

Exercise 8.1

Question 1:

In $\triangle ABC$ right angled at B, $AB = 24$ cm, $BC = 7$ m. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Answer:

Applying Pythagoras theorem for $\triangle ABC$, we obtain

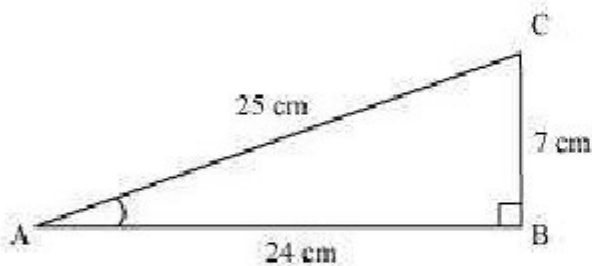
$$AC^2 = AB^2 + BC^2$$

$$= (24 \text{ cm})^2 + (7 \text{ cm})^2$$

$$= (576 + 49) \text{ cm}^2$$

$$= 625 \text{ cm}^2$$

$$\therefore AC = \sqrt{625} \text{ cm} = 25 \text{ cm}$$

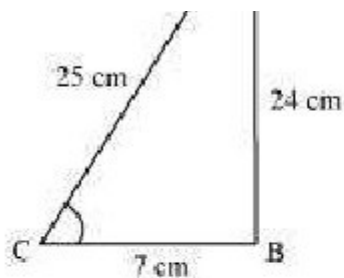


$$\begin{aligned} \text{(i) } \sin A &= \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} \\ &= \frac{7}{25} \end{aligned}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

(ii)





$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

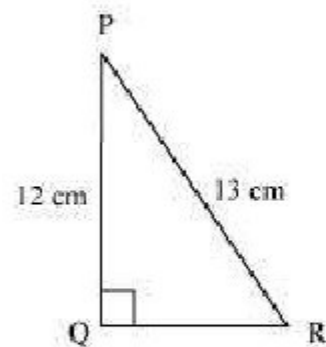
$$= \frac{24}{25}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

Question 2:

In the given figure find $\tan P - \cot R$



Answer:

Applying Pythagoras theorem for ΔPQR , we obtain

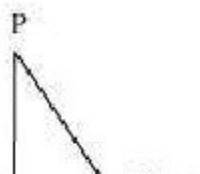
$$PR^2 = PQ^2 + QR^2$$

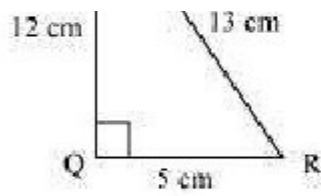
$$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$$

$$169 \text{ cm}^2 = 144 \text{ cm}^2 + QR^2$$

$$25 \text{ cm}^2 = QR^2$$

$$QR = 5 \text{ cm}$$





$$\begin{aligned}\tan P &= \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} \\ &= \frac{5}{12}\end{aligned}$$

$$\begin{aligned}\cot R &= \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ} \\ &= \frac{5}{12}\end{aligned}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

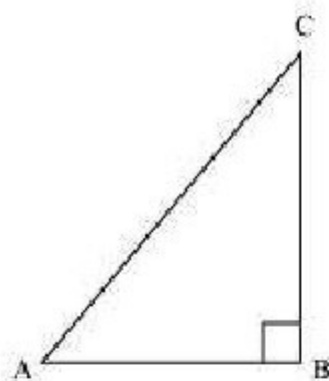


Question 3:

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Answer:

Let $\triangle ABC$ be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be $3k$. Therefore, AC will be $4k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = AB^2 + (3k)^2$$

$$16k^2 - 9k^2 = AB^2$$

$$7k^2 = AB^2$$

$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

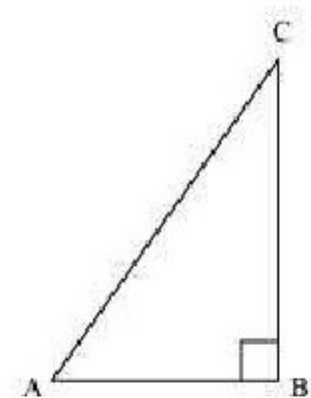
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

