

Exercise 14.1

Question 1:

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

$$\text{Class mark } (x_i) = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class mark (x_i) =

2

x_i and $f_i x_i$ can be calculated as follows.

Number of plants	Number of houses (f_i)	x_i	$f_i x_i$
0 – 2	1	1	$1 \times 1 = 1$
2 – 4	2	3	$2 \times 3 = 6$
4 – 6	1	5	$1 \times 5 = 5$
6 – 8	5	7	$5 \times 7 = 35$
8 – 10	6	9	$6 \times 9 = 54$
10 – 12	2	11	$2 \times 11 = 22$
12 – 14	3	13	$3 \times 13 = 39$

Total	20		162
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From the table, it can be observed that

$$\sum f_i = 20$$

$$\sum f_i x_i = 162$$

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{162}{20} = 8.1$$

Therefore, mean number of plants per house is 8.1.

Here, direct method has been used as the values of class marks (x_i) and f_i are small.

Question 2:

Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer:

To find the class mark for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 20

Taking 150 as assured mean (a), d_i , u_i and $f_i u_i$ can be calculated as follows.

Daily wages (in Rs)	Number of workers (f_i)	x_i	$d_i = x_i - 150$	$u_i = \frac{d_i}{20}$	$f_i u_i$
100 – 120	12	110	– 40	– 2	– 24
120 – 140	14	130	– 20	– 1	– 14
140 – 160	8	150	0	0	0
160 – 180	6	170	20	1	6

180 – 200	10	190	40	2	20
Total	50				- 12

From the table, it can be observed that

$$\sum f_i = 50$$

$$\sum f_i u_i = -12$$

$$\begin{aligned}\text{Mean } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h \\ &= 150 + \left(\frac{-12}{50} \right) 20 \\ &= 150 - \frac{24}{5} \\ &= 150 - 4.8 \\ &= 145.2\end{aligned}$$

Therefore, the mean daily wage of the workers of the factory is Rs 145.20.

Question 3:

The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency f .

Daily pocket allowance (in Rs)	11 – 13	13 – 15	15 –17	17 – 19	19 – 21	21 – 23	23 – 25
Number of workers	7	6	9	13	f	5	4

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Given that, mean pocket allowance, $\bar{x} = \text{Rs } 18$

Taking 18 as assured mean (a), d_i and $f_i d_i$ are calculated as follows.

Daily pocket allowance (in Rs)	Number of children f	Class mark x_i	$d_i = x_i - 18$	$f_i d_i$
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(MRS)	u_i			
11 – 13	7	12	– 6	– 42
13 – 15	6	14	– 4	– 24
15 – 17	9	16	– 2	– 18
17 – 19	13	18	0	0
19 – 21	f	20	2	$2f$
21 – 23	5	22	4	20
23 – 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$2f - 40$

From the table, we obtain

$$\sum f_i = 44 + f$$

$$\sum f_i u_i = 2f - 40$$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f} \right)$$

$$0 = \left(\frac{2f - 40}{44 + f} \right)$$

$$2f - 40 = 0$$

$$2f = 40$$

$$f = 20$$

Hence, the missing frequency, f , is 20.



Question 4:

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart

beats per minute. (Use the assumed mean method to find the mean.)

beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	65 – 68	68 – 71	71 – 74	74 – 77	77 – 80	80 – 83	83 – 86
Number of women	2	4	3	8	7	4	2

Answer:

To find the class mark of each interval (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size, h , of this data = 3

Taking 75.5 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Number of heart beats per minute	Number of women f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
65 – 68	2	66.5	– 9	– 3	– 6
68 – 71	4	69.5	– 6	– 2	– 8
71 – 74	3	72.5	– 3	– 1	– 3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	30				4

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = 4$$

$$\begin{aligned}\text{Mean } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 75.5 + \left(\frac{4}{30} \right) \times 3 \\ &= 75.5 + 0.4 = 75.9\end{aligned}$$

Therefore, mean hear beats per minute for these women are 75.9 beats per minute.

