Class : 11th
Subject : Maths
Chapter : 13
Chapter Name : Limits and Derivatives

Exercise 13.1

Q1 Evaluate the Given limit: $\lim_{x\to 3} x+3$

Answer. $\lim_{x\to 3} x+3 = 3+3 = 6$

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Q2 Evaluate the Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Answer. $\lim_{x\to \pi}\left(x-\frac{22}{7}\right) = \left(\pi-\frac{22}{7}\right)$

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Q3 Evaluate the Given limit: $\lim_{r\to 1} \pi r^2$

Answer. $\lim_{r\to 1} \pi r^2 = \pi(1)^2 = \pi$

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Q4 Evaluate the Given limit: $\lim_{x\to 4} \frac{4x+3}{x-2}$

Answer. $\lim_{x\to 4} \frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$

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Q5 Evaluate the Given limit: $\lim_{x\to -1} \frac{x^{10}+x^5+1}{x-1}$

Answer. $\lim_{x\to -1} \frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$

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Q6 Evaluate the Given limit: $\lim_{x\to 0} \frac{(x+1)^5-1}{x}$

Answer. $\lim_{x\to 0} \frac{(x+1)^5-1}{x}$
Put x + 1 = y so that y → 1 as x → 0.

$$\text{Accordingly, } \lim \frac{(x+1)^5-1}{x} = \lim_{y\to 1} \frac{y^5-1}{y-1}$$

$$= \lim_{y\to 1} \frac{y^5-1^5}{y-1}$$

$= 5.1^{5-1} \qquad \left[\lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$
=5
$\therefore \lim_{x\to 0} \frac{(x+5)^5-1}{x} = 5$

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Q7 Evaluate the Given limit: $\lim_{x\to 2} \frac{3x^2-x-10}{x^2-4}$

Answer. At x = 2, the value of the given rational function takes the form $\frac{0}{0}$

$$\begin{aligned}\therefore \lim_{x\to 2}\frac{3x^2-x-10}{x^2-4} &= \lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(x+2)}\\ &= \lim_{x\to 2}\frac{3x+5}{x+2}\\ &= \frac{3(2)+5}{2+2}\\ &= \frac{11}{4}\end{aligned}$$

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Q8 Evaluate the Given limit: $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

Answer. At x = 2, the value of the given rational function takes the form $\frac{0}{0}$

$$\begin{aligned}\therefore \lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3} &= \lim_{x\to 3}\frac{(x-3)(x+3)\left(x^2+9\right)}{(x-3)(2x+1)}\\ &= \lim_{x\to 3}\frac{(x+3)\left(x^2+9\right)}{2x+1}\\ &= \frac{(3+3)\left(3^2+9\right)}{2(3)+1}\\ &= \frac{6\times 18}{7}\\ &= \frac{108}{7}\end{aligned}$$

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Q9 Evaluate the Given limit: $\lim_{x\to 0}\frac{ax+b}{cx+1}$

Answer. $\lim_{x\to 0}\frac{ax+b}{cx+1}=\frac{a(0)+b}{c(0)+1}=b$

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Q10 Evaluate the Given limit: $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Answer. $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

At z = 1, the value of the given function takes the form $\frac{0}{0}$

Put $z^{\frac{1}{6}}=x$ so that z → 1 as x → 1.

$$\begin{aligned}\text{Accordingly, }\lim\frac{z^{\frac{1}{3}}}{z\to 1}\frac{1}{z^{\frac{1}{6}}-1} &= \lim_{x\to 1}\frac{x^2-1}{x-1}\\ &= \lim_{x\to 1}\frac{x^2-1^2}{x-1}\end{aligned}$$

$$\begin{aligned}&= 2.1^{2-1} \quad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]\\ &= 2\end{aligned}$$

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Q11 Evaluate the Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Answer.

$$\begin{aligned}\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a} &= \frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}\\ &= \frac{a+b+c}{a+b+c}\\ &= 1 \qquad [a+b+c\neq 0]\end{aligned}$$

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Q12 Evaluate the Given limit: $\lim_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Answer. $\lim_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$
At x = −2, the value of the given function takes the form $\frac{0}{0}$

$$\begin{aligned}\text{Now, } \lim_{x \to -2} \frac{1}{x \to -2} \frac{1}{x+2} &= \lim_{x \to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2} \\ &= \lim_{x \to -2} \frac{1}{2x} \\ &= \frac{1}{2(-2)} = \frac{-1}{4}\end{aligned}$$

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Q13 Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{bx}$

Answer. $\lim_{x \to 0} \frac{\sin ax}{bx}$
At x = 0, the value of the given function takes the form $\frac{0}{0}$

$$\begin{aligned}\text{Now, } \lim_{x \to 0} \frac{\sin ax}{bx} &= \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx} \\ &= \lim_{x \to 0} \left(\frac{\sin ax}{ax}\right) \times \left(\frac{a}{b}\right) \\ &= \frac{a}{b} \lim_{ax \to 0} \left(\frac{\sin ax}{ax}\right) \qquad [x \to 0 \Rightarrow ax \to 0] \\ &= \frac{a}{b} \times 1 \\ &= \frac{a}{b}\end{aligned}$$

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Q14 Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

Answer. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

Now, $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\left(\frac{\sin ax}{ax}\right) \times ax}{\left(\frac{\sin bx}{bx}\right) \times bx}$

$= \left(\frac{a}{b}\right) \times \frac{\lim_{a \to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx \to 0}\left(\frac{\sin bx}{bx}\right)} \quad \left[\begin{array}{l} x \to 0 \Rightarrow ax \to 0 \\ \text{and } x \to 0 \Rightarrow bx \to 0 \end{array}\right]$

$= \left(\frac{a}{b}\right) \times \frac{1}{1} \quad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1\right]$

$= \frac{a}{b}$

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Q15 Evaluate the Given limit: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Answer. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$
It is seen that x → π ⇒ (π − x) → 0

$$\begin{aligned}\therefore \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} &= \frac{1}{\pi} \lim_{(\pi - x) \to 0} \frac{\sin(\pi - x)}{(\pi - x)} \\ &= \frac{1}{\pi} \times 1 \quad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1\right] \\ &= \frac{1}{\pi}\end{aligned}$$

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Q16 Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\cos x}{\pi-x}$

Answer. $\lim_{x \rightarrow 0} \frac{\cos x}{\pi-x}=\frac{\cos 0}{\pi-0}=\frac{1}{\pi}$

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Q17 Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\cos 2x-1}{\cos x-1}$

Answer. $\lim_{x \rightarrow 0} \frac{\cos 2x-1}{\cos x-1}$
At x = 0, the value of the given function takes the form $\frac{0}{0}$.
Now,
$\lim_{x \rightarrow 0} \frac{\cos 2x-1}{\cos x-1}=\lim_{x \rightarrow 0} \frac{1-2 \sin^2 x-1}{1-2 \sin^2 \frac{x}{2}-1} \quad \left[\cos x=1-2 \sin^2 \frac{x}{2}\right]$
$=\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}}=\lim_{x \rightarrow 0} \frac{\left(\frac{\sin^2 x}{x^2}\right) \times x^2}{\left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right) \times \frac{x^2}{4}}$
$=4 \frac{\lim_{x \rightarrow 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x \rightarrow 0}\left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$
$=4 \frac{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x}\right)^2}{\left(\lim_{x \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2} \quad \left[x \rightarrow 0 \Rightarrow \frac{x}{2} \rightarrow 0\right]$
$=4 \frac{1^2}{1^2} \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y}=1\right]$
$=4$

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Q18 Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{ax+x \cos x}{b \sin x}$

Answer. $\lim_{x \rightarrow 0} \frac{ax+x \cos x}{b \sin x}$
At x = 0, the value of the given function takes the form $\frac{0}{0}$.
Now,

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{ax+x \cos x}{b \sin x} & =\frac{1}{b} \lim_{x \rightarrow 0} \frac{x(a+\cos x)}{\sin x} \\
& =\frac{1}{b} \lim_{x \rightarrow 0}\left(\frac{x}{\sin x}\right) \times \lim_{x \rightarrow 0}(a+\cos x) \\
& =\frac{1}{b} \times \frac{1}{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x}\right)} \times \lim_{x \rightarrow 0}(a+\cos x)
\end{aligned}$$

$=\frac{1}{b} \times(a+\cos 0) \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x}=1\right]$
$=\frac{a+1}{b}$

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Q19 Evaluate the Given limit: $\lim_{x \rightarrow 0} x \sec x$

Answer. $\lim_{x \rightarrow 0} x \sec x=\lim_{x \rightarrow 0} \frac{x}{\cos x}=\frac{0}{\cos 0}=\frac{0}{1}=0$

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Q20 Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\sin ax+bx}{ax+\sin bx} a, b, a+b \neq 0$

Answer. At x = 0, the value of the given function takes the form $\frac{0}{0}$
Now,
$\lim_{x \rightarrow 0} \frac{\sin ax+bx}{ax+\sin bx}$

$= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax}\right) ax+bx}{ax+bx\left(\frac{\sin bx}{bx}\right)}$

$= \frac{\left(\lim_{a \rightarrow 0} \frac{\sin ax}{ax}\right) \times \lim_{x \rightarrow 0}(ax)+\lim_{x \rightarrow 0} bx}{\lim_{x \rightarrow 0} ax+\lim_{x \rightarrow 0} bx\left(\lim_{hx \rightarrow 0} \frac{\sin bx}{bx}\right)} \quad [\text{ As } x \rightarrow 0 \Rightarrow ax \rightarrow 0 \text{ and } bx \rightarrow 0]$

$= \frac{\lim_{x \rightarrow 0}(ax)+\lim bx}{\lim_{x \rightarrow 0} ax+\lim bx} \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x}=1\right]$

$= \frac{\lim_{x \rightarrow 0}(ax+bx)}{\lim_{x \rightarrow 0}(ax+bx)}$
$= \lim_{r \rightarrow 0}(1)$
$= 1$
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Q21 Evaluate the Given limit: $\lim_{x \rightarrow 0}(\csc x-\cot x)$

Answer. At x = 0, the value of the given function takes the form $\frac{0}{0}$
Now,

$$\begin{aligned}
\lim_{x \rightarrow 0}(\, cosecx-\cot x) & \\
& =\lim_{x \rightarrow 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right) \\
& =\lim_{x \rightarrow 0}\left(\frac{1-\cos x}{\sin x}\right) \\
& =\lim_{x \rightarrow 0} \frac{\left(\frac{1-\cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)} \\
& =\frac{\lim_{x \rightarrow 0} \frac{1-\cos x}{x}}{\lim_{x \rightarrow 0} \frac{\sin x}{x}}
\end{aligned}$$