Given $15 \cot A = 8$. Find $\sin A$ and $\sec A$

Answer:

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$AB$$

$$= \frac{8}{15}$$

It is given that,

$$\cot A = \frac{8}{15}$$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$. Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$$AC = 17k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

$$= \frac{AC}{AB} = \frac{17}{8}$$



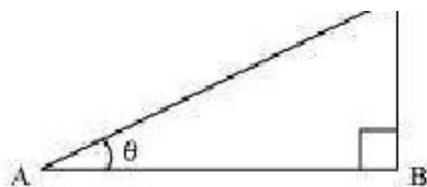
Question 5:

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Answer:

Consider a right-angle triangle $\triangle ABC$, right-angled at point B.





$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is $13k$, AB will be $12k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + BC^2$$

$$25k^2 = BC^2$$

$$BC = 5k$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

Question 6:

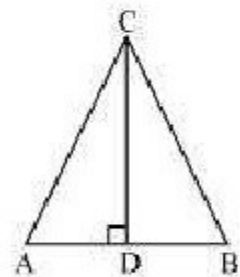
If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that

$$\angle A = \angle B.$$

Answer:

Answer :

Let us consider a triangle ABC in which $CD \perp AB$.

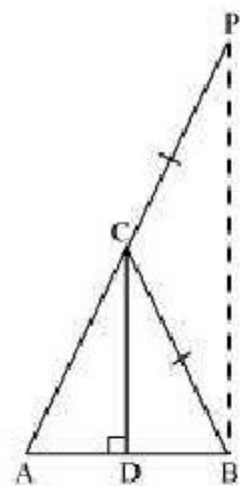


It is given that

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \dots (1)$$

We have to prove $\angle A = \angle B$. To prove this, let us extend AC to P such that $BC = CP$.



From equation (1), we obtain

$$\begin{aligned} \frac{AD}{BD} &= \frac{AC}{BC} \\ \Rightarrow \frac{AD}{BD} &= \frac{AC}{CP} \quad \text{(By construction, we have } BC = CP) \quad \dots (2) \end{aligned}$$

By using the converse of B.P.T,

$$CD \parallel BP$$

$$\Rightarrow \angle ACD = \angle CPB \text{ (Corresponding angles)} \dots (3)$$

$$\text{And, } \angle BCD = \angle CBP \text{ (Alternate interior angles)} \dots (4)$$

By construction, we have $BC = CP$.

$$\therefore \angle CBP = \angle CPB \text{ (Angle opposite to equal sides of a triangle)} \dots (5)$$

From equations (3), (4), and (5), we obtain

$$\angle ACD = \angle BCD \dots (6)$$

In $\triangle CAD$ and $\triangle CBD$,

$$\angle ACD = \angle BCD \text{ [Using equation (6)]}$$

$$\angle CDA = \angle CDB \text{ [Both } 90^\circ]$$

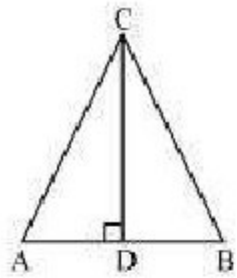
Therefore, the remaining angles should be equal.

$$\therefore \angle CAD = \angle CBD$$

$$\Rightarrow \angle A = \angle B$$

Alternatively,

Let us consider a triangle ABC in which $CD \perp AB$.



