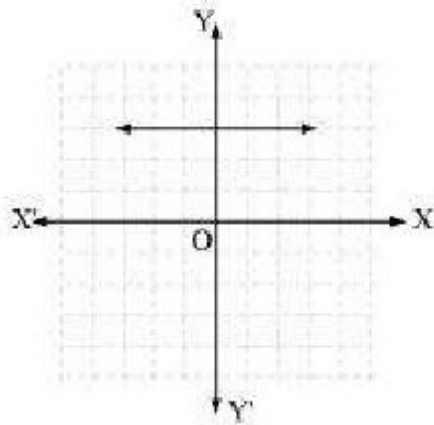


## Exercise 2.1

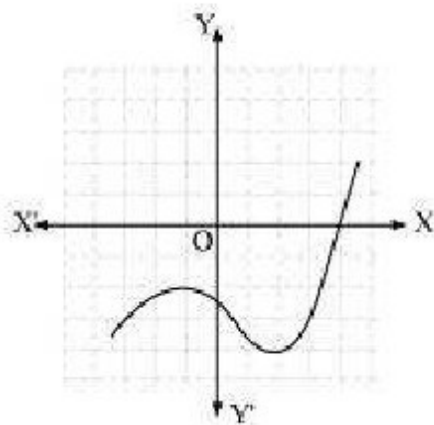
### Question 1:

The graphs of  $y = p(x)$  are given in following figure, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$ , in each case.

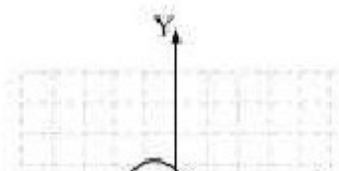
(i)

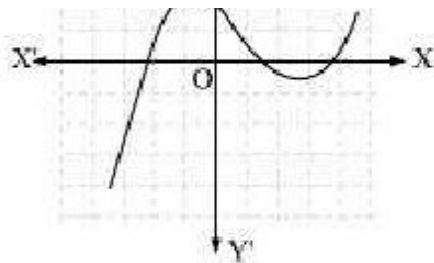


(ii)

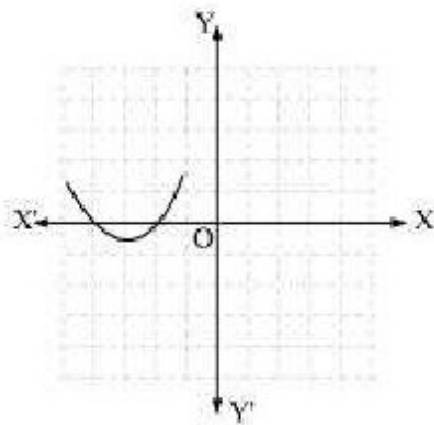


(iii)

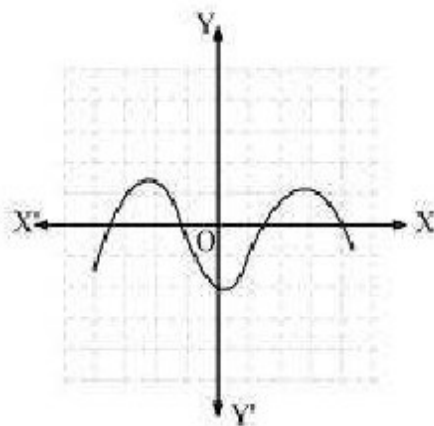




(iv)



(v)



(v)

## Exercise 2.2

### Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i)  $x^2 - 2x - 8$  (ii)  $4s^2 - 4s + 1$  (iii)  $6x^2 - 3 - 7x$

$$(iv) 4u^2 + 8u \quad (v) t^2 - 15 \quad (vi) 3x^2 - x - 4$$

**Answer:**

$$(i) \quad x^2 - 2x - 8 = (x - 4)(x + 2)$$

The value of  $x^2 - 2x - 8$  is zero when  $x - 4 = 0$  or  $x + 2 = 0$ , i.e., when  $x = 4$  or  $x = -2$

Therefore, the zeroes of  $x^2 - 2x - 8$  are 4 and -2.

