$$\text{Probability of getting a defective bulb} = \frac{6}{30} = \frac{1}{5}$$

$$\text{Probability of getting a good bulb} = 1 - \frac{1}{5} = \frac{4}{5}$$

Let X denote variable of defective bulbs in a sample of 4 bulbs

$$P(X=0) = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$$

$$P(X=1) = {}^4C_1 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right) = 4 \times \frac{64}{125} \times \frac{1}{5} = \frac{256}{625}$$

$$P(X=2) = {}^4C_2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^2 = 6 \times \frac{16}{25} \times \frac{1}{5} = \frac{96}{625}$$

$$P(X=3) = {}^4C_3 \left(\frac{4}{5}\right) \left(\frac{1}{5}\right)^3 = 4 \times \frac{4}{5} \times \frac{1}{25} = \frac{16}{625}$$

$$P(X=4) = \left(\frac{1}{5}\right)^4 = \frac{1}{625}$$

∴ Probability distribution of defective bulb is

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Question 7.

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of heads.

Solution:

Now $q = 3p$ As $p + q = 1 \Rightarrow p + 3p = 1$

$$\Rightarrow p = \frac{1}{4} \text{ and } q = \frac{3}{4}$$

 $n = 2$ (number of tosses), $r = 0, 1, 2$

$$P(X=0) = {}^2C_0 \cdot q^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$P(X=1) = {}^2C_1 \quad qp = 2 \times \frac{3}{4} \times \frac{1}{4} = \frac{6}{16}$$

$$P(X=2) = {}^2C_2 p^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Probability distribution

$X_i:$	0	1	2
$P_i:$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Question 8

A random variable X has the following probability distribution:

Determine

(i) k

(ii) $P(X < 3)$ (iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

Solution:

(i) Sum of probabilities = 1

$$\text{i.e., } 0+k+2k+2k+3k+2k^2+7k^2+k=1$$

$$10k^2+9k=1 \quad \text{or} \quad 10k^2+9k-1=0$$

$$(k+1)(10k-1)=0, \quad k=-1 \text{ or } k=\frac{1}{10}$$

$$k \neq -1 \quad \therefore k = \frac{1}{10}$$

\therefore The probability distribution is

X	0	1	2	3	4	5	6	7
P(X)	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$

$$(ii) \quad P(X < 3) = P(0) + P(1) + P(2) = 0$$

$$+\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$$

$$(iii) \quad P(X > 6) = P(7) = \frac{7}{100} + \frac{1}{10} = \frac{7+10}{100} = \frac{17}{100}$$

$$(iv) \quad P(0 < X < 3) = P(1) + P(2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$$

Question 9.

The random variable X has a probability distribution $P(X)$ of the following form, where k is some number

(a) Determine the value of k

(b) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$

Solution:

(a) Sum of probabilities = 1

$$k + 2k + 3k = 1 \text{ or } 6k = 1, k = \frac{1}{6}$$

The probability distribution is as given below

X	0	1	2
P(X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

(b) (i) $P(X < 2) = P(0) + P(1)$

$$= \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

(ii) $P(X \leq 2) = P(0) + P(1) + P(2)$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

(iii) $P(X \geq 2) = P(2) + P(3) + P(4) + \dots$

$$= \frac{3}{6} + 0 + 0 + \dots = \frac{3}{6} = \frac{1}{2}$$

Question 10.

Find the mean number of heads in three tosses of a fair coin.

Solution:

$$S = \{H, T\}, n(S) = 2$$

Let A denotes the appearance of head on a toss $A = \{H\}$

$$p = \frac{n(A)}{n(S)} = \frac{1}{2}, q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = 3, r = 0, 1, 2, 3,$$

$$P(X=0) = q^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(X=1) = 3q^2p = 3 \times \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{3}{8}$$

$$P(X=2) = 3qp^2 = 3 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P(X=3) = p^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Probability distribution

x_i	0	1	2	3
p_i	1/8	3/8	3/8	1/8

$$\begin{aligned} \text{Mean} &= \sum p_i x_i = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} \\ &= \frac{12}{8} = \frac{3}{2} \end{aligned}$$

Question 11.

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Solution:

Two dice thrown simultaneously is the same the die thrown 2 times.