It is given that,

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

$$\text{Let } \frac{AD}{BD} = \frac{AC}{BC} = k$$

$$\Rightarrow AD = k \cdot BD \dots (1)$$

$$\text{And, } AC = k \cdot BC \dots (2)$$

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$$CD^2 = AC^2 - AD^2 \dots (3)$$

$$\text{And, } CD^2 = BC^2 - BD^2 \dots (4)$$

From equations (3) and (4), we obtain

$$AC^2 - AD^2 = BC^2 - BD^2$$

$$\Rightarrow (k \cdot BC)^2 - (k \cdot BD)^2 = BC^2 - BD^2$$

$$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$$

$$\Rightarrow k^2 = 1$$

$$\Rightarrow k = 1$$

Putting this value in equation (2), we obtain

$$AC = BC$$

$\Rightarrow \angle A = \angle B$ (Angles opposite to equal sides of a triangle)

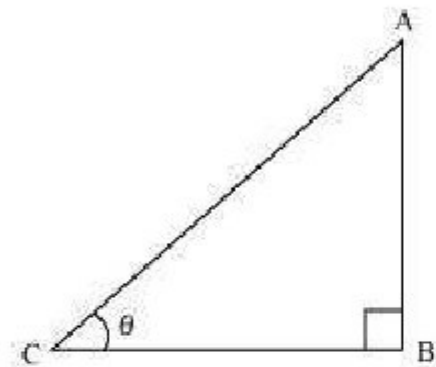
Question 7:

If $\cot \theta = \frac{7}{8}$, evaluate

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \quad (ii) \cot^2 \theta$$

Answer:

Let us consider a right triangle ABC, right-angled at point B.



$$\begin{aligned} \cot \theta &= \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{BC}{AB} \\ &= \frac{7}{8} \end{aligned}$$

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (8k)^2 + (7k)^2 \\ &= 64k^2 + 49k^2 \\ &= 113k^2 \end{aligned}$$

$$AC = \sqrt{113}k$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113k}} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113k}} = \frac{7}{\sqrt{113}}$$

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1 - \sin^2 \theta)}{(1 - \cos^2 \theta)}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

$$(ii) \cot^2 \theta = (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Question 8:

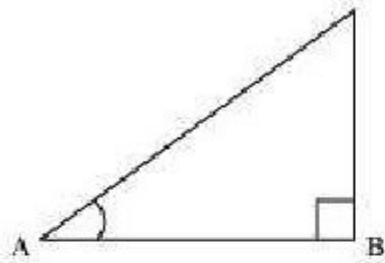
If $3 \cot A = 4$, Check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer:

It is given that $3 \cot A = 4$

$$\text{Or, } \cot A = \frac{4}{3}$$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is $4k$, then BC will be $3k$, where k is a positive integer.

In $\triangle ABC$,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (4k)^2 + (3k)^2$$

$$= 16k^2 + 9k^2$$

$$= 25k^2$$

$$AC = 5k$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$\frac{7}{25}$$

$$= \frac{16}{25} = \frac{7}{25}$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

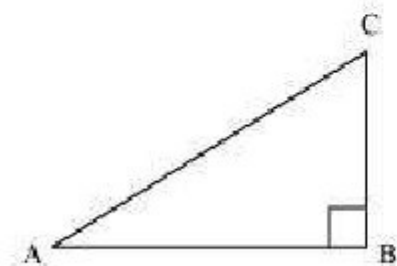


Question 9:

In $\triangle ABC$, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

- (i) $\sin A \cos C + \cos A \sin C$
- (ii) $\cos A \cos C - \sin A \sin C$

Answer:



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is k , then AB will be $\sqrt{3}k$, where k is a positive integer.

In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{3}k)^2 + (k)^2$$

$$= (\sqrt{3}k)^2 + (k)^2$$

$$= 3k^2 + k^2 = 4k^2$$

$$\therefore AC = 2k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$(i) \sin A \cos C + \cos A \sin C$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR , right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Answer:

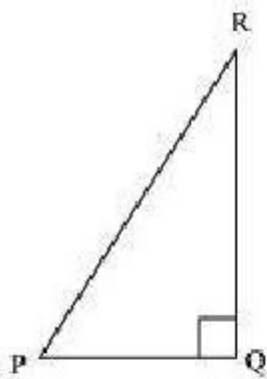
Given that, $PR + QR = 25$

$PQ = 5$

Let PR be x .