$$\text{Sum of zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

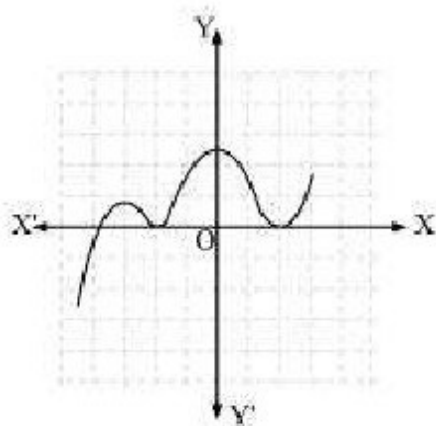
$$\text{Product of zeroes} = 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$(ii) \quad 4s^2 - 4s + 1 = (2s - 1)^2$$

The value of  $4s^2 - 4s + 1$  is zero when  $2s - 1 = 0$ , i.e.,  $s = \frac{1}{2}$

Therefore, the zeroes of  $4s^2 - 4s + 1$  are  $\frac{1}{2}$  and  $\frac{1}{2}$ .

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$



**Answer:**

(i) The number of zeroes is 0 as the graph does not cut the x-axis at

any point.

(ii) The number of zeroes is 1 as the graph intersects the  $x$ -axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the  $x$ -axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the  $x$ -axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the  $x$ -axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the  $x$ -axis at 3 points.

The value of  $t^2 - 15$  is zero when  $t - \sqrt{15} = 0$  or  $t + \sqrt{15} = 0$ , i.e., when  $t = \sqrt{15}$  or  $t = -\sqrt{15}$

Therefore, the zeroes of  $t^2 - 15$  are  $\sqrt{15}$  and  $-\sqrt{15}$ .

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

$$\text{Product of zeroes} = (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\begin{aligned} \text{(vi)} \quad 3x^2 - x - 4 \\ = (3x - 4)(x + 1) \end{aligned}$$

The value of  $3x^2 - x - 4$  is zero when  $3x - 4 = 0$  or  $x + 1 = 0$ , i.e.,

when  $x = \frac{4}{3}$  or  $x = -1$

Therefore, the zeroes of  $3x^2 - x - 4$  are  $\frac{4}{3}$  and  $-1$ .

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

### Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$$(i) \quad \frac{1}{4}, -1 \quad (ii) \quad \sqrt{2}, \frac{1}{3} \quad (iii) \quad 0, \sqrt{5}$$

$$(iv) \quad 1, 1 \quad (v) \quad -\frac{1}{4}, \frac{1}{4} \quad (vi) \quad 4, 1$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

$$(iii) \quad 6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x + 1)(2x - 3)$$

The value of  $6x^2 - 3 - 7x$  is zero when  $3x + 1 = 0$  or  $2x - 3 = 0$ , i.e.,

$$x = \frac{-1}{3} \text{ or } x = \frac{3}{2}$$

Therefore, the zeroes of  $6x^2 - 3 - 7x$  are  $\frac{-1}{3}$  and  $\frac{3}{2}$ .

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\begin{aligned} (iv) \quad 4u^2 + 8u &= 4u^2 + 8u + 0 \\ &= 4u(u + 2) \end{aligned}$$

The value of  $4u^2 + 8u$  is zero when  $4u = 0$  or  $u + 2 = 0$ , i.e.,  $u = 0$  or

$$u = -2$$

Therefore, the zeroes of  $4u^2 + 8u$  are 0 and  $-2$ .

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

$$\begin{aligned} \text{(v)} \quad t^2 - 15 \\ = t^2 - 0t - 15 \\ = (t - \sqrt{15})(t + \sqrt{15}) \end{aligned}$$

**Answer:**

$$\text{(i)} \quad \frac{1}{4}, -1$$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If  $a = 4$ , then  $b = -1$ ,  $c = -4$

Therefore, the quadratic polynomial is  $4x^2 - x - 4$ .

$$\text{(ii)} \quad \sqrt{2}, \frac{1}{3}$$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If  $a = 3$ , then  $b = -3\sqrt{2}$ ,  $c = 1$

Therefore, the quadratic polynomial is  $3x^2 - 3\sqrt{2}x + 1$ .

$$(iii) \quad 0, \sqrt{5}$$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If  $a = 1$ , then  $b = 0$ ,  $c = \sqrt{5}$