$= \frac{0}{1} \quad \left[\lim_{x \rightarrow 0} \frac{1-\cos x}{x}=0 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x}=1\right]$
$= 0$

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Q22 Evaluate the Given limit: $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}$

Answer. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}$
At $x=\frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$
Now, put $\mathrm{x}-\frac{\pi}{2}=\mathrm{y}$ so that $\mathrm{x} \rightarrow \frac{\pi}{2}, \mathrm{y} \rightarrow 0$
$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}}=\lim_{y \rightarrow 0} \frac{\tan 2\left(y+\frac{\pi}{2}\right)}{y}$
$= \lim_{y \rightarrow 0} \frac{\tan(\pi+2y)}{y}$
$= \lim_{y \rightarrow 0} \frac{\tan 2y}{y} \quad [\tan(\pi+2\mathrm{y})=\tan 2\mathrm{y}]$
$\lim_{y \rightarrow 0} \frac{\sin 2y}{y \cos 2y}$
$= \lim_{y \rightarrow 0}\left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y}\right)$
$= \left(\lim_{2y \rightarrow 0} \frac{\sin 2y}{2y}\right) \times \lim_{y \rightarrow 0}\left(\frac{2}{\cos 2y}\right) \quad [y \rightarrow 0 \Rightarrow 2y \rightarrow 0] \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x}=1\right]$
$= 1 \times \frac{2}{\cos 0}$
$= 1 \times \frac{2}{1}$
$= 2$

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Q23 Find $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$, where $f(x)=\begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x>0 \end{cases}$

Answer. The given function is

$$f(x)=\begin{cases}2x+3, & x\leq 0\\ 3(x+1), & x>0\end{cases}$$

$\lim_{x\rightarrow 0} f(x)=\lim_{x\rightarrow 0}[2x+3]=2(0)+3=3$

$\lim_{x\rightarrow 0^+} f(x)=\lim_{x\rightarrow 0} 3(x+1)=3(0+1)=3$

$\therefore \lim_{x\rightarrow 0^+} f(x)=\lim_{x\rightarrow 0^+} f(x)=\lim_{x\rightarrow 0} f(x)=3$

$\lim_{x\rightarrow 1} f(x)=\lim_{x\rightarrow 1} 3(x+1)=3(1+1)=6$

$\lim_{x\rightarrow 1^+} f(x)=\lim_{x\rightarrow 1} 3(x+1)=3(1+1)=6$

$\therefore \lim_{x\rightarrow 1^-} f(x)=\lim_{x\rightarrow 1^+} f(x)=\lim_{x\rightarrow 1} f(x)=6$

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Q24 Find $\lim_{x\rightarrow 1} f(x)$, where $f(x)=\begin{cases}x^2-1, & x\leq 1\\ -x^2-1, & x>1\end{cases}$

Answer. The given function is

$$f(x)=\begin{cases}x^2-1, x\leq 1\\ -x^2-1, x>1\end{cases}$$

$\lim_{x\rightarrow 1^+} f(x)=\lim_{x\rightarrow 1}\left[x^2-1\right]=1^2-1=1-1=0$

$\lim_{x\rightarrow 1^+} f(x)=\lim_{x\rightarrow 1}\left[-x^2-1\right]=-1^2-1=-1-1=-2$

It is observed that $\lim_{x\rightarrow 1^+} f(x)\neq \lim_{x\rightarrow 1^+} f(x)$

Hence, $\lim_{x\rightarrow 1} f(x)$ does not exist.

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Q25 Evaluate $\lim_{x\rightarrow 0} f(x)$, where $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$

Answer. The given function is

$$f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\begin{aligned}\lim_{x\rightarrow 0^+} f(x)&=\lim_{x\rightarrow 0}\left[\frac{|x|}{x}\right]\\ &=\lim_{x\rightarrow 0}\left(\frac{-x}{x}\right) \quad [\text{When } x \text{ is negaitve } |x|=-x]\\ &=\lim_{x\rightarrow 0}(-1)\\ &=-1\end{aligned}$$

$$\begin{aligned}\lim_{x\rightarrow 0^+} f(x)&=\lim_{x\rightarrow 0^+}\left[\frac{|x|}{x}\right]\\ &=\lim_{x\rightarrow 0^+}\left[\frac{x}{x}\right] \quad [\text{When } x \text{ is positive, } |x|=x]\\ &=\lim_{x\rightarrow 0}(1)\\ &=\lim_{x\rightarrow 0}(1)\\ &=1\end{aligned}$$

It is observed that $\lim_{x\rightarrow 0^0} f(x)\neq \lim_{x\rightarrow 0^+} f(x)$

Hence, $\lim_{x\rightarrow 0} f(x)$ does not exist.

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Q26 Find $\lim_{x \to 0} f(x),$ where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Answer. The given function is

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^0} \left[\frac{x}{|x|}\right] \\ &= \lim_{x \to 0} \left[\frac{x}{-x}\right] \quad [\text{ When } x < 0, |x| = -x] \\ &= \lim_{x \to 0} (-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} \left[\frac{x}{|x|}\right] \\ &= \lim_{x \to 0^+} \left[\frac{x}{x}\right] \quad [\text{ When } x > 0, |x| = x] \\ &= \lim_{x \to 0} (1) \\ &= 1 \end{aligned}$$

It is observed that $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^+} f(x)$

Hence, $\lim_{x \to 0} f(x)$ does not exist.

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Q27 Find $\lim_{x \to 5} f(x),$ where $f(x) = |x| - 5$

Answer. The given function is f(x) = $\lim_{x \to 5} f(x),$ where $f(x) = |x| - 5$

$$\begin{aligned} \lim_{x \to 5} f(x) &= \lim_{x \to 5} [|x| - 5] \\ &= \lim_{x \to 5} (x - 5) \quad [\text{ When } x > 0, |x| = x] \end{aligned}$$

= 5 - 5

= 0

$$\begin{aligned} \lim_{x \to -5} f(x) &= \lim_{x \to 5^+} (|x| - 5) \\ &= \lim_{x \to 4} (x - 5) \quad [\text{ When } x > 0, |x| = x] \end{aligned}$$

$= 5 - 5$

$= 0$

$\therefore \lim_{x \to 5} f(x) = \lim_{x \to 5^+} f(x) = 0$

Hence, $\lim_{x \to 5} f(x) = 0$

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Q28 Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$

$\lim_{x \to 1} f(x) = f(1)$ what are possible values of a and b?

Answer. The given function is

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax & x > 1 \end{cases}$$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (a + bx) = a + b$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (b - ax) = b - a$

$f(1) = 4$

It is given that $\lim_{x \to 1} f(x) = f(1)$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = f(1)$

$\Rightarrow a + b = 4$ and $b - a = 4$

On solving these two equations, we obtain $a = 0$ and $b = 4$

Thus, the respective possible values of a and b are 0 and 4.

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Q29

Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define a function

$f(x) = (x - a_1)(x - a_2) \ldots (x - a_n)$

What is $\lim_{x \to a_1} f(x)$? For some $a \neq a_{11} a_2, \ldots, a_n$, compute $\lim_{x \to a} f(x)$

Answer. The given function is $f(x) = (x - a_1)(x - a_2) \ldots (x - a_n)$

$$\begin{aligned} \lim_{x \to a_1} f(x) &= \lim_{x \to a_1} [(x - a_1)(x - a_2) \ldots (x - a_n)] \\ &= \left[\lim_{x \to a_1} (x - a_1)\right] \left[\lim_{x \to a_1} (x - a_2)\right] \ldots \left[\lim_{x \to a_1} (x - a_n)\right] \\ &= (a_1 - a_1)(a_1 - a_2) \ldots (a_1 - a_n) = 0 \end{aligned}$$

$\therefore \lim_{x \to a_1} f(x) = 0$

Now, $\lim_{x \to a} f(x) = \lim_{x \to a} [(x - a_1)(x - a_2) \ldots (x - a_n)]$

$$= \left[\lim_{x \to a} (x - a_1)\right] \left[\lim_{x \to a} (x - a_2)\right] \ldots \left[\lim_{x \to a} (x - a_n)\right]$$

$$= (a - a_1)(a - a_2) \ldots (a - a_n)$$

$$\therefore \lim_{x \to a} f(x) = (a - a_1)(a - a_2) \ldots (a - a_n)$$

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Q30 If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$

For what value (s) of a does $\lim_{x \to a} f(x)$ exists?

Answer. The given function is

$$f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$$

When $a = 0$

$$\begin{aligned} \lim_{x \to 0} f(x) &= \lim_{x \to 0^+} (|x| + 1) \quad [\text{If } x < 0, |x| = -x] \\ &= \lim_{x \to 0} (-x + 1) \\ &= -0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} (|x| - 1) \quad [\text{If } x > 0, |x| = x] \\ &= \lim_{x \to 0^+} (x - 1) \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

Here, it is observed that $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} f(x)$ does not exist.

When a < 0,

$$\begin{aligned} \lim_{x \to \sigma} f(x) &= \lim_{x \to \infty} (|x| + 1) \\ &= \lim_{x \to a} (-x + 1) \quad [x < a < 0 \Rightarrow |x| = -x] \\ &= -a + 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \to a^-} f(x) &= \lim_{x \to a^*} (|x| + 1) \quad [a < x < 0 \Rightarrow |x| = -x] \\ &= \lim_{x \to \infty} (-x + 1) \\ &= -a + 1 \end{aligned}$$

$\therefore \lim_{x \to a} f(x) = \lim_{x \to a'} f(x) = -a + 1$

Thus, limit of $f(x)$ exists at $x = a$, where $a < 0$

When a > 0

$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} (|x| - 1)$

$= \lim_{x \to a} (x - 1) \quad [0 < x < a \Rightarrow |x| = x]$

= a-1

$\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x| - 1)$
$= \lim_{x\to a}(x - 1) \quad [0 < a < x \Rightarrow |x| = x]$
= a - 1
$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^{\bullet}} f(x) = a - 1$
Thus, limit of $f(x)$ exists at $x = a,$ where $a > 0$
Thus, $\lim_{x\to a} f(x)$ exists for all a ≠ 0.