Then $QR = 25 - PR = 25 - x$

Therefore, $QR = 25 - x$



Applying Pythagoras theorem in ΔPQR , we obtain

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = (5)^2 + (25 - x)^2$$

$$x^2 = 25 + 625 + x^2 - 50x$$

$$50x = 650$$

$$x = 13$$

Therefore, $PR = 13$ cm

$$QR = (25 - 13) \text{ cm} = 12 \text{ cm}$$

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A .

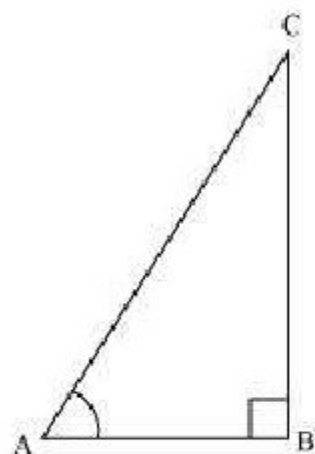
(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

(iv) $\cot A$ is the product of $\cot$ and A

(v) $\sin \theta = \frac{4}{3}$, for some angle θ

Answer:

(i) Consider a $\triangle ABC$, right-angled at B.



$$\begin{aligned}\tan A &= \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} \\ &= \frac{12}{5}\end{aligned}$$

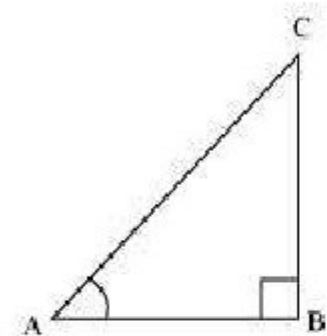
$$\text{But } \frac{12}{5} > 1$$

$$\therefore \tan A > 1$$

So, $\tan A < 1$ is not always true.

Hence, the given statement is false.

$$(ii) \sec A = \frac{12}{5}$$



$$\begin{aligned}\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} &= \frac{12}{5} \\ \frac{AC}{AB} &= \frac{12}{5}\end{aligned}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$(12k)^2 = (5k)^2 + BC^2$$

$$144k^2 = 25k^2 + BC^2$$

$$BC^2 = 119k^2$$

$$BC = 10.9k$$

It can be observed that for given two sides $AC = 12k$ and $AB = 5k$,
BC should be such that,

$$AC - AB < BC < AC + AB$$

$$12k - 5k < BC < 12k + 5k$$

$$7k < BC < 17k$$

However, $BC = 10.9k$. Clearly, such a triangle is possible and hence, such value of $\sec A$ is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) $\cot A$ is not the product of cot and A. It is the cotangent of $\angle A$.

Hence, the given statement is false.

$$(v) \sin \theta = \frac{4}{3}$$

We know that in a right-angled triangle,

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of $\sin \theta$ is not possible.

Hence, the given statement is false

Exercise 8.2

Question 1:

Evaluate the following

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$\begin{aligned} &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$\begin{aligned} &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 2 + \frac{3}{4} - \frac{3}{4} = 2 \end{aligned}$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$\begin{aligned}
&= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}} \\
&= \frac{\sqrt{3}}{\sqrt{2}(2 + 2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} \\
&= \frac{\sqrt{3}(2\sqrt{6} - 2\sqrt{2})}{(2\sqrt{6} + 2\sqrt{2})(2\sqrt{6} - 2\sqrt{2})} \\
&= \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{24 - 8} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{16} \\
&= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}
\end{aligned}$$

$$\text{(iv) } \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$$

$$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

$$(v) \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$



Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

- (A). $\sin 60^\circ$
- (B). $\cos 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

- (A). $\tan 90^\circ$
- (B). 1
- (C). $\sin 45^\circ$
- (D). 0

(iii) $\sin 2A = 2 \sin A$ is true when $A =$

- (A). 0°
- (B). 30°
- (C). 45°
- (D). 60°

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

- (A). $\cos 60^\circ$
- (B). $\sin 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

Answer:

$$\begin{aligned}
 & \text{(i) } \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \\
 &= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}
 \end{aligned}$$

$$1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{4}{3}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$$

$$= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only $A = 0^\circ$ is correct.

$$\text{As } \sin 2A = \sin 0^\circ = 0$$

$$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$$

Hence, (A) is correct.