Therefore, the quadratic polynomial is  $x^2 + \sqrt{5}$ .

$$(iv) \quad 1, 1$$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If  $a = 1$ , then  $b = -1$ ,  $c = 1$

Therefore, the quadratic polynomial is  $x^2 - x + 1$ .

$$(v) \quad -\frac{1}{4}, \frac{1}{4}$$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .

$$\alpha + \beta = -\frac{1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

If  $a = 4$ , then  $b = 1$ ,  $c = 1$

Therefore, the quadratic polynomial is  $4x^2 + x + 1$ .

$$(vi) \quad 4, 1$$

Let the polynomial be  $ax^2 + bx + c$ .

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If  $a = 1$ , then  $b = -4$ ,  $c = 1$

Therefore, the quadratic polynomial is  $x^2 - 4x + 1$ .

### Exercise 2.3

#### Question 1:

Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following:

(i)  $p(x) = x^3 - 3x^2 + 5x - 3$ ,  $g(x) = x^2 - 2$

(ii)  $p(x) = x^4 - 3x^2 + 4x + 5$ ,  $g(x) = x^2 + 1 - x$

(iii)  $p(x) = x^4 - 5x + 6$ ,  $g(x) = 2 - x^2$

**Answer:**

(i)  $p(x) = x^3 - 3x^2 + 5x - 3$   
 $q(x) = x^2 - 2$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3 \quad \quad -2x} \phantom{-3} \\ - \phantom{x^3} \phantom{-2x} + \phantom{-3} \\ \phantom{-} -3x^2 + 7x - 3 \\ \phantom{-} -3x^2 \phantom{+7x} + 6 \\ \phantom{-} \phantom{-3x^2} + \phantom{+7x} - \phantom{+6} \\ \hline \phantom{-} \phantom{-3x^2} \phantom{+7x} - 9 \end{array}$$

Quotient =  $x - 3$

Remainder =  $7x - 9$

(ii)  $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$   
 $q(x) = x^2 + 1 - x = x^2 - x + 1$



$$\begin{array}{r}
 x^2 + x - 3 \\
 x^2 - x + 1 \overline{) x^4 + 0x^3 - 3x^2 + 4x + 5} \\
 \underline{x^4 - x^3 + x^2} \phantom{+ 5} \\
 - \phantom{x^4} + \phantom{x^3} - \phantom{x^2} \phantom{+ 5} \\
 \underline{x^3 - 4x^2 + 4x + 5} \\
 x^3 - x^2 + x \phantom{+ 5} \\
 \underline{- \phantom{x^3} + \phantom{x^2} -} \\
 -3x^2 + 3x + 5 \\
 -3x^2 + 3x - 3 \\
 \underline{\phantom{-3x^2} + \phantom{-3x} +} \\
 8
 \end{array}$$

Quotient =  $x^2 + x - 3$

Remainder = 8

(iii)  $p(x) = x^4 - 5x + 6 = x^4 + 0x^3 + 0x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r}
 -x^2 - 2 \\
 -x^2 + 2 \overline{) x^4 + 0x^3 + 0x^2 - 5x + 6} \\
 \underline{x^4 - 2x^2} \phantom{- 5x + 6} \\
 - \phantom{x^4} + \phantom{x^3} \phantom{+ 0x^2} - 5x + 6 \\
 \underline{2x^2 - 5x + 6} \\
 2x^2 \phantom{- 5x} - 4 \\
 \underline{- \phantom{2x^2} +} \\
 -5x + 10
 \end{array}$$

Quotient =  $-x^2 - 2$

Remainder =  $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i)  $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii)  $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii)  $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