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Q31 If the function $f(x)$ satisfies $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi,$ evaluate $\lim_{x\to 1} f(x)$

Answer.
$\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$
$\Rightarrow \frac{\lim(f(x)-2)}{\lim(x^2-1)} = \pi$
$\Rightarrow \lim_{x\to 1}(\mathrm{f}(\mathrm{x}) - 2) = \pi \lim_{x\to 1}\left(\mathrm{x}^2 - 1\right)$
$\Rightarrow \lim_{x\to 1}(\mathrm{f}(\mathrm{x}) - 2) = \pi\left(1^2 - 1\right)$
$\Rightarrow \lim_{x\to 1}(f(x) - 2) = 0$
$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = 0$
$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0$
$\therefore \lim_{x\to 1} f(x) = 2$

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Q32 If $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1 \\ nx^3 + m, & x > 1 \end{cases}$. For what integers m and n does both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist?

Answer. The given function is

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1 \\ nx^3 + m, & x > 1 \end{cases}$$

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0}\left(mx^2 + n\right) \\ &= m(0)^2 + n \\ &= n \end{aligned}$$

$$\begin{aligned} \lim_{x\to 0^+} f(x) &= \lim_{x\to 0}(nx + m) \\ &= n(0) + m \\ &= m \end{aligned}$$

$\lim_{x\to 0} f(x)$ exists if $\mathrm{m} = \mathrm{n}$

Thus, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(nx + m)$
$= n(1) + m$
= m + n

$$\begin{aligned} \lim_{x\to 1^+} f(x) &= \lim_{x\to 1}\left(nx^3 + m\right) \\ &= n(1)^3 + m \\ &= m + n \end{aligned}$$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x)$
Thus, $\lim_{x\to 1} f(x)$ exists for any integral value of m and n.

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Exercise 13.2

Q1 Find the derivative of $x^2 - 2$ at x = 10.

Answer.

Let $f(x)=x^2-2$. Accordingly

$$f'(10)=\lim_{h\to 0}\frac{f(10+h)-f(10)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(10+h)^2-2\right]-\left(10^2-2\right)}{h}$$

$=\lim_{h\to 0}\frac{10^2+2\cdot 10\cdot h+h^2-2-10^2+2}{h}$

$=\lim_{h\to 0}\frac{20h+h^2}{h}$

$=\lim_{h\to 0}(20+h)=(20+0)=20$

Thus, the derivative of x^2-2 at x = 10 is 20.

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Q2 Find the derivative of x at x = 1.

Answer. Let f(x) = x. Accordingly,

$$\begin{aligned}f'(1)&=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}\\&=\lim_{h\to 0}\frac{(1+h)-1}{h}\\&=\lim_{h\to 0}\frac{h}{h}\\&=\lim_{h\to 0}(1)\\&=1\end{aligned}$$

Thus, the derivative of x at x = 1 is 1.

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Q3 Find the derivative of 99x at x = 100.

Answer. Let f(x) = 99x. Accordingly,

$$f'(100)=\lim_{h\to 0}\frac{f(100+h)-f(100)}{h}$$

$$=\lim_{h\to 0}\frac{99(100+h)-99(100)}{h}$$

$=\lim_{h\to 0}\frac{99\times 100+99h-99\times 100}{h}$

$=\lim_{h\to 0}\frac{99h}{h}$

$=\lim_{h\to 0}(99)=99$

Thus, the derivative of 99x at x = 100 is 99.

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Q4 Find the derivative of the following functions from first principle.

(i) x^3-27 (ii) $(x-1)(x-2)$

(iii) $\frac{1}{x^2}$ (iv) $\frac{x+1}{x-1}$

Answer. (i) Let $f(x)=x^3-27$.. Accordingly, from the first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(x+h)^3-27\right]-\left(x^3-27\right)}{h}$$

$=\lim_{h\to 0}\frac{x^3+h^3+3x^2h+3xh^2-x^3}{h}$

$=\lim_{h\to 0}\frac{h^3+3x^2h+3xh^2}{h}$

$=\lim_{h\to 0}\left(h^2+3x^2+3xh\right)$

$$= 0 + 3x^2 + 0 = 3x^2$$

(ii) Let f(x) = (x – 1) (x – 2). Accordingly, from the first principle,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h-1)(x+h-2)-(x-1)(x-2)}{h} \\ &= \lim_{h\to 0} \frac{\left(x^2+hx-2x+hx+h^2-2h-x-h+2\right)-\left(x^2-2x-x+2\right)}{h} \end{aligned}$$

$$= \lim_{h\to 0} \frac{(hx+hx+h^2-2h-h)}{h}$$

$$= \lim_{h\to 0} \frac{2hx+h^2-3h}{h}$$

$$= \lim_{h\to 0}(2x+h-3)$$

$$= (2x+0-3)$$

$$= 2x-3$$

(iii) Let $f(x) = \frac{1}{x^2}$. Accordingly, from the first principle,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\left[\frac{x^2-(x+h)^2}{x^2(x+h)^2}\right] \end{aligned}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{x^2-x^2-h^2-2hx}{x^2(x+h)^2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h^2-2hx}{x^2(x+h)^2}\right]$$

$$= \lim_{h\to 0} \left[\frac{-h-2x}{x^2(x+h)^2}\right]$$

$$= \frac{0-2x}{x^2(x+0)^2} = \frac{-2}{x^3}$$

(iv) Let $f(x) = \frac{x+1}{x-1}$. Accordingly, from the first principle,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\left(\frac{x+h+1}{x+h-1}-\frac{x+1}{x-1}\right)}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x-1)(x+h+1)-(x+1)(x+h-1)}{(x-1)(x+h-1)}\right] \end{aligned}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x^2+hx+x-x-h-1)-(x^2+hx-x+x+h-1)}{(x-1)(x+h-1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-2h}{(x-1)(x+h-1)}\right]$$

$$= \lim_{h\to 0} \left[\frac{-2}{(x-1)(x+h-1)}\right]$$

$$= \frac{-2}{(x-1)(x-1)} = \frac{-2}{(x-1)^2}$$

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Q5 For the function:
$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$
Prove That $f'(1) = 100f'(0)$

Answer. The given function is

$f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots+\frac{x^{2}}{2}+x+1$

$\frac{d}{dx}f(x)=\frac{d}{dx}\left[\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots+\frac{x^{2}}{2}+x+1\right]$

$\frac{d}{dx}f(x)=\frac{d}{dx}\left(\frac{x^{100}}{100}\right)+\frac{d}{dx}\left(\frac{x^{99}}{99}\right)+\ldots+\frac{d}{dx}\left(\frac{x^{2}}{2}\right)+\frac{d}{dx}(x)+\frac{d}{dx}(1)$

On using theorem $\frac{d}{dx}(x^n)=nx^{n-1}$, we obtain

$\frac{d}{dx}f(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+\ldots+\frac{2x}{2}+1+0$

$\therefore f'(x)=x^{\text{yp}}+x^{98}+\ldots+x+1$

At $x=0$

$f'(0)=1$

At $x=1$

$f'(1)=1^{99}+1^{98}+\ldots+1+1=[1+1+\ldots+1+1]_{100\text{ terms}}$

= 1x 100 = 100

Thus, $f'(1)=100\times f^{1}(0)$

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Q6 Find the derivative of $x^n+ax^{n-1}+a^2x^{n-2}+\ldots+a^{n-1}x+a^n$ for some fixed real number a.

Answer.

Let $f(x)=x^n+ax^{n-1}+a^2x^{n-2}+\ldots+a^{n-1}x+a^n$

$\therefore f'(x)=\frac{d}{dx}\left(x^n+ax^{n-1}+a^2x^{n-2}+\ldots+a^{n-1}x+a^n\right)$

$=\frac{d}{dx}(x^n)+a\frac{d}{dx}(x^{n-1})+a^2\frac{d}{dx}(x^{n-2})+\ldots+a^{n-1}\frac{d}{dx}(x)+a^n\frac{d}{dx}(1)$

On using theorem $\frac{d}{dx}x^n=nx^{n-1}$, we obtain

$f'(x)=nx^{n-1}+a(n-1)x^{n-2}+a^2(n-2)x^{n-3}+\ldots+a^{n-1}+a^n(0)$

$=nx^{n-1}+a(n-1)x^{n-2}+a^2(n-2)x^{n-3}+\ldots+a^{n-1}$

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Q7 For some constants a and b, find the derivative of:

(i) (x-a) (x-b) (ii) $\left(ax^2+b\right)^2$ (iii) $\frac{x-a}{x-b}$

Answer. (i) Let f (x) = (x – a) (x – b)

$\Rightarrow f(x)=x^2-(a+b)x+ab$

$$\therefore f'(x)=\frac{d}{dx}\left(x^2-(a+b)x+ab\right)$$

$$=\frac{d}{dx}(x^2)-(a+b)\frac{d}{dx}(x)+\frac{d}{dx}(ab)$$

On using theorem $\frac{d}{dx}(x^n)=nx^{n-1}$, we obtain

$f'(x)=2x-(a+b)+0=2x-a-b$

(ii) Let $f(x)=\left(ax^2+b\right)^2$

$\Rightarrow f(x)=a^2x^4+2abx^2+b^2$

$\therefore f'(x)=\frac{d}{dx}\left(a^2x^4+2abx^2+b^2\right)=a^2\frac{d}{dx}\left(x^4\right)+2ab\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(b^2\right)$

On using theorem $\frac{d}{dx}x^n=nx^{n-1}$, we obtain

$$\begin{aligned}f'(x)&=a^2\left(4x^3\right)+2ab(2x)+b^2(0)\\&=4a^2x^3+4abx\\&=4ax\left(ax^2+b\right)\end{aligned}$$

(iii)

Let $f(x)=\frac{(x-a)}{(x-b)}$

$\Rightarrow f'(x)=\frac{d}{dx}\left(\frac{x-a}{x-b}\right)$

By quotient rule,

$$f'(x)=\frac{(x-b)\frac{d}{dx}(x-a)-(x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$$
$$=\frac{(x-b)(1)-(x-a)(1)}{(x-b)^2}$$
$$=\frac{x-b-x+a}{(x-b)^2}$$
$$=\frac{a-b}{(x-b)^2}$$

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Q8 Find the derivative of $\frac{x^n-a^n}{x-a}$ for some constant a.