Let $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Let A denotes the number 6

$$A = \{6\}, n(A) = 1, P(A) = \frac{n(A)}{n(S)} = \frac{1}{6},$$

$$P(\bar{A}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Now $n = 2, r = 0, 1, 2, P(X=0)$

$$= P(\bar{A})P(\bar{A}) = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$P(X=1) = 2P(A)P(\bar{A}) = 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{10}{36},$$

$$P(X=2) = P(A)P(A) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$E(X) = \sum_{i=1}^2 x_i P(x_i) = 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$$

$$= \frac{12}{36} = \frac{1}{3}$$

Question 12.

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find $E(X)$

Solution:

There are six numbers 1,2,3,4,5,6 one of them is selected in 6 ways

When one of the numbers has been selected, 5 numbers are left, one number out of 5 may be select in 5 ways

\therefore No. of ways of selecting two numbers without replacement out of 6 positive integers = $6 \times 5 = 30$

X	Favourable cases	No. of ways Probability
2	(1, 2), (2, 1)	2 $\frac{2}{30}$
3	(1, 3), (2, 3), (3, 1) (3, 2)	4 $\frac{4}{30}$
4	(1, 4), (2, 4), (3, 4), (4, 1), (4, 2) (4, 3)	6 $\frac{6}{30}$
5	(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4)	8 $\frac{8}{30}$
6	(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)	10 $\frac{10}{30}$

Expected value = $E(X) = \sum p x$

$$= 2 \times \frac{2}{30} + 3 \times \frac{4}{30} + 4 \times \frac{6}{30} + 5 \times \frac{8}{30} + 6 \times \frac{10}{30}$$

$$= 4 \frac{2}{3}$$

Question 13.

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Solution:

$$S = \begin{bmatrix} 11 & 21 & 31 & 41 & 51 & 61 \\ 12 & 22 & 32 & 42 & 52 & 62 \\ 13 & 23 & 33 & 43 & 53 & 63 \\ 14 & 24 & 34 & 44 & 54 & 64 \\ 15 & 25 & 35 & 45 & 55 & 65 \\ 16 & 26 & 36 & 46 & 56 & 66 \end{bmatrix}$$

$$n(S) = 36$$

Let A denotes the sum of the numbers = 2

B denotes the sum of the numbers = 3

C denotes the sum of the numbers = 4

D denotes the sum of the numbers = 5

E denotes the sum of the numbers = 6

F denotes the sum of the numbers = 7

G denotes the sum of the numbers = 8

H denotes the sum of the numbers = 9

I denotes the sum of the numbers = 10

J denotes the sum of the numbers = 11

K denotes the sum of the numbers = 12

$$A = \{11\}, P(A) = \frac{1}{36}$$

$$B = \{12, 21\}, P(B) = \frac{2}{36}$$

$$C = \{13, 22, 31\}, P(C) = \frac{3}{36}$$

$$D = \{14, 23, 32, 41\}, P(D) = \frac{4}{36}$$

$$E = \{15, 24, 33, 42, 51\}, P(E) = \frac{5}{36}$$

$$F = \{16, 25, 34, 43, 52, 61\}, P(F) = \frac{6}{36}$$

$$G = \{26, 35, 44, 53, 62\}, P(G) = \frac{5}{36}$$

$$H = \{36, 45, 54, 63\}, P(H) = \frac{4}{36}$$

$$I = \{46, 55, 64\}, P(I) = \frac{3}{36}$$

$$J = \{56, 65\}, P(J) = \frac{2}{36}$$

$$K = \{6, 6\}, P(K) = \frac{1}{36}$$

Probability distribution:

x_i	$P(x_i)$	x_i	$P(x_i)$
2	$\frac{1}{36}$	8	$\frac{5}{36}$
3	$\frac{2}{36}$	9	$\frac{4}{36}$
4	$\frac{3}{36}$	10	$\frac{3}{36}$
5	$\frac{4}{36}$	11	$\frac{2}{36}$
6	$\frac{5}{36}$	12	$\frac{1}{36}$
7	$\frac{6}{36}$		

$$\text{Mean} = \mu = \sum p_i x_i$$

$$= \frac{1}{36} (2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12)$$

$$= \frac{252}{36} = 7$$

$$\text{Now, } \sum p_i x_i^2$$

$$= \frac{1}{36} [4 + 18 + 48 + 100 + 180 + 294 + 320 +$$

$$324 + 300 + 242 + 144]$$

$$= \frac{1}{36} \times 1974 = \frac{329}{6}$$

$$\text{Variance} = \sum p_i x_i^2 - (\sum p_i x_i)^2 = 54.83 - 49 = 5.83$$

$$\text{Standard deviation} = \sqrt{5.83} = 2.4 \text{ (nearly).}$$

Question 14.