**Answer:**

(i)  $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0t - 3$

$$\begin{array}{r}
 \phantom{t^2 + 0t - 3} \overline{2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 t^2 + 0t - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 + 0t^3 - 6t^2} \phantom{- 9t - 12} \\
 - \phantom{2t^4} - \phantom{0t^3} + \phantom{- 9t - 12} \\
 \underline{\phantom{2t^4} 3t^3 + 4t^2 - 9t - 12} \\
 \phantom{2t^4} 3t^3 + 0t^2 - 9t \phantom{- 12} \\
 \underline{\phantom{2t^4} 3t^3 + 0t^2 - 9t} \phantom{- 12} \\
 \phantom{2t^4} \phantom{3t^3} - \phantom{0t^2} + \phantom{- 9t} - 12 \\
 \phantom{2t^4} \phantom{3t^3} 4t^2 + 0t - 12 \\
 \phantom{2t^4} \phantom{3t^3} 4t^2 + 0t - 12 \\
 \underline{\phantom{2t^4} \phantom{3t^3} 4t^2 + 0t - 12} \\
 \phantom{2t^4} \phantom{3t^3} \phantom{4t^2} - \phantom{0t} - \phantom{12} + \phantom{0} \\
 \underline{\phantom{2t^4} \phantom{3t^3} \phantom{4t^2} - \phantom{0t} - \phantom{12} + \phantom{0}} \\
 \phantom{2t^4} \phantom{3t^3} \phantom{4t^2} \phantom{- 0t} \phantom{- 12} 0
 \end{array}$$

Since the remainder is 0,

Hence,  $t^2 - 3$  is a factor of  $2t^4 + 3t^3 - 2t^2 - 9t - 12$ .

(ii)  $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r}
 \phantom{x^2 + 3x + 1} \overline{3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \phantom{+ 2x + 2} \\
 - \phantom{3x^4} - \phantom{9x^3} - \phantom{3x^2} \phantom{+ 2x + 2}
 \end{array}$$

$$\begin{array}{r}
 -4x^3 - 10x^2 + 2x + 2 \\
 -4x^3 - 12x^2 - 4x \\
 + \quad + \quad + \\
 \hline
 2x^2 + 6x + 2 \\
 2x^2 + 6x + 2 \\
 \hline
 0
 \end{array}$$

Since the remainder is 0,

Hence,  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ .

(iii)  $x^3 - 3x + 1$ ,  $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 x^2 - 1 \\
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 x^5 - 3x^3 + x^2 \\
 - \quad + \quad - \\
 \hline
 -x^3 \quad + 3x + 1 \\
 -x^3 \quad + 3x - 1 \\
 + \quad - \quad + \\
 \hline
 2
 \end{array}$$

Since the remainder  $\neq 0$ ,

Hence,  $x^3 - 3x + 1$  is not a factor of  $x^5 - 4x^3 + x^2 + 3x + 1$ .

### Question 3:

Obtain all other zeroes of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ , if two of its zeroes are

$$\sqrt{\frac{5}{3}} \text{ and } -\sqrt{\frac{5}{3}}.$$

**Answer:**

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since the two zeroes are  $\sqrt{\frac{5}{3}}$  and  $-\sqrt{\frac{5}{3}}$ ,

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$  is a factor of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ .

Therefore, we divide the given polynomial by  $x^2 - \frac{5}{3}$ .

$$\begin{array}{r}
 x^2 + 0x - \frac{5}{3} \overline{) \begin{array}{l} 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ 3x^4 + 0x^3 - 5x^2 \\ \hline 6x^3 + 3x^2 - 10x - 5 \\ 6x^3 + 0x^2 - 10x \\ \hline 3x^2 + 0x - 5 \\ 3x^2 + 0x - 5 \\ \hline 0 \end{array}} \\
 \hline
 \end{array}$$

$$\begin{aligned}
 3x^4 + 6x^3 - 2x^2 - 10x - 5 &= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) \\
 &= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)
 \end{aligned}$$

We factorize  $x^2 + 2x + 1$

$$= (x+1)^2$$

Therefore, its zero is given by  $x + 1 = 0$

$$x = -1$$

As it has the term  $(x+1)^2$ , therefore, there will be 2 zeroes at  $x = -1$ .

Hence, the zeroes of the given polynomial are  $\sqrt{\frac{5}{3}}$ ,  $-\sqrt{\frac{5}{3}}$ ,  $-1$  and  $-1$ .

**Question 4:**

On dividing  $x^3 - 3x^2 + x + 2$  by a polynomial  $g(x)$ , the quotient and remainder were  $x - 2$  and  $-2x + 4$ , respectively. Find  $g(x)$ .