Answer.

Let $f(x)=\frac{x^n-a^n}{x-a}$

$\Rightarrow f'(x)=\frac{d}{dx}\left(\frac{x^n-a^n}{x-a}\right)$

By quotient rule,

$$f'(x)=\frac{(x-a)\frac{d}{dx}(x^n-a^n)-(x^n-a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$
$$=\frac{(x-a)\left(nx^{n-1}-0\right)-(x^n-a^n)}{(x-a)^2}$$
$$=\frac{nx^n-anx^{n-1}-x^n+a^n}{(x-a)^2}$$

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Q9 Find the derivative of

(i) $2x-\frac{3}{4}$ (ii) $\left(5x^3+3x-1\right)(x-1)$

(iii) $x^{-3}(5+3x)$ (iv) $x^5\left(3-6x^{-9}\right)$

(iv) $x^{-4}\left(3-4x^{-5}\right)$ (vi) $\frac{2}{x+1}-\frac{x^2}{3x-1}$

Answer. (i) Let

$$f(x)=2x-\frac{3}{4}$$
$$f'(x)=\frac{d}{dx}\left(2x-\frac{3}{4}\right)$$
$$=2\frac{d}{dx}(x)-\frac{d}{dx}\left(\frac{3}{4}\right)$$
$$=2-0$$
$$=2$$

(ii) Let $f(x)=\left(5x^3+3x-1\right)(x-1)$

By Leibnitz product rule,

$$f'(x)=\left(5x^3+3x-1\right)\frac{d}{dx}(x-1)+(x-1)\frac{d}{dx}\left(5x^3+3x-1\right)$$
$$=\left(5x^3+3x-1\right)(1)+(x-1)\left(5.3x^2+3-0\right)$$
$$=\left(5x^3+3x-1\right)+(x-1)\left(15x^2+3\right)$$
$$=5x^3+3x-1+15x^3+3x-15x^2-3$$
$$=20x^3-15x^2+6x-4$$

(iii) Let $f(x)=x^{-3}(5+3x)$

By Leibnitz product rule,

$$\begin{aligned} f'(x) &= x^{-3}\frac{d}{dx}(5+3x)+(5+3x)\frac{d}{dx}\left(x^{-3}\right) \\ &= x^{-3}(0+3)+(5+3x)\left(-3x^{-3-1}\right) \\ &= x^{-3}(3)+(5+3x)\left(-3x^{-4}\right) \\ &= 3x^{-3}-15x^{-4}-9x^{-3} \\ &= -6x^{-3}-15x^{-4} \end{aligned}$$

$$= -3x^{-3}\left(2+\tfrac{5}{x}\right)$$

$$= \tfrac{-3x^{-3}}{x}(2x+5)$$

$$= \tfrac{-3}{x^4}(5+2x)$$

(iv) Let $f(x) = x^5\left(3-6x^{-9}\right)$

By Leibnitz product rule,

$$\begin{aligned} f'(x) &= x^5\frac{d}{dx}\left(3-6x^{-9}\right)+\left(3-6x^{-9}\right)\frac{d}{dx}\left(x^5\right) \\ &= x^5\left\{0-6(-9)x^{-9-1}\right\}+\left(3-6x^{-9}\right)\left(5x^4\right) \\ &= x^5\left(54x^{-10}\right)+15x^4-30x^{-5} \\ &= 54x^{-5}+15x^4-30x^{-5} \\ &= 24x^{-5}+15x^4 \\ &= 15x^4+\frac{24}{x^5} \end{aligned}$$

(v) Let $f(x) = x^{-4}\left(3-4x^{-5}\right)$

By Leibnitz product rule,

$$\begin{aligned} f'(x) &= x^{-4}\frac{d}{dx}\left(3-4x^{-5}\right)+\left(3-4x^{-5}\right)\frac{d}{dx}\left(x^{-4}\right) \\ &= x^{-4}\left\{0-4(-5)x^{-5-1}\right\}+\left(3-4x^{-5}\right)(-4)x^{-4-1} \\ &= x^{-4}\left(20x^{-6}\right)+\left(3-4x^{-5}\right)\left(-4x^{-5}\right) \\ &= 20x^{-10}-12x^{-5}+16x^{-10} \\ &= 36x^{-10}-12x^{-5} \\ &= -\frac{12}{x^5}+\frac{36}{x^{10}} \end{aligned}$$

(vi) Let f(x) =

$$\tfrac{2}{x+1}-\tfrac{x^2}{3x-1}$$

$$f'(x) = \tfrac{d}{dx}\left(\tfrac{2}{x+1}\right)-\tfrac{d}{dx}\left(\tfrac{x^2}{3x-1}\right)$$

By quotient rule,

$$\begin{aligned} f'(x) &= \left[\frac{(x+1)\frac{d}{dx}(2)-2\frac{d}{dx}(x+1)}{dx}\right]-\left[\frac{(3x-1)\frac{d}{dx}\left(x^2\right)-x^2\frac{d}{dx}(3x-1)}{dx}\right] \\ &= \left[\frac{(x+1)(0)-2(1)}{(x+1)^2}\right]-\left[\frac{(3x-1)(2x)-\left(x^2\right)(3)}{(3x-1)^2}\right] \end{aligned}$$

$$= \frac{-2}{(x+1)^2} - \left[\frac{6x^2 - 2x - 3x^2}{(3x-1)^2}\right]$$
$$= \frac{-2}{(x+1)^2} - \left[\frac{3x^2 - 2x^2}{(3x-1)^2}\right]$$
$$= \frac{-2}{(x+1)^2} - \frac{x(3x-2)}{(3x-1)^2}$$

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Q10 Find the derivative of cos x from first principle.

Answer. Let f (x) = cos x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$
$$= \lim_{h\to 0}\left[\tfrac{\cos x \cos h - \sin x \sin h - \cos x}{h}\right]$$
$$= \lim_{h\to 0}\left[\tfrac{-\cos x(1-\cosh h) - \sin x \sin h}{h}\right]$$
$$= \lim_{h\to 0}\left[\tfrac{-\cos x(1-\cos h)}{h} - \tfrac{\sin x \sin h}{h}\right]$$
$$= -\cos x\left(\lim_{h\to 0}\tfrac{1-\cos h}{h}\right) - \sin x \lim_{h\to 0}\left(\tfrac{\sin h}{h}\right)$$
$$= -\cos x(0) - \sin x(1) \quad \left[\lim_{h\to 0}\tfrac{1-\cos h}{h} = 0 \text{ and } \lim_{h\to 0}\tfrac{\sin h}{h} = 1\right]$$
$$\therefore f'(x) = -\sin x$$

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Q11 Find the derivative of the following functions:
(i) sin x cos x (ii) sec x (iii) 5sec x + 4cos x
(iv) cosec x (v) 3cot + 5cosec x
(vi) 5sin - 6cos x + 7 (vii) 2tan x - 7sec x

Answer. (i) Let f (x) = sin x cos x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sin(x+h)\cos(x+h) - \sin x \cos x}{h}$$
$$= \lim_{h\to 0} \frac{1}{2h}[2\sin(x+h)\cos(x+h) - 2\sin x \cos x]$$
$$= \lim_{h\to 0} \tfrac{1}{2h}[\sin 2(x+h) - \sin 2x]$$
$$= \lim_{h\to 0} \tfrac{1}{2h}\left[2\cos\tfrac{2x+2h+2x}{2} \cdot \sin\tfrac{2x+2h-2x}{2}\right]$$
$$= \lim_{h\to 0} \tfrac{1}{h}\left[\cos\tfrac{4x+2h}{2}\sin\tfrac{2h}{2}\right]$$

$$= \lim_{h\to 0} \tfrac{1}{h}[\cos(2x+h)\sin h]$$
$$= \lim_{h\to 0}\cos(2x+h) \cdot \lim_{h\to 0}\tfrac{\sin h}{h}$$
$$= \cos(2x+0).1$$
$$= \cos 2x$$

(ii) Let f (x) = sec x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\cos(x+h)} - \frac{1}{\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cos x - \cos(x+h)}{\cos x \cos(x+h)}\right]$$

$$= \frac{1}{\cos x} \lim_{h\to 0} \frac{1}{h}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos(x+h)}\right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h\to 0} \frac{1}{h}\left[\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos(x+h)}\right]$$

$$= \frac{1}{\cos x} \lim_{h\to 0} \frac{\left[\sin\left(\frac{2x+h}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right]}{\cos(x+h)}$$

$$= \frac{1}{\cos x} \cdot \lim_{\frac{h}{2}\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \cdot \lim_{h\to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)}$$

$$= \frac{1}{\cos x} \cdot 1 \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \tan x$$

(iii) Let f (x) = 5 sec x + 4 cos x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{5\sec(x+h)+4\cos(x+h)-[5\sec x+4\cos x]}{h}$$

$$= 5\lim_{h\to 0} \frac{[\sec(x+h)-\sec x]}{h} + 4\lim_{h\to 0} \frac{[\cos(x+h)-\cos x]}{h}$$

$$= 5\lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\cos(x+h)} - \frac{1}{\cos x}\right] + 4\lim_{h\to 0} \frac{1}{h}[\cos(x+h) - \cos x]$$

$$= 5\lim_{h\to 0} \frac{1}{h}\left[\frac{\cos x-\cos(x+h)}{\cos x\cos(x+h)}\right] + 4\lim_{h\to 0} \frac{1}{h}[\cos x\cos h - \sin x\sin h - \cos x]$$

$$= \frac{5}{\cos x} \lim_{h\to 0} \frac{1}{h}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos(x+h)}\right] + 4\lim_{h\to 0} \frac{1}{h}[-\cos x(1-\cos h) - \sin x\sin h]$$

$$= \frac{5}{\cos x} \cdot \lim_{h\to 0} \frac{1}{h}\left[\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos(x+h)}\right] + 4\left[-\cos x \lim_{h\to 0} \frac{(1-\cos h)}{h} - \sin x \lim_{h\to 0} \frac{\sin h}{h}\right]$$

$$= \frac{5}{\cos x} \cdot \left[\lim_{h\to 0} \frac{\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)} \cdot \lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right] - 4\sin x$$

$$= \frac{5}{\cos x} \cdot \frac{\sin x}{\cos x} \cdot 1 - 4\sin x$$

$$= 5\sec x\tan x \cdot -4\sin x$$

(iv) Let f (x) = cosec x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{1}{h}[\csc(x+h) - \ cosec x]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sin(x+h)} - \frac{1}{\sin x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\cdot\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \frac{-\cos\left(\frac{2x+h}{2}\right)\cdot\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}}{\sin(x+h)\sin x}$$

$$= \lim_{h\to 0} \left(\frac{-\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x}\right) \cdot \lim_{\frac{h}{2}\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= \left(\frac{-\cos x}{\sin x\sin x}\right).1$$

$$= -\csc x\cot x$$

(v) Let f (x) = 3cot x + 5cosec x. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3\cot(x+h) + 5\,cosec(x+h) - 3\cot x - 5\,cosecx}{h} \qquad ....(1)$$

$$= 3\lim_{h\to 0} \frac{1}{h}[\cot(x+h) - \cot x] + 5\lim_{h\to 0} \frac{1}{h}[\,cosec(x+h) -\ cosecx]$$