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct.

Question 3:

$$\text{If } \tan(A + B) = \sqrt{3} \text{ and } \tan(A - B) = \frac{1}{\sqrt{3}};$$

$0^\circ < A + B \leq 90^\circ$, $A > B$ find A and B.

Answer:

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60$$

$$\Rightarrow A + B = 60 \dots (1)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30$$

$$\Rightarrow A - B = 30 \dots (2)$$

On adding both equations, we obtain

$$2A = 90$$

$$\Rightarrow A = 45$$

From equation (1), we obtain

$$45 + B = 60$$

$$B = 15$$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

Question 4:

State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin \theta$ increases as θ increases

(iii) The value of $\cos \theta$ increases as θ increases

(iv) $\sin \theta = \cos \theta$ for all values of θ

(v) $\cot A$ is not defined for $A = 0^\circ$

Answer:

(i) $\sin(A + B) = \sin A + \sin B$

Let $A = 30^\circ$ and $B = 60^\circ$

$$\sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Clearly, $\sin (A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$ as

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

$$(iii) \cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

It can be observed that the value of $\cos \theta$ does not increase in the interval of $0^\circ < \theta < 90^\circ$.

Hence, the given statement is false.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

This is true when $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ .

As $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) $\cot A$ is not defined for $A = 0^\circ$

As $\cot A = \frac{\cos A}{\sin A}$,

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined}$$

Hence, the given statement is true.

Exercise 8.3

Question 1:

Evaluate

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) $\cos 48^\circ - \sin 42^\circ$

(IV) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Answer:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$$\cot 64^\circ \dots$$

$$= \frac{\cancel{\cos 64^\circ}}{\cot 64^\circ} = 1$$

$$\begin{aligned} \text{(III)} \cos 48^\circ - \sin 42^\circ &= \cos (90^\circ - 42^\circ) - \sin 42^\circ \\ &= \sin 42^\circ - \sin 42^\circ \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{(IV)} \operatorname{cosec} 31^\circ - \sec 59^\circ &= \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ \\ &= \sec 59^\circ - \sec 59^\circ \\ &= 0 \end{aligned}$$

Question 2:

Show that

$$\text{(I)} \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

$$\text{(II)} \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

Answer:

$$\begin{aligned} \text{(I)} \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ &= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ \\ &= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \\ &= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ) \\ &= (1)(1) \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(II)} \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ &= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ \\ &= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ \\ &= 0 \end{aligned}$$

Question 3:

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Answer:

Given that,

$$\begin{aligned} \tan 2A &= \cot (A - 18^\circ) \\ \cot (90^\circ - 2A) &= \cot (A - 18^\circ) \\ 90^\circ - 2A &= A - 18^\circ \\ 108^\circ &= 3A \end{aligned}$$

$$A = 36^\circ$$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90^\circ$

Answer:

Given that,

$$\tan A = \cot B$$

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

Question 5:

If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Answer:

Given that,

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

$$\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

$$90^\circ - 4A = A - 20^\circ$$

$$110^\circ = 5A$$

$$A = 22^\circ$$

Question 6:

If A , B and C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer:

We know that for a triangle ABC ,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\begin{aligned}\sin\left(\frac{B+C}{2}\right) &= \sin\left(90^\circ - \frac{A}{2}\right) \\ &= \cos\left(\frac{A}{2}\right)\end{aligned}$$

Question 7:

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Answer:

$$\sin 67^\circ + \cos 75^\circ$$

$$= \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ)$$

$$= \cos 23^\circ + \sin 15^\circ$$

Exercise 8.4

Question 1:

Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Answer:

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

$\sqrt{1 + \cot^2 A}$ will always be positive as we are adding two positive quantities.

$$\text{Therefore, } \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\text{We know that, } \tan A = \frac{\sin A}{\cos A}$$

$$\text{However, } \cot A = \frac{\cos A}{\sin A}$$

$$\text{Therefore, } \tan A = \frac{1}{\cot A}$$

$$\text{Also, } \sec^2 A = 1 + \tan^2 A$$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Answer:

We know that,

$$\cos A = \frac{1}{\sec A}$$

$$\text{Also, } \sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\begin{aligned}\sin A &= \sqrt{1 - \left(\frac{1}{\sec A}\right)^2} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}\end{aligned}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\begin{aligned}\cot A &= \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} \\ &= \frac{1}{\sqrt{\sec^2 A - 1}}\end{aligned}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3:

Evaluate

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

Answer:

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(i) \cos^2 17^\circ + \cos^2 73^\circ$$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1} \text{ (As } \sin^2 A + \cos^2 A = 1 \text{)}$$

$$= 1$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= (\sin 25^\circ) \{\cos(90^\circ - 25^\circ)\} + \cos 25^\circ \{\sin(90^\circ - 25^\circ)\}$$

$$= (\sin 25^\circ)(\sin 25^\circ) + (\cos 25^\circ)(\cos 25^\circ)$$

$$= \sin^2 25^\circ + \cos^2 25^\circ$$

$$= 1 \text{ (As } \sin^2 A + \cos^2 A = 1 \text{)}$$

Question 4:

Choose the correct option. Justify your choice.

$$(i) 9 \sec^2 A - 9 \tan^2 A =$$

$$(A) 1$$

$$(B) 9$$

$$(C) 8$$

$$(D) 0$$

$$(ii) (1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$$

$$(A) 0$$

$$(B) 1$$

$$(C) 2$$

$$(D) -1$$

$$(iii) (\sec A + \tan A) (1 - \sin A) =$$

$$(A) \sec A$$

$$(B) \sin A$$

(C) cosecA

(D) cosA

$$(iv) \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer:

$$(i) 9 \sec^2 A - 9 \tan^2 A$$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 (1) \text{ [As } \sec^2 A - \tan^2 A = 1]$$

$$= 9$$

Hence, alternative (B) is correct.

(ii)

$$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$$

$$= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \right)$$

$$= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta} \right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta} \right)$$

$$= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

Hence, alternative (C) is correct.

$$(iii) (\sec A + \tan A) (1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) (1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A} \right) (1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

Hence, alternative (D) is correct.

$$(iv) \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer:

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\begin{aligned}
 \text{L.H.S.} &= (\operatorname{cosec} \theta - \cot \theta)^2 \\
 &= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\
 &= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\
 &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} \\
 &= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)(\cos A)} \\
 &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)} \\
 &= \frac{\sin^2 A + \cos^2 A + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} \\
 &= \frac{1 + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} = \frac{2 + 2 \sin A}{(1 + \sin A)(\cos A)} \\
 &= \frac{2(1 + \sin A)}{(1 + \sin A)(\cos A)} = \frac{2}{\cos A} = 2 \sec A \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\begin{aligned}
\text{L.H.S.} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\
&= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
&= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\
&= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\
&= \left(\frac{1}{\sin \theta - \cos \theta} \right) \left[\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right] \\
&= \left(\frac{1}{\sin \theta - \cos \theta} \right) \left[\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\sin \theta \cos \theta} \right] \\
&= \frac{(1 + \sin \theta \cos \theta)}{(\sin \theta \cos \theta)}
\end{aligned}$$

$$= \sec \theta \operatorname{cosec} \theta +$$

$$= \text{R.H.S.}$$

$$(iv) \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\begin{aligned} \text{L.H.S.} &= \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} \\ &= \frac{\cos A + 1}{\frac{1}{\cos A}} = (\cos A + 1) \cos A \\ &= \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)} \\ &= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A} \end{aligned}$$