**Answer:**

$$p(x) = x^3 - 3x^2 + x + 2 \quad (\text{Dividend})$$

$$g(x) = ? \quad (\text{Divisor})$$

$$\text{Quotient} = (x - 2)$$

$$\text{Remainder} = (-2x + 4)$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

$g(x)$  is the quotient when we divide  $(x^3 - 3x^2 + 3x - 2)$  by  $(x - 2)$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2 \phantom{+ 3x} - 2} \phantom{+ 3x} \\
 x^2 - 3x \phantom{+ 3x} - 2 \\
 \underline{x^2 - 2x \phantom{+ 3x} - 2} \\
 x \phantom{+ 3x} \phantom{- 2}
 \end{array}$$

$$\begin{array}{r}
 x^3 - 2x^2 \\
 \underline{- \quad +} \\
 -x^2 + 3x - 2 \\
 \underline{-x^2 + 2x} \\
 + \quad - \\
 \underline{\hspace{1.5cm} x - 2} \\
 \hspace{1.5cm} x - 2 \\
 \underline{- \quad +} \\
 \hspace{1.5cm} 0
 \end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

### Question 5:

Give examples of polynomial  $p(x)$ ,  $g(x)$ ,  $q(x)$  and  $r(x)$ , which satisfy the division algorithm and

(i)  $\deg p(x) = \deg q(x)$

(ii)  $\deg q(x) = \deg r(x)$

(iii)  $\deg r(x) = 0$

Answer:

According to the division algorithm, if  $p(x)$  and  $g(x)$  are two polynomials with

$g(x) \neq 0$ , then we can find polynomials  $q(x)$  and  $r(x)$  such that

$$p(x) = g(x) \times q(x) + r(x),$$

where  $r(x) = 0$  or degree of  $r(x) < \text{degree of } g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i)  $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant ( i.e., when any polynomial is divided by a constant).

Let us assume the division of  $6x^2 + 2x + 2$  by 2.

Here  $p(x) = 6x^2 + 2x + 2$

Here,  $p(x) = 6x^2 + 2x + 2$

$$g(x) = 2$$

$$q(x) = 3x^2 + x + 1 \text{ and } r(x) = 0$$

Degree of  $p(x)$  and  $q(x)$  is the same i.e., 2.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$$

$$= 6x^2 + 2x + 2$$

Thus, the division algorithm is satisfied.

$$(ii) \deg q(x) = \deg r(x)$$

Let us assume the division of  $x^3 + x$  by  $x^2$ ,

$$\text{Here, } p(x) = x^3 + x$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = x$$

Clearly, the degree of  $q(x)$  and  $r(x)$  is the same i.e., 1.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + x = (x^2) \times x + x$$

$$x^3 + x = x^3 + x$$

Thus, the division algorithm is satisfied.

$$(iii) \deg r(x) = 0$$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of  $x^3 + 1$  by  $x^2$ .

$$\text{Here, } p(x) = x^3 + 1$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = 1$$

Clearly, the degree of  $r(x)$  is 0.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + 1 = (x^2) \times x + 1$$

$$x^3 + 1 = x^3 + 1$$

Thus, the division algorithm is satisfied.

## Exercise 2.4

### Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

$$(i) \quad 2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$$

$$(ii) \quad x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$$

**Answer:**

$$(i) \quad p(x) = 2x^3 + x^2 - 5x + 2.$$

Zeroes for this polynomial are  $\frac{1}{2}, 1, -2$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 \\ &= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 5 \times 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2 \\ &= -16 + 4 + 10 + 2 = 0 \end{aligned}$$

Therefore,  $\frac{1}{2}, 1$ , and  $-2$  are the zeroes of the given polynomial.



Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we obtain  $a = 2$ ,  
 $b = 1$ ,  $c = -5$ ,  $d = 2$

We can take  $\alpha = \frac{1}{2}$ ,  $\beta = 1$ ,  $\gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

$$(ii) \quad p(x) = x^3 - 4x^2 + 5x - 2$$

Zeroes for this polynomial are 2, 1, 1.

$$\begin{aligned} p(2) &= 2^3 - 4(2^2) + 5(2) - 2 \\ &= 8 - 16 + 10 - 2 = 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 1^3 - 4(1)^2 + 5(1) - 2 \\ &= 1 - 4 + 5 - 2 = 0 \end{aligned}$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we obtain  $a = 1$ ,  
 $b = -4$ ,  $c = 5$ ,  $d = -2$ .