Now, $\lim_{h\to 0} \frac{1}{h}[\cot(x+h) - \cot x]$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cos(x+h)\sin x - \cos x\sin(x+h)}{\sin x\sin(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x-x-h)}{\sin x\sin(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(-h)}{\sin x\sin(x+h)}\right]$$

$$= -\left(\lim_{h\to 0} \frac{\sin h}{h}\right)\cdot\left(\lim_{h\to 0} \frac{1}{\sin x\cdot\sin(x+h)}\right)$$

$$= -1\cdot\tfrac{1}{\sin x\cdot\sin(x+0)} = \tfrac{-1}{\sin^2 x} = -\,cosec^2 x\ldots(2)$$

$$\lim_{h\to 0} \tfrac{1}{h}[\,cosec(x+h) -\ cosecx]$$

$$= \lim_{h\to 0} \tfrac{1}{h}\left[\tfrac{1}{\sin(x+h)} - \tfrac{1}{\sin x}\right]$$

$$= \lim_{h\to 0} \tfrac{1}{h}\left[\tfrac{\sin x - \sin(x+h)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \tfrac{1}{h}\left[\tfrac{2\cos\left(\frac{x+x+h}{2}\right)\cdot\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \tfrac{1}{h}\left[\tfrac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\sin(x+h)\sin x}\right]$$

$$= \lim_{h\to 0} \tfrac{-\cos\left(\frac{2x+h}{2}\right)\cdot\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}}{\sin(x+h)\sin x}$$

$$= \lim_{h\to 0}\left(\tfrac{-\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x}\right)\cdot\lim_{h\to 0}\tfrac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= \left(\frac{-\cos x}{\sin x\sin x}\right)\cdot 1$$

$$= -\operatorname{cosec}\cot x\ldots\ldots(3)$$

From (1), (2), and (3), we obtain

$$f'(x) = -3\,cosec^2 x - 5\,cosecx\cot x$$

(vi) Let f (x) = 5sin x – 6cos x + 7. Accordingly, from the first principle,

$$f'(x) = \lim_{h\to 0} \tfrac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \tfrac{1}{h}[5\sin(x+h) - 6\cos(x+h) + 7 - 5\sin x + 6\cos x - 7]$$

$$= \lim_{h\to 0} \tfrac{1}{h}[5\{\sin(x+h) - \sin x\} - 6\{\cos(x+h) - \cos x\}]$$

$$= 5\lim_{h\to 0} \tfrac{1}{h}\left[2\cos\left(\tfrac{x+h+x}{2}\right)\sin\left(\tfrac{x+h-x}{2}\right)\right] - 6\lim_{h\to 0} \tfrac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= 5\lim_{h\to 0} \tfrac{1}{h}\left[2\cos\left(\tfrac{2x+h}{2}\right)\sin\tfrac{h}{2}\right] - 6\lim_{h\to 0}\left[\tfrac{-\cos x(1-\cos h) - \sin x\sin h}{h}\right]$$

$$= 5\left[\lim_{h\to 0}\cos\left(\tfrac{2x+h}{2}\right)\right]\left[\lim_h \tfrac{\sin\frac{h}{2}}{\frac{h}{2}}\right] - 6\left[(-\cos x)\left(\lim_{h\to 0}\tfrac{1-\cos h}{h}\right) - \sin x\lim_{h\to 0}\left(\tfrac{\sin h}{h}\right)\right]$$

$$= 5\cos x\cdot 1 - 6[(-\cos x)\cdot(0) - \sin x.1]$$

$$= 5\cos x + 6\sin x$$

(vii) Let f (x) = 2 tan x – 7 sec x. Accordingly, from the first principle,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}[2\tan(x+h)-7\sec(x+h)-2\tan x+7\sec x]\\
&= \lim_{h\to 0}\frac{1}{h}[2\{\tan(x+h)-\tan x\}-7\{\sec(x+h)-\sec x\}]
\end{aligned}
$$

$= 2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{1}{\cos(x+h)}-\frac{1}{\cos x}\right]$

$= 2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)\cos x-\sin x\cos(x+h)}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{\cos x-\cos(x+h)}{\cos x\cos(x+h)}\right]$

$= 2\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\cos x\cos(x+h)}\right]$

$= 2\lim_{h\to 0}\left[\left(\frac{\sin h}{h}\right)\frac{1}{\cos x\cos(x+h)}\right]-7\lim_{h\to 0}\frac{1}{h}\left[\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos x\cos(x+h)}\right]$

$= 2.1\cdot\frac{1}{\cos x\cos x}-7.1\left(\frac{\sin x}{\cos x\cos x}\right)$

$= 2\sec^2 x-7\sec x\tan x$

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Miscellaneous Exercise

Q1 Find the derivative of the following functions from first principle:

$\text{(i)}\ -x\ (\text{ ii})\ (-x)^{-1}(\text{ iii})\ \sin(x+1)$

$\text{(iii)}\cos\left(x-\frac{\pi}{8}\right)$

Answer. (i) Let f(x) = −x. Accordingly, f(x + h) = -(x + h)

By first principle,

$$
\begin{aligned}
\mathrm{f}'(\mathrm{x}) &= \lim_{h\to 0}\frac{\mathrm{f}(\mathrm{x}+\mathrm{h})-\mathrm{f}(\mathrm{x})}{\mathrm{h}}\\
&= \lim_{h\to 0}\frac{-(\mathrm{x}+\mathrm{h})-(-\mathrm{x})}{\mathrm{h}}\\
&= \lim_{h\to 0}\frac{-\mathrm{x}-\mathrm{h}+\mathrm{x}}{\mathrm{h}}\\
&= \lim_{h\to 0}\frac{-\mathrm{h}}{\mathrm{h}}\\
&= \lim_{h\to 0}(-1)=-1
\end{aligned}
$$

(ii) Let $f(x)=(-x)^{-1}=\frac{1}{-x}=\frac{-1}{x}$. Accordingly, $\mathrm{f}(\mathrm{x}+\mathrm{h})=\frac{-1}{(\mathrm{x}+\mathrm{h})}$

By first principle,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{-1}{x+h}-\left(\frac{-1}{x}\right)\right]\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{-x+(x+h)}{x+h}+\frac{1}{x}\right]\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{-x+(x+h)}{x(x+h)}\right]\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{-x+(x+h)}{x(x+h)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{h}{x(x+h)}\right]\\
&= \lim_{h\to 0}\frac{1}{x(x+h)}\\
&= \frac{1}{x\cdot x}=\frac{1}{x^2}
\end{aligned}
$$

(iii) Let f(x) = sin (x + 1). Accordingly, $f(x+h)=\sin(x+h+1)$

By first principle,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}[\sin(x+h+1)-\sin(x+1)]\\
&= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{x+h+1+x+1}{2}\right)\sin\left(\frac{x+h+1-x-1}{2}\right)\right]\\
&= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{2x+h+2}{2}\right)\sin\left(\frac{h}{2}\right)\right]
\end{aligned}
$$

$$= \lim_{h\to 0}\left[\cos\left(\frac{2x+h+2}{2}\right)\cdot\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right]$$

$$= \lim_{h\to 0}\cos\left(\frac{2x+h+2}{2}\right)\cdot\lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{b}{2}+0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}+0}\quad\left[\text{As } h\to 0\Rightarrow\frac{h}{2}\to 0\right]$$

$$= \cos\left(\frac{2x+0+2}{2}\right)\cdot 1\qquad\left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$= \cos(x+1)$$

(iv) Let $f(x)=\cos\left(x-\frac{\pi}{8}\right)$ Accordingly, $f(x+h)=\cos\left(x+h-\frac{\pi}{8}\right)$

By first principle,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}\left[\cos\left(x+h-\frac{\pi}{8}\right)-\cos\left(x-\frac{\pi}{8}\right)\right]
\end{aligned}
$$

$$= \lim_{h\to 0}\frac{1}{h}\left[-2\sin\frac{\left(x+h-\frac{\pi}{8}+x-\frac{\pi}{8}\right)}{2}\sin\left(\frac{x+h-\frac{\pi}{8}-x+\frac{\pi}{8}}{2}\right)\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[-2\sin\left(\frac{2x+h-\frac{\pi}{4}}{2}\right)\sin\frac{h}{2}\right]$$

$$= \lim_{h\to 0}\left[-\sin\left(\frac{2x+h-\frac{\pi}{4}}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{2}\right]\Bigg]$$

$$= \lim_{h\to 0}\left[-\sin\left(\frac{2x+h-\frac{\pi}{4}}{2}\right)\right]\cdot\lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\quad\left[\text{As h}\to 0\Rightarrow\frac{h}{2}\to 0\right]$$

$$= -\sin\left(\frac{2x+0-\frac{\pi}{4}}{2}\right).1$$

$$= -\sin\left(x-\frac{\pi}{8}\right)$$

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Q2 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): (x + a)

Answer. Let f(x) = x + a. Accordingly, f(x + h) = -(x + h)

By first principle,

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{x+h+a-x-a}{h}\\
&= \lim_{h\to 0}\left(\frac{h}{h}\right)\\
&= \lim_{h\to 0}(1)\\
&= 1
\end{aligned}
$$

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Q3 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(px+q)\left(\frac{r}{x}+s\right)$

Answer. $f(x) = (px+q)\left(\frac{r}{x}+s\right)$

By Leibnitz product rule,

$$f'(x) = (px+q)\left(\frac{r}{x}+s\right) + \left(\frac{r}{x}+s\right)(px+q)'$$
$$= (px+q)\left(rx^{-1}+s\right)' + \left(\frac{r}{x}+s\right)(p)$$
$$= (px+q)\left(-rx^{-2}\right) + \left(\frac{r}{x}+s\right)p$$

$$= (px+q)\left(\frac{-r}{x^2}\right) + \left(\frac{r}{x}+s\right)p$$
$$= \frac{-pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + ps$$
$$= ps - \frac{qr}{x^2}$$

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Q4 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):$(ax+b)(cx+d)^2$

Answer.