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean, variance and standard deviation of X ?

Solution:

There are 15 students in a class. Each has the same chance of being chosen.

The probability of each student to be selected

$\frac{1}{15}$				
X_i	F_i	P_i	$P_i x_i$	$P_i x_i^2$
14	2	$\frac{2}{15}$	$\frac{28}{15}$	$\frac{392}{15}$
15	1	$\frac{1}{15}$	$\frac{15}{15}$	$\frac{225}{15}$
16	2	$\frac{2}{15}$	$\frac{32}{15}$	$\frac{512}{15}$
17	3	$\frac{3}{15}$	$\frac{51}{15}$	$\frac{867}{15}$
18	1	$\frac{1}{15}$	$\frac{18}{15}$	$\frac{324}{15}$
19	2	$\frac{2}{15}$	$\frac{38}{15}$	$\frac{722}{15}$
20	3	$\frac{3}{15}$	$\frac{60}{15}$	$\frac{1200}{15}$
21	1	$\frac{1}{15}$	$\frac{21}{15}$	$\frac{441}{15}$
Total			$\frac{263}{15}$	$\frac{4683}{15}$

First and third columns from the probability distribution

$$\text{Mean} = E(X) = \sum p_i x_i = \frac{293}{15} = 17.53333$$

$$\text{Variance} = E(X^2) - [E(X)]^2 = \sum p_i x_i^2 - (\sum p_i x_i)^2$$

$$= \frac{4683}{15} - (17.53333)^2 = 312.20 - 307.41766 = 4.78$$

Standard deviation

$$= \sqrt{\text{Variance}} = \sqrt{4.78} = 2.19$$

probability distribution is

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean = 17.53, variance = 4.78, S.D. (X) = 2.19

Question 15.

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X = 0$, if he opposed, and $X = 1$, if he is in favour Find $E(X)$ and $\text{Var}(X)$.

Solution:

Here the variable values are 1 and 0 and the probability of occurrence is 70% = 0.7 and 30% = 0.3

Probability distribution is

X	0	1
P(X)	0.3	0.70

$$E(X) = \sum p_i x_i = 0.30 \times 0 + 0.70 \times 1 = 0.7$$

$$E(X^2) = \sum p_i x_i^2 = 0.3 \times 0 + 0.70 \times 1^2 = 0.7$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= 0.7 - (.7)^2 = 0.7(1 - 0.7) = 0.21.$$

Choose the correct answer in each of the following:

Question 16.

The mean of the number obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

- (a) 1
(b) 2
(c) 5
(d) $\frac{8}{3}$

Solution:

x_i	p_i	$p_i x_i$
1	$\frac{3}{6}$	$\frac{3}{6}$
2	$\frac{2}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
		$\sum p_i x_i = \frac{12}{6} = 2$

Mean 2

Option (b) is correct

Question 17.

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. What is the value of E (X)?

- (a) $\frac{37}{221}$
(b) $\frac{5}{13}$
(c) $\frac{1}{13}$
(d) $\frac{2}{13}$

Solution:

$$n(S) = 52, n(A) = 4$$

$$P(X=0) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$$P(X=1) = \frac{{}^{48}C_1 \times {}^4C_1}{{}^{52}C_2} = \frac{32}{221}$$

$$P(X=2) = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

x_i	p_i	$p_i x_i$
0	$\frac{188}{221}$	0
1	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$
		$\sum p_i x_i = \frac{34}{221} = \frac{2}{13}$

$$\text{Now } E(X) = \frac{2}{13}$$

Option (d) is correct

NCERT Solutions for Class 12 Maths Chapter 13 Probability Ex 13.5

Question 1.

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

- (i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes?

Solution:

There are 3 odd numbers on a die

∴ Probability of getting an odd number on a die = $\frac{3}{6} = \frac{1}{2}$

(i) Probability of getting 5 success

$$P(5) = {}^6C_5$$

$$= \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^5 = 6 \cdot \left(\frac{1}{2}\right)^6 = \frac{6}{64} = \frac{3}{32}$$

(ii) Probability of getting at least 5 successes

$$P(\text{at least 5 success}) = P(x=5) + P(x=6)$$

$$= \left(\frac{1}{2}\right)^6 (6+1) = \frac{7}{64}$$

(iii) P (at most 5 success)

$$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$

$$= [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)] - P(6)$$

$$= 1 - P(6) = 1 - \left(\frac{1}{2}\right)^6 = 1 - \frac{1}{64} = \frac{63}{64}$$

Question 2.