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time =  $(2)(1) + (1)(1) + (2)(1)$

$$= 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$$

$$\text{Multiplication of zeroes} = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$$

Hence, the relationship between the zeroes and the coefficients is verified.

### Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer:

Let the polynomial be  $ax^3 + bx^2 + cx + d$  and the zeroes be  $\alpha, \beta$ , and  $\gamma$ .

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If  $a = 1$ , then  $b = -2$ ,  $c = -7$ ,  $d = 14$

Hence, the polynomial is  $x^3 - 2x^2 - 7x + 14$ .

### Question 3:

If the zeroes of polynomial  $x^3 - 3x^2 + x + 1$  are  $a - b, a, a + b$ , find  $a$  and  $b$ .

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are  $a - b, a, a + b$

Comparing the given polynomial with  $px^3 + qx^2 + rx + t$ , we obtain

$$p = 1, q = -3, r = 1, t = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are  $1-b, 1, 1+b$ .

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$

$$\frac{-t}{p} = 1-b^2$$

$$\frac{-1}{1} = 1-b^2$$

$$1-b^2 = -1$$

$$1+1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence,  $a = 1$  and  $b = \sqrt{2}$  or  $-\sqrt{2}$ .

**Question 4:**

**]It two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find other zeroes.**

**Answer:**

Given that  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  are zeroes of the given polynomial.

$$\begin{aligned} \text{Therefore, } (x-2-\sqrt{3})(x-2+\sqrt{3}) &= x^2 + 4 - 4x - 3 \\ &= x^2 - 4x + 1 \text{ is a factor of the given polynomial} \end{aligned}$$

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing  $x^4 - 6x^3 - 26x^2 + 138x - 35$  by  $x^2 - 4x + 1$ .

$$\begin{array}{r} x^2 - 2x - 35 \\ x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\ \underline{x^4 - 4x^3 + \phantom{-26}x^2} \phantom{+ 138x - 35} \end{array}$$

$$\begin{array}{r}
 \begin{array}{r}
 - \quad + \quad - \\
 -2x^3 - 27x^2 + 138x - 35 \\
 -2x^3 + 8x^2 - 2x \\
 + \quad - \quad + \\
 \hline
 -35x^2 + 140x - 35 \\
 -35x^2 + 140x - 35 \\
 + \quad - \quad + \\
 \hline
 0
 \end{array}
 \end{array}$$

Clearly,  $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that  $(x^2 - 2x - 35)$  is also a factor of the given polynomial.

And  $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when  $x - 7 = 0$  or  $x + 5 = 0$

Or  $x = 7$  or  $-5$

Hence, 7 and  $-5$  are also zeroes of this polynomial.

### Question 5:

If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .

Answer:

By division algorithm,

Dividend = Divisor  $\times$  Quotient + Remainder

Dividend – Remainder = Divisor  $\times$  Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$  will be perfectly divisible by  $x^2 - 2x + k$ .

Let us divide  $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$  by  $x^2 - 2x + k$

$$\begin{array}{r}
 \phantom{x^4 - 6x^3 + 16x^2 - 26x + 10 - a} x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 26x + 10 - a} \\
 \underline{x^4 - 2x^3 + kx^2} \phantom{- 26x + 10 - a} \\
 -4x^3 + (16 - k)x^2 - 26x \phantom{+ 10 - a} \\
 \underline{-4x^3 + 8x^2 - 4kx} \phantom{+ 10 - a} \\
 (8 - k)x^2 - (26 - 4k)x + 10 - a \\
 \underline{(8 - k)x^2 - (16 - 2k)x + (8k - k^2)} \\
 (-10 + 2k)x + (10 - a - 8k + k^2)
 \end{array}$$

It can be observed that  $(-10 + 2k)x + (10 - a - 8k + k^2)$  will be 0.

Therefore,  $(-10 + 2k) = 0$  and  $(10 - a - 8k + k^2) = 0$

For  $(-10 + 2k) = 0$ ,

$$2k = 10$$

And thus,  $k = 5$

For  $(10 - a - 8k + k^2) = 0$

$$10 - a - 8 \times 5 + 25 = 0$$

$$10 - a - 40 + 25 = 0$$

$$-5 - a = 0$$

Therefore,  $a = -5$

Hence,  $k = 5$  and  $a = -5$