Let $f(x) = (ax+b)(cx+d)^2$

By Leibnitz product rule,

$$f'(x) = (ax+b)\frac{d}{dx}(cx+d)^2 + (cx+d)^2\frac{d}{dx}(ax+b)$$
$$= (ax+b)\frac{d}{dx}\left(c^2x^2+2cdx+d^2\right) + (cx+d)^2\frac{d}{dx}(ax+b)$$
$$= (ax+b)\left[\frac{d}{dx}\left(c^2x^2\right) + \frac{d}{dx}(2cdx) + \frac{d}{dx}d^2\right] + (cx+d)^2\left[\frac{d}{dx}ax + \frac{d}{dx}b\right]$$
$$= (ax+b)\left(2c^2x+2cd\right) + \left(cx+d^2\right)a$$
$$= 2c(ax+b)(cx+d) + a(cx+d)^2$$

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Q5 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{cx+d}$

Answer. Let $f(x) = \frac{ax+b}{cx+d}$

By quotient rule,

$$f'(x) = \frac{(cx+d)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}(cx+d)$$
$$= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2}$$
$$= \frac{acx+ad-acx-bc}{(cx+d)^2}$$
$$= \frac{ad-bc}{(cx+d)^2}$$

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Q6 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Answer. $f(x) = \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$, where $x \neq 0$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(x-1)-(x-1)-(x+1)-(x+1)}{(x-1)^2}, x \neq 0,1 \\ &= \frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}, x \neq 0,1 \\ &= \frac{x-1-x-1}{(x-1)^2}, x \neq 0,1 \\ &= \frac{-2}{(x-1)^2}, x \neq 0,1 \end{aligned}$$

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Q7 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1}{ax^2+bx+c}$

Answer. Let $f(x) = \frac{1}{ax^2+bx+c}$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{\left(ax^2+bx+c\right)\frac{d}{dx}(1)-\frac{d}{dx}\left(ax^2+bx+c\right)}{dx} \\ &= \frac{\left(ax^2+bx+c\right)(0)-(2ax+b)}{\left(ax^2+bx+c\right)^2} \\ &= \frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2} \end{aligned}$$

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Q8 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{px^2+qx+r}$

Answer. Let $f(x) = \frac{ax+b}{px^2+qx+r}$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2} \\ &= \frac{\left(px^2+qx+r\right)(a)-(ax+b)(2px+q)}{\left(px^2+qx+r\right)^2} \end{aligned}$$

$$\begin{aligned} &= \frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{\left(px^2+qx+r\right)^2} \\ &= \frac{-apx^2-2bpx+ar-bq}{\left(px^2+qx+r\right)^2} \end{aligned}$$

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Q9 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{px^2+qx+r}{ax+b}$

Answer.

Let $f(x) = \frac{px^2+qx+r}{ax+b}$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)-\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)}{(ax+b)^2} \\ &= \frac{(ax+b)(2px+q)-\left(px^2+qx+r\right)(a)}{(ax+b)^2} \end{aligned}$$

$$\begin{aligned} &= \frac{2apx^2+aqx+2bpx+bq-apx^2-aqx-ar}{(ax+b)^2} \\ &= \frac{apx^2+2bpx+bq-ar}{(ax+b)^2} \end{aligned}$$

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Q10 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{a}{x^4} - \frac{b}{x^2} + \cos$

Answer.

$$\text{Let } f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{a}{x^4}\right) - \frac{d}{dx}\left(\frac{b}{x^2}\right) + \frac{d}{dx}(\cos x) \\ &= a\frac{d}{dx}\left(x^{-4}\right) - b\frac{d}{dx}\left(x^{-2}\right) + \frac{d}{dx}(\cos x) \end{aligned}$$

$$= a\left(-4x^{-5}\right) - b\left(-2x^{-3}\right) + (-\sin x) \quad \left[\frac{d}{dx}(x^n) = nx^{n-1} \text{ and } \frac{d}{dx}(\cos x) = -\sin x\right]$$

$$= \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$$

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Q11 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $4\sqrt{x} - 2$

Answer.

$$\text{Let } f(x) = 4\sqrt{x} - 2$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}(4\sqrt{x} - 2) = \frac{d}{dx}(4\sqrt{x}) - \frac{d}{dx}(2) \\ &= 4\frac{d}{dx}\left(x^{\frac{1}{2}}\right) - 0 = 4\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) \\ &= \left(2x^{-\frac{1}{2}}\right) = \frac{2}{\sqrt{x}} \end{aligned}$$

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Q12 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)^n$

Answer. $f(x) = (ax+b)^n$. Accordingly, $f(x+h) = \{a(x+h)+b\}^n = (ax+ah+b)^n$

By first principle,

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(ax+ah+b)^n - (ax+b)^n}{h} \\ &= \lim_{h\to 0} \frac{(ax+b)^n\left(1+\frac{ah}{ax+b}\right)^n - (ax+b)^n}{h} \\ &= (ax+b)^n \lim_{h\to 0} \frac{\left(1+\frac{ah}{ax+b}\right)^n - 1}{h} \end{aligned}$$

$$= (ax+b)^n \lim_{h\to 0} \frac{1}{n}\left[\left\{1 + n\left(\frac{ah}{ax+b}\right) + \frac{n(n-1)}{\lfloor 2}\left(\frac{ah}{ax+b}\right)^2 + \ldots\right\} - 1\right] \text{(Using binomial theorem)}$$

$$= (ax+b)^n \lim_{h\to 0} \frac{1}{h}\left[n\left(\frac{ah}{ax+b}\right) + \frac{n(n-1)a^2h^2}{\lfloor 2(ax+b)^2} + \ldots \text{ (Terms containing higher degrees of } h\right)\Big]$$

$$= (ax+b)^n \lim_{b\to 0}\left[\frac{na}{(ax+b)} + \frac{n(n-1)a^2h}{\lfloor 2(ax+b)^2} + \ldots\right]$$

$$= (ax+b)^n\left[\frac{na}{(ax+b)} + 0\right]$$

$$= na\frac{(ax+b)^n}{(ax+b)}$$

$$= na(ax+b)^{n-1}$$

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Q13 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)^n(cx+d)^m$

Answer.

Let $f(x)=(ax+b)^n(cx+d)^m$

By Leibnitz product rule,

$f'(x)=(ax+b)^n\frac{d}{dx}(cx+d)^m+(cx+d)^m\frac{d}{dx}(ax+b)^n$

Now, let $f_1(x)=(cx+d)^m$

$f_1(x+h)=(cx+ch+d)^m$

$f_1'(x)=\lim_{h\to 0}\frac{f_1(x+h)-f_1(x)}{h}$

$=\lim_{h\to 0}\frac{(cx+ch+d)^m-(cx+d)^m}{h}$

$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\left(1+\frac{ch}{cx+d}\right)^m-1\right]$

$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\left(1+\frac{mch}{(cx+d)}+\frac{m(m-1)}{2}\frac{(c^2h^2)}{(cx+d)^2}+\ldots\right)-1\right]$

$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\frac{mch}{(cx+d)}+\frac{m(m-1)c^2h^2}{2(cx+d)^2}+\ldots \text{ (Terms containing higher degrees of } h\text{)}\right]$

$=(cx+d)^m\lim_{h\to 0}\left[\frac{mc}{(cx+d)}+\frac{m(m-1)c^2h}{2(cx+d)^2}+\ldots\right]$

$=(cx+d)^m\left[\frac{mc}{cx+d}+0\right]$

$=\frac{mc(cx+d)^m}{(cx+d)}$

$=mc(cx+d)^{m-1}$

$\frac{d}{dx}(cx+d)^m=mc(cx+d)^{m-1} \qquad \ldots(2)$

Similarly, $\frac{d}{dx}(ax+b)^n=na(ax+b)^{n-1} \qquad \ldots(3)$

Therefore, from (1), (2), and (3), we obtain

$f'(x)=(ax+b)^n\left\{mc(cx+d)^{m-1}\right\}+(cx+d)^m\left\{na(ax+b)^{n-1}\right\}$

$\quad=(ax+b)^{n-1}(cx+d)^{m-1}[mc(ax+b)+na(cx+d)]$

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Q14 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): sin (x + a)

Answer.

Let $f(x)=\sin(x+a)$

$f(x+h)=\sin(x+h+a)$

By first principle,

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\sin(x+h+a)-\sin(x+a)}{h}$

$=\lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{x+h+a+x+a}{2}\right)\sin\left(\frac{x+h+a-x-a}{2}\right)\right]$

$=\lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{2x+2a+h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$

$=\lim_{h\to 0}\left[\cos\left(\frac{2x+2a+h}{2}\right)\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\}\right]$

$=\lim_{h\to 0}\cos\left(\frac{2x+2a+h}{2}\right)\lim_{\frac{b}{2}\to 0}\left\{\frac{\sin\left(\frac{n}{2}\right)}{\left(\frac{h}{2}\right)}\right\} \qquad \left[\text{As } h\to 0\Rightarrow\frac{h}{2}\to 0\right]$

$=\cos\left(\frac{2x+2a}{2}\right)\times 1 \qquad \left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$

$=\cos(x+a)$

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Q15 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): cosec x cot x

Answer.