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ?

Solution:

Probability of getting one defective item = 5%

$$= \frac{5}{100}$$

$$= \frac{1}{20}$$

Probability of getting a good item = $1 - \frac{1}{20} = \frac{19}{20}$

A sample of 10 item include not more than one defective item.

⇒ sample contains at most (me defective item Its probability = $P(0) + P(1)$

$$= \left(\frac{19}{20}\right)^{10} + {}^{10}C_1 \left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^9 \left(\frac{19}{20} + 10 \times \frac{1}{20}\right) = \frac{29}{20} \left(\frac{19}{20}\right)^9$$

Question 3.

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability, of two successes.

Solution:

$$n(S) = 36, A = \{11, 22, 33, 44, 55, 66\}$$

$$n(A) = 6, p = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}, n = 4, r = 2$$

$$P(X=r) = C(n, r) p^r q^{n-r}$$

$$p(X=2) = C(4, 2) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$= 6 \times 25 \times \left(\frac{1}{6}\right)^4 = \frac{25}{216}$$

Question 4.

Five cards are drawn successively with replacement from a well- shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is spade?

Solution:

$S = \{52 \text{ cards}\}$, $n(S) = 52$

Let A denotes the favourable events

$A = \{13 \text{ spade}\}$, $n(A) = 13$

$$p = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}, q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

(i) $n = 5, r = 5$

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=5) = {}^5C_5 \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$$

(ii) $n = 5, r = 3$

$$P(X=3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = \frac{45}{512}$$

(iii) $n = 5, r = 0$

$$P(X=0) = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

Question 5.

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs.

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one will fuse after 150 days of use

Solution:

Probability that a bulb gets fuse after 150 days of its use = 0.05

Probability that the bulb will not fuse after 150 days of its use = $1 - 0.05 = 0.95$

(i) Probability that no bulb will fuse after 150

$$\text{days of its use} = P(\text{none}) = (0.95)^5 = 0.7738 \\ = 0.77 \text{ (approx)}$$

$$\begin{aligned} \text{(ii) } P(\text{not more than one}) &= P(0) + P(1) \\ &= (0.95)^5 + {}^5C_1 \times (0.95)^4 \times (0.05) \\ &= (0.95)^4 [0.95 + 5 \times 0.05] = (0.95)^4 \times 1.2 \end{aligned}$$

$$\begin{aligned} \text{(iii) } P(\text{more than one}) &= P(2) + P(3) + P(4) + P(5) \\ &= [P(0) + P(1) + P(2) + P(3) + P(4) + P(5)] - \\ &\quad [P(0) + P(1)] \\ &= 1 - [P(0) + P(1)] = 1 - (0.95)^4 \times 1.2 \end{aligned}$$

Question 6.

A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Solution:

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $n(S) = 10$

Let A represents that the ball is marked with the digit 0.

$A = \{0\}$, $n(A) = 1$

$$p = \frac{1}{10}, q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$n = 4, r = 0$

$$\begin{aligned} P(\text{none is marked with its digit 0}) \\ &= {}^4C_0 (p)^0 (q)^4 = \left(\frac{9}{10}\right)^4 = \frac{6561}{10000} \end{aligned}$$

Question 7.

In an examination, 20 questions of true – false type are asked. Suppose a student tosses fair coin to determine his answer to each question. If the coin falls heads, he answers 'true,' if it falls tails, he answers " false". Find the probability that he answers at least 12 questions correctly.

Solution:

Probability that student answers a question true = $\frac{1}{2}$

i.e., when a coin is thrown, probability that a head is obtained = $\frac{1}{2}$

Probability that his answer is false = $1 - \frac{1}{2} = \frac{1}{2}$

Probability that his answer at least 12 questions correctly = $P(12) + P(13) + P(14) + \dots + P(20)$

$$\begin{aligned} &= {}^{20}C_{12} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{12} + {}^{20}C_{13} \left(\frac{1}{2}\right)^7 \\ &\quad \left(\frac{1}{2}\right)^{13} + \dots + {}^{20}C_{20} \left(\frac{1}{2}\right)^{20} \\ &= \left(\frac{1}{2}\right)^{20} \left[{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + \dots + {}^{20}C_{20} \right] \end{aligned}$$

Question 8

Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome.