$$= \text{R.H.S.}$$

$$(v) \quad \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$,

$$\text{L.H.S.} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

$$\begin{aligned}
&= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}} \\
&= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 + \operatorname{cosec} A} \\
&= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}} \\
&= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \\
&= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2 \cot A - 2 \operatorname{cosec} A + 2 \cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2 \operatorname{cosec} A)} \\
&= \frac{2 \operatorname{cosec}^2 A + 2 \cot A \operatorname{cosec} A - 2 \cot A - 2 \operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2 \operatorname{cosec} A} \\
&= \frac{2 \operatorname{cosec} A (\operatorname{cosec} A + \cot A) - 2 (\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2 \operatorname{cosec} A} \\
&= \frac{(\operatorname{cosec} A + \cot A) (2 \operatorname{cosec} A - 2)}{-1 - 1 + 2 \operatorname{cosec} A} \\
&= \frac{(\operatorname{cosec} A + \cot A) (2 \operatorname{cosec} A - 2)}{(2 \operatorname{cosec} A - 2)} \\
&= \operatorname{cosec} A + \cot A \\
&= \text{R.H.S}
\end{aligned}$$

$$\text{(vi) } \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\begin{aligned}
\text{L.H.S.} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
&= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
&= \frac{(1 + \sin A)}{\sqrt{1 - \sin^2 A}} = \frac{1 + \sin A}{\sqrt{\cos^2 A}} \\
&= \frac{1 + \sin A}{\cos A} = \sec A + \tan A \\
&= \text{R.H.S.}
\end{aligned}$$

$$(vii) \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$\begin{aligned}
\text{L.H.S.} &= \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} \\
&= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\
&= \frac{\sin \theta \times (1 - 2 \sin^2 \theta)}{\cos \theta \times \{2(1 - \sin^2 \theta) - 1\}} \\
&= \frac{\sin \theta \times (1 - 2 \sin^2 \theta)}{\cos \theta \times (1 - 2 \sin^2 \theta)} \\
&= \tan \theta = \text{R.H.S}
\end{aligned}$$

$$(viii) \quad (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$\begin{aligned}
\text{L.H.S} &= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\
&= \sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A \sec A \\
&= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2 \sin A \left(\frac{1}{\sin A} \right) + 2 \cos A \left(\frac{1}{\cos A} \right) \\
&= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2) \\
&= 7 + \tan^2 A + \cot^2 A \\
&= \text{R.H.S}
\end{aligned}$$

$$(ix) \quad (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\begin{aligned}
\text{L.H.S} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
&= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \\
&= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \\
&= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A} \\
&= \sin A \cos A
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S} &= \frac{1}{\tan A + \cot A} \\
&= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} \\
&= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A
\end{aligned}$$

Hence, L.H.S = R.H.S

$$(x) \quad \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} + 1} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A + \sin^2 A}{\sin^2 A}}$$

$$\begin{aligned}
 1 + \cot^2 A &= 1 + \frac{\cos^2 A}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A} \\
 &= \frac{1}{\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} \\
 &= \tan^2 A
 \end{aligned}$$

$$\begin{aligned}
 \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 &= \frac{1 + \tan^2 A - 2 \tan A}{1 + \cot^2 A - 2 \cot A} \\
 &= \frac{\sec^2 A - 2 \tan A}{\operatorname{cosec}^2 A - 2 \cot A} \\
 &= \frac{\frac{1}{\cos^2 A} - \frac{2 \sin A}{\cos A}}{\frac{1}{\sin^2 A} - \frac{2 \cos A}{\sin A}} = \frac{\frac{1 - 2 \sin A \cos A}{\cos^2 A}}{\frac{1 - 2 \sin A \cos A}{\sin^2 A}} \\
 &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
 \end{aligned}$$