Let $f(x)=\csc x\cot x$

By Leibnitz product rule,

$f'(x)=\csc x(\cot x)'+\cot x(\csc x)'$

Let $f_1(x)=\cot x$. Accordingly , $f_1(x+h)=\cot(x+h)$

By first principle,

$$\begin{aligned}f_1'(x)&=\lim_{h\to0}\frac{f_1(x+h)-f_1(x)}{h}\\&=\lim_{h\to0}\frac{\cot(x+h)-\cot x}{h}\\&=\lim_{h\to0}\frac{1}{h}\left(\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}\right)\end{aligned}$$

$=\lim_{h\to0}\frac{1}{h}\left[\frac{\sin x\cos(x+h)-\cos x\sin(x+h)}{\sin x\sin(x+h)}\right]$

$=\lim_{h\to0}\frac{1}{h}\left[\frac{\sin(x-x-h)}{\sin x\sin(x+h)}\right]$

$=\frac{1}{\sin x}\lim_{h}\frac{1}{h}\left[\frac{\sin(-h)}{\sin(x+h)}\right]$

$=\frac{-1}{\sin x}\cdot\left(\lim_{h\to0}\frac{\sin h}{h}\right)\left(\lim_{h\to0}\frac{1}{\sin(x+h)}\right)$

$=\frac{-1}{\sin x}\cdot1\cdot\left(\frac{1}{\sin(x+0)}\right)$

$=\frac{-1}{\sin^2x}$

$=-cosec^2x$

$\therefore(\cot x)'=-\csc^2x$......(2)

Now, let $f_2(x)=\csc x$. Accordingly, $f_2(x+h)=cosec(x+h)$

By first principle,

$f_2'(x)=\lim_{h\to0}\frac{f_2(x+h)-f_2(x)}{h}$

$=\lim_{h\to0}\frac{1}{h}[cosec(x+h)-cosecx]$

$=\lim_{h\to0}\frac{1}{h}\left[\frac{1}{\sin(x+h)}-\frac{1}{\sin x}\right]$

$=\lim_{h\to0}\frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)}\right]$

$=\frac{1}{\sin x}\lim_{h\to0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)}\right]$

$=\frac{1}{\sin x}\lim_{h\to0}\frac{1}{h}=\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\sin(x+h)}]$

$$=\frac{1}{\sin x}\lim_{h\to0}\left[\frac{-\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\cdot\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}\right]$$

$$=\frac{-1}{\sin x}\lim_{h\to0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\cdot\lim_{h\to0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$=\frac{-1}{\sin x}.1.\frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)}$

$=\frac{-1}{\sin x}\cdot\frac{\cos x}{\sin x}$

$=-\cos\mathrm{ec}\,x\cdot\cot x$

$\therefore(cosecx)'=-cosecx\cot x\ldots(3)$

From (1), (2), and (3), we obtain

$f'(x)=cosecx\left(-cosec^2x\right)+\cot x(-cosecx\cot x)$

$=-cosec^3x-\cot^2x\,cosecx$

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Q16 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\cos x}{1+\sin x}$

Answer. Let

$f(x)=\frac{\cos x}{1+\sin x}$

By quotient rule,

$$f'(x)=\frac{(1+\sin x)\frac{d}{dx}(\cos x)-(\cos x)\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$=\frac{(1+\sin x)(-\sin x)-(\cos x)(\cos x)}{(1+\sin x)^2}$$

$$=\frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$$

$$=\frac{-\sin x-\left(\sin^2 x+\cos^2 x\right)}{(1+\sin x)^2}$$

$$=\frac{-\sin x-1}{(1+\sin x)^2}$$

$$=\frac{-(1+\sin x)}{(1+\sin x)^2}$$

$$=\frac{-1}{(1+\sin x)}$$

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Q17 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{\sin x+\cos x}{\sin x-\cos x}$$

Answer. Let $f(x)=\frac{\sin x+\cos x}{\sin x-\cos x}$

By quotient rule,

$$f'(x)=\frac{(\sin x-\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$=\frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$$

$$=\frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$$

$$=\frac{-\left[\sin^2 x+\cos^2 x-2\sin x\cos x+\sin^2 x+\cos^2 x+2\sin x\cos x\right]}{(\sin x-\cos x)^2}$$

$$=\frac{-[1+1]}{(\sin x-\cos x)^2}$$

$$=\frac{-2}{(\sin x-\cos x)^2}$$

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Q18 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

Answer. Let

$$f(x)=\frac{\sec x-1}{\sec x+1}$$

$$f(x)=\frac{\frac{1}{\cos x}-1}{\frac{1}{\cos x}+1}=\frac{1-\cos x}{1+\cos x}$$

By quotient rule,

$$f'(x)=\frac{(1+\cos x)\frac{d}{dx}(1-\cos x)-(1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)(\sin x)-(1-\cos x)^2}{(1+\cos x)^2}$$

$$=\frac{\sin x+\cos x\sin x+\sin x-\sin x\cos x}{(1+\cos x)^2}$$

$$=\frac{2\sin x}{(1+\cos x)^2}$$

$$= \frac{2\sin x}{\left(1+\frac{1}{\sec x}\right)^2} = \frac{2\sin x}{\frac{(\sec x+1)^2}{\sec^2 x}}$$

$$= \frac{2\sin x\sec^2 x}{(\sec x+1)^2}$$

$$= \frac{2\sin x}{(\sec x+1)^2}$$

$$= \frac{2\sec x + 1\mathrm{n}x}{(\sec x+1)^2}$$

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Q19 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):
$\sin^n x$

Answer.

Let $y = \sin^n x$

Accordingly, for $n = 1, y = \sin x$

$$\therefore \frac{dy}{dx} = \cos x,\ \text{i.e., } \frac{d}{dx}\sin x = \cos x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin x\sin x)$$

$$= (\sin x)'\sin x + \sin x(\sin x)' \quad \text{[By leibnitz product rule]}$$

$$= \cos x\sin x + \sin x\cos x$$

$$= 2\sin x\cos x\ldots(1)$$

For $\mathrm{n} = 3, \mathrm{y} = \sin^3 \mathrm{x}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\sin x\sin^2 x\right)$$

$$= (\sin x)'\sin^2 x + \sin x\left(\sin^2 x\right)' \qquad \text{[By Leibnitz product rule]}$$

$$= \cos x\sin^2 x + \sin x(2\sin x\cos x)\text{[Using (1)]}$$

$$= \cos x\sin^2 x + 2\sin^2 x\cos x$$

$$= 3\sin^2 x\cos x$$

$$\frac{d}{dx}(\sin^n x) = n\sin^{(n-1)} x\cos x$$

Let our assertion be true for $\mathrm{n} = \mathrm{k}$.

$$i.e., \frac{d}{dx}\left(\sin^k x\right) = k\sin^{(k-1)} x\cos x\ldots.(2)$$

Consider

$$\frac{d}{dx}\left(\sin^{k+1} x\right) = \frac{d}{dx}\left(\sin x\sin^k x\right)$$

$$= (\sin x)'\sin^t x + \sin x\left(\sin^k x\right)' \text{[By leibnitz product rule]}$$

$$= \cos x\sin^k x + \sin x\left(k\sin^{(k-1)} x\cos x\right)\text{[Using (2)]}$$

$$= \cos x\sin^k x + k\sin^k x\cos x$$

$$= (k+1)\sin^k x\cos x$$

Thus, our assertion is true for $\mathrm{n} = \mathrm{k}+1$

Hence, by mathematical induction, $\frac{d}{dx}(\sin^n x) = n\sin^{(n-1)} x\cos x$

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Q20 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

Answer.

Let $f(x) = \frac{a+b\sin x}{c+d\cos x}$

By quotient rule,

$$f'(x) = \frac{(c+d\cos x)\frac{d}{dx}(a+b\sin x)-(a+b\sin x)\frac{d}{dx}(c+d\cos x)}{(c+d\cos x)^2}$$

$$= \frac{(c+d\cos x)(b\cos x)-(a+b\cos x)(-d\sin x)}{(c+d\cos x)^2}$$

$$= \frac{cb\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c+d\cos x)^2}$$

$$= \frac{bc\cos x + ad\sin x + bd\left(\cos^2 x + \sin^2 x\right)}{(c + d\cos x)^2}$$
$$= \frac{bc\cos x + ad\sin x + bd}{(c + d\cos x)^2}$$

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Q21 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin(x+a)}{\cos x}$

Answer. Let
$f(x) = \frac{\sin(x+a)}{\cos x}$
By quotient rule,
$f'(x) = \frac{\cos x \frac{d}{dx}[\sin(x+a)] - \sin(x+a)\frac{d}{dx}\cos x}{\cos^2 x}$
$f'(x) = \frac{\cos x \frac{d}{dx}[\sin(x+a)] - \sin(x+a)(-\sin x)}{\cos^2 x} \ldots\ldots (i)$
Let $g(x) = \sin(x+a) \cdot$ Accordingly, $g(x+h) = \sin(x+h+a)$

By first principle,

$$\begin{aligned} g'(x) &= \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0}\frac{1}{h}[\sin(x+h+a) - \sin(x+a)] \\ &= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{x+h+a+x+a}{2}\right)\sin\left(\frac{x+h+a-x-a}{2}\right)\right] \\ &= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{2x+h+a+h}{2}\right)\sin\left(\frac{h}{2}\right)\right] \end{aligned}$$

$= \lim_{h\to 0}\left[\cos\left(\frac{2x+2a+h}{2}\right)\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\}\right]$

$= \lim_{h\to 0}\cos\left(\frac{2x+2a+h}{2}\right)\lim_{\frac{h}{2},0}\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\} \quad \left[\text{As } h \to 0 \Rightarrow \frac{h}{2} \to 0\right]$

$= \cos(x+a)$ ....(ii)
From (i) and (ii), we obtain

$$\begin{aligned} f'(x) &= \frac{\cos x \cdot \cos(x+a) + \sin x \sin(x+a)}{\cos^2 x} \\ &= \frac{\cos(x+a-x)}{\cos^2 x} \\ &= \frac{\cos a}{\cos^2 x} \end{aligned}$$

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Q22 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): x^4 (5 sin x – 3 cos x)

Answer.
Let $f(x) = x^4(5\sin x - 3\cos x)$
By product rule,
$f'(x) = x^4\frac{d}{dx}(5\sin x - 3\cos x) + (5\sin x - 3\cos x)\frac{d}{dx}\left(x^4\right)$
$= x^4\left[5\frac{d}{dx}(\sin x) - 3\frac{d}{dx}(\cos x)\right] + (5\sin x - 3\cos x)\frac{d}{dx}\left(x^4\right)$
$= x^4[5\cos x - 3(-\sin x)] + (5\sin x - 3\cos x)\left(4x^3\right)$
$= x^3[5x\cos x + 3x\sin x + 20\sin x - 12\cos x]$

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Q23 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\left(x^2+1\right) \cos x$

Answer.