(Hint: $P(X = 3)$ is the maximum among all $P(X_i)$, $x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution:

$$\begin{aligned} &\left(\frac{1}{2} + \frac{1}{2}\right)^6 \\ &= {}^6C_0 \left(\frac{1}{2}\right)^6 + {}^6C_1 \left(\frac{1}{2}\right)^6 \\ &\quad + \dots + {}^6C_6 \left(\frac{1}{2}\right)^6 \\ &= \left(\frac{1}{2}\right)^6 \left[{}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 \right] \\ &= \left(\frac{1}{2}\right)^6 \left[{}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_2 + {}^6C_1 + {}^6C_0 \right] \\ &\quad {}^6C_3 \text{ has the maximum value in} \\ &\quad {}^6C_0, {}^6C_1, {}^6C_2, {}^6C_3. \\ &\text{Hence } {}^6C_3 \left(\frac{1}{2}\right)^6 = P(X = 3) \text{ is maximum.} \\ &\Rightarrow P(X = 3) \text{ is most likely outcome.} \end{aligned}$$

Question 9.

On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution:

$$\begin{aligned} P &= \frac{1}{3}, q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3} \\ n &= 5, r = 4, 5, P(X=r) = {}^nC_r p^r q^{(n-r)} \\ P(\text{Four or more successes}) \\ &= P(X=4) + P(X=5) \\ &= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + {}^5C_5 \left(\frac{1}{3}\right)^5 = \frac{11}{243} \end{aligned}$$

Question 10.

A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize?

(a) at least once,

(b) exactly once,

(c) at least twice?

Solution:

Probability that the person wins the prize = $\frac{1}{100}$

Probability of losing = $1 - \frac{1}{100} = \frac{99}{100}$

(a) Probability that he loses in all the loteries

$$= \left(\frac{99}{100}\right)^{50}$$

Probability that he wins at least in one lottery

$$= 1 - \left(\frac{99}{100}\right)^{50} = 1 - (.99)^{50}$$

(b) Probability that he wins exactly once

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 = \frac{1}{2} \left(\frac{99}{100}\right)^{49}$$

(c) Probability that he wins at least twice

$$= P(2) + P(3) + \dots + P(50)$$

$$= [P(0) + P(1) + \dots + P(50)] - [P(0) + P(1)]$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \left(\frac{99}{100} + \frac{50}{100}\right) = 1 - \left(\frac{99}{100}\right)^{50} \left(\frac{149}{100}\right)$$

Question 11.

Find the probability of getting 5 exactly twice in 7 throws of a die.

Solution:

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

$$A = \{5\} \Rightarrow n(A) = 1$$

$$p = \frac{n(A)}{n(S)} = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$$

$$n = 7, r = 2$$

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=2) = {}^7C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 = \frac{7}{12} \left(\frac{5}{6}\right)$$

Question 12.

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution:

When a die is thrown,

$$\text{Probability of getting a six} = \frac{1}{6}$$

$$\text{Probability of not getting a six} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\text{Probability of getting at most 2 sixes in 6 throws of a single die} = P(0) + P(1) + P(2)$$

$$= \left(\frac{5}{6}\right)^6 + {}^6C_1 \left(\frac{5}{6}\right)^5 \frac{1}{6} + {}^6C_2 \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^6 + 6 \times \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \frac{6 \times 5}{2} \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{5}{12} \times \left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^4 \left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12} \right]$$

$$= \left(\frac{5}{6}\right)^4 \left(\frac{25+30+15}{36} \right) = \frac{35}{18} \left(\frac{5}{6}\right)$$

Question 13.

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles 9 are defective?

Solution:

$$p = \frac{1}{100} = \frac{1}{10}$$

$$q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$n = 12, r = 9, P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=9) = {}^{12}C_9 \left(\frac{1}{10}\right)^9 \left(\frac{9}{10}\right)^3 = \frac{220 \times 729}{(10)^{12}}$$

In each of the following choose the correct answer.

Question 14.

In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(a) 10^{-1}

(b) $\left(\frac{1}{2}\right)^5$

(c) $\left(\frac{9}{10}\right)^5$

(d) $\frac{9}{10}$

Solution:

$$p = \frac{1}{10}$$

$$q = \frac{9}{10}, n = 5, r = 0, P(X=0) = \left(\frac{9}{10}\right)^5$$

Option (c) is correct

Question 15.

The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is:

Solution:

$$p = \frac{4}{5}, q = \frac{1}{5}, n = 5, r = 4$$

$$P(X=4) = {}^5C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)$$

Option (a) is true