Let $f(x)=\left(x^2+1\right) \cos x$

By product rule,

$f'(x)=\left(x^2+1\right) \frac{d}{dx}(\cos x)+\cos x \frac{d}{dx}\left(x^2+1\right)$

$=\left(x^2+1\right)(-\sin x)+\cos x(2x)$

$=-x^2 \sin x-\sin x+2x \cos x$

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Q24 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):
$\left(ax^2+\sin x\right)(p+q \cos x)$

Answer.

Let $f(x)=\left(ax^2+\sin x\right)(p+q \cos x)$

By product rule,

$f'(x)=\left(ax^2+\sin x\right) \frac{d}{dx}(p+q \cos x)+(p+q \cos x) \frac{d}{dx}\left(ax^2+\sin x\right)$

$=\left(ax^2+\sin x\right)(-q \sin x)+(p+q \cos x)(2ax+\cos x)$

$=-q \sin x\left(ax^2+\sin x\right)+(p+q \cos x)(2ax+\cos x)$

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Q25 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x+\cos x)(x-\tan x)$

Answer.

Let $f(x)=(x+\cos x)(x-\tan x)$

By product rule,

$f'(x)=(x+\cos x) \frac{d}{dx}(x-\tan x)+(x-\tan x) \frac{d}{dx}(x+\cos x)$

$=(x+\cos x)\left[\frac{d}{dx}(x)-\frac{d}{dx}(\tan x)\right]+(x-\tan x)(1-\sin x)$

$=(x+\cos x)\left[1-\frac{d}{dx} \tan x\right]+(x-\tan x)(1-\sin x)$

Let $g(x)=\tan x$. Accordingly, $g(x+h)=\tan(x+h)$

By first principle,

$g'(x)=\lim_{h \rightarrow 0} \frac{g(x+h)-g(x)}{h}$

$=\lim_{h \rightarrow 0}\left(\frac{\tan(x+h)-\tan x}{h}\right)$

$=\lim_{h \rightarrow 0} \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}\right]$

$=\lim_{h \rightarrow 0} \frac{1}{h}\left[\frac{\sin(x+h) \cos x-\sin x \cos(x+h)}{\cos(x+h) \cos x}\right]$

$=\frac{1}{\cos x} \cdot \lim_{h \rightarrow \frac{1}{h}}\left[\frac{\sin(x+h-x)}{\cos(x+h)}\right]$

$$=\frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \frac{1}{h}\left[\frac{\sin h}{\cos(x+h)}\right]$$

$$=\frac{1}{\cos x} \cdot\left(\lim_{h \rightarrow 0} \frac{\sin h}{h}\right) \cdot\left(\lim_{h \rightarrow \infty} \frac{1}{\cos(x+h)}\right)$$

$$=\frac{1}{\cos x} \cdot 1 \cdot \frac{1}{\cos(x+0)}$$

1

$$= \frac{}{\cos^2 x}$$

$= \sec^2 x \ldots\ldots (ii)$

Therefore, from (i) and (ii), we obtain

$$\begin{aligned} f'(x) &= (x+\cos x)\left(1-\sec^2 x\right)+(x-\tan x)(1-\sin x) \\ &= (x+\cos x)\left(-\tan^2 x\right)+(x-\tan x)(1-\sin x) \\ &= -\tan^2 x(x+\cos x)+(x-\tan x)(1-\sin x) \end{aligned}$$

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Q26 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$\frac{4x+5\sin x}{3x+7\cos x}$

Answer.

$f(x) = \frac{4x+5\sin x}{3x+7\cos x}$

By quotient rule,

$f'(x) = \frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$

$$= \frac{(3x+7\cos x)\left[4\frac{d}{dx}(x)+5\frac{d}{dx}(\sin x)\right]-(4x+5\sin x)\left[3\frac{d}{dx}x+7\frac{d}{dx}\cos x\right]}{(3x+7\cos x)^2}$$

$$= \frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$

$$= \frac{12x+15x\cos x+28\cos x+35\cos^2 x-12x+28x\sin x-15\sin x+35\sin^2 x}{(3x+7\cos x)^2}$$

$$= \frac{15x\cos x+28\cos x+28x\sin x-15\sin x+35\left(\cos^2 x+\sin^2 x\right)}{(3x+7\cos x)^2}$$

$$= \frac{35+15x\cos x+28\cos x+28x\sin x-15\sin x}{(3x+7\cos x)^2}$$

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Q27 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

Answer.

$f(x) = \frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

By quotient rule,

$f'(x) = \cos\frac{\pi}{4}\cdot\left[\frac{\sin x\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(\sin x)}{\sin^2 x}\right]$

$$= \cos\frac{\pi}{4}\cdot\left[\frac{\sin x\cdot 2x-x^2\cos x}{\sin^2 x}\right]$$

$$= \frac{x\cos\frac{\pi}{4}[2\sin x-x\cos x]}{\sin^2 x}$$

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Q28 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{x}{1+\tan x}$

Answer. Let

$f(x) = \frac{x}{1+\tan x}$

$f'(x) = \frac{(1+\tan x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$

$f'(x) = \frac{(1+\tan x)-x\cdot\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}\ldots\ldots(i)$

Let $g(x) = 1+\tan x$. Accordingly, $g(x+h) = 1+\tan(x+h)$

By first principle,

$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= \lim_{h\to 0} \left[\frac{1+\tan(x+h)-1-\tan x}{h}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos(x+h)\cos x}\right]$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos(x+h)\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin h}{\cos(x+h)\cos x}\right]$$

$$= \left(\lim_{h\to 0}\frac{\sin h}{h}\right)\cdot\left(\lim_{h\to 0}\frac{1}{\cos(x+h)\cos x}\right)$$

$= 1 \times \frac{1}{\cos^2 x} = \sec^2 x$

$\Rightarrow \frac{d}{dx}(1+\tan x) = \sec^2 x \ldots\ldots(ii)$

From (i) and (ii), we obtain

$f'(x) = \frac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

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Q29 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): (x + sec x) (x − tan x)

Answer.

Let $f(x) = (x+\sec x)(x-\tan x)$

By product rule,

$f'(x) = (x+\sec x)\frac{d}{dx}(x-\tan x) + (x-\tan x)\frac{d}{dx}(x+\sec x)$

$= (x+\sec x)\left[\frac{d}{dx}(x) - \frac{d}{dx}\tan x\right] + (x-\tan x)\left[\frac{d}{dx}(x) + \frac{d}{dx}\sec x\right]$

$= (x+\sec x)\left[1 - \frac{d}{dx}\tan x\right] + (x-\tan x)\left[1 + \frac{d}{dx}\sec x\right]$.....(i)

Let $f_1(x) = \tan x, f_2(x) = \sec x$

Accordingly, $f_1(x+h) = \tan(x+h)$ and $f_2(x+h) = \sec(x+h)$

$f_1'(x) = \lim_{h\to 0}\left(\frac{f_1(x+h)-f_1(x)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\tan(x+h)-\tan x}{h}\right)$

$= \lim_{h\to 0}\left[\frac{\tan(x+h)-\tan x}{h}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h)\cos x - \sin x\cos(x+h)}{\cos(x+h)\cos x}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos(x+h)\cos x}\right]$

$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin h}{\cos(x+h)\cos x}\right]$

$= \left(\lim_{h\to 0}\frac{\sin h}{h}\right)\cdot\left(\lim_{h\to 0}\frac{1}{\cos(x+h)\cos x}\right)$

$= 1 \times \frac{1}{\cos^2 x} = \sec^2 x$

$\Rightarrow \frac{d}{dx}\tan x = \sec^2 x \ldots(ii)$

$$
\begin{aligned}
f_2'(x) &= \lim_{h\to 0}\left(\frac{f_2(x+h)-f_2(x)}{h}\right)\\
&= \lim_{h\to 0}\left(\frac{\sec(x+h)-\sec x}{h}\right)\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{1}{\cos(x+h)}-\frac{1}{\cos x}\right]\\
&= \lim_{h\to 0}\frac{1}{h}\left[\frac{\cos x-\cos(x+h)}{\cos(x+h)\cos x}\right]
\end{aligned}
$$

$$= \tfrac{1}{\cos x}\cdot \lim_{h\to 0}\tfrac{1}{h}\left[\tfrac{-2\sin\left(\frac{x+x+h}{2}\right)\cdot\sin\left(\frac{x-x-h}{2}\right)}{\cos(x+h)}\right]$$

$$= \tfrac{1}{\cos x}\cdot \lim_{h\to 0}\tfrac{1}{h}\left[\tfrac{-2\sin\left(\frac{2x+h}{2}\right)\cdot\sin\left(\frac{-h}{2}\right)}{\cos(x+h)}\right]$$

$$= \tfrac{1}{\cos x}\cdot \lim_{h\to 0}\left[\tfrac{\sin\left(\frac{2x+h}{2}\right)\left\{\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right\}}{\cos(x+h)}\right]$$

$$= \sec x\cdot\tfrac{\left\{\lim_{h\to 0}\sin\left(\frac{2x+h}{2}\right)\right\}\left\{\lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}\to 0}\frac{h}{2}\right\}}{\lim_{h\to 0}\cos(x+h)}$$

$$= \sec x\cdot\tfrac{\sin x\cdot 1}{\cos x}$$

$$\Rightarrow \tfrac{d}{dx}\sec x = \sec x\tan x$$

From (i), (ii), and (iii), we obtain

$$f'(x) = (x+\sec x)\left(1-\sec^2 x\right)+(x-\tan x)(1+\sec x\tan x)$$

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Q30 Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$\frac{x}{\sin^n x}$

Answer. Let

$f(x) = \frac{x}{\sin^n x}$

By quotient rule,

$f'(x) = \frac{\sin^n x\frac{d}{dx}x - x\frac{d}{dx}\sin^n x}{\sin^{2n} x}$

It can be easily shown that $\frac{d}{dx}\sin^n x = n\sin^{n-1} x\cos x$

Therefore,

$$
\begin{aligned}
f'(x) &= \frac{\sin^n x\frac{d}{dx}x - x\frac{d}{dx}\sin^n x}{\sin^{2n} x}\\
&= \frac{\sin^n x\cdot 1 - x\left(n\sin^{n-1} x\cos x\right)}{\sin^{2n} x}
\end{aligned}
$$

$$= \frac{\sin^{n-1} x(\sin x - nx\cos x)}{\sin^{2n} x}$$

$$= \frac{\sin x - nx\cos x}{\operatorname{cin}^{n+1} x}$$

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