Question 5:

In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Answer:

Number of mangoes	Number of boxes f_i
50 – 52	15
53 – 55	110
56 – 58	135
59 – 61	115
62 – 64	25

It can be observed that class intervals are not continuous. There is a gap of 1

between two class intervals. Therefore, $\frac{1}{2}$ has to be added to the upper class limit

and $\frac{1}{2}$ has to be subtracted from the lower class limit of each interval.

Class mark (x_i) can be obtained by using the following relation

Class mark (x_i) can be obtained by using the following relation.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 3

Taking 57 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Class interval	f_i	x_i	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	$f_i u_i$
49.5 – 52.5	15	51	– 6	– 2	– 30
52.5 – 55.5	110	54	– 3	– 1	– 110
55.5 – 58.5	135	57	0	0	0
58.5 – 61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
Total	400				25

It can be observed that

$$\sum f_i = 400$$

$$\sum f_i u_i = 25$$

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\
 &= 57 + \left(\frac{25}{400} \right) \times 3 \\
 &= 57 + \frac{3}{16} = 57 + 0.1875 \\
 &= 57.1875 \\
 &\approx 57.19
 \end{aligned}$$

Mean number of mangoes kept in a packing box is 57.19.

Step deviation method is used here as the values of f_i , d_i are big and also, there is a common multiple between all d_i .

Question 6:

The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size = 50

Taking 225 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Daily expenditure (in Rs)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	$f_i u_i$
100 – 150	4	125	– 100	– 2	– 8
150 – 200	5	175	– 50	– 1	– 5
200 – 250	12	225	0	0	0
250 – 300	2	275	50	1	2
300 – 350	2	325	100	2	4
Total	25				– 7

From the table, we obtain

$$\sum f_i = 25$$

$$\sum f_i u_i = -7$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 225 + \left(\frac{-7}{25} \right) \times (50)\end{aligned}$$

$$= 225 - 14$$

$$= 211$$

Therefore, mean daily expenditure on food is Rs 211.

Question 7:

To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

concentration of SO_2 (in ppm)	Frequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

Find the mean concentration of SO_2 in the air.

Answer:

To find the class marks for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size of this data = 0.04

Taking 0.14 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated as follows.

Concentration of SO_2 (in ppm)	Frequency f_i	Class mark x_i	$d_i = x_i - 0.14$	$u_i = \frac{d_i}{0.04}$	$f_i u_i$
0.00 – 0.04	4	0.02	- 0.12	- 3	- 12
0.04 – 0.08	9	0.06	- 0.08	- 2	- 18

					18
0.08 – 0.12	9	0.10	– 0.04	– 1	– 9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	30				– 31

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_i u_i = -31$$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 0.14 + \left(\frac{-31}{30} \right) (0.04)$$

$$= 0.14 - 0.04133$$

$$= 0.09867$$

$$\approx 0.099 \text{ ppm}$$

Therefore, mean concentration of SO₂ in the air is 0.099 ppm.

Question 8:

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 – 6	6 – 10	10 – 14	14 – 20	20 – 28	28 – 38	38 – 40
Number of students	11	10	7	4	4	3	1

Answer:

To find the class mark of each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 17 as assumed mean (a), d_i and $f_i d_i$ are calculated as follows.

Number of days	Number of students f_i	x_i	$d_i = x_i - 17$	$f_i d_i$
0 – 6	11	3	– 14	– 154
6 – 10	10	8	– 9	– 90
10 – 14	7	12	– 5	– 35

14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
Total	40			– 181

From the table, we obtain

$$\sum f_i = 40$$

$$\sum f_i d_i = -181$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i d_i}{\sum f_i} \right) \\ &= 17 + \left(\frac{-181}{40} \right) \\ &= 17 - 4.525 \\ &= 12.475 \\ &\approx 12.48 \end{aligned}$$

Therefore, the mean number of days is 12.48 days for which a student was absent.



Question 9:

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Answer:

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) for this data = 10

Taking 70 as assumed mean (a), d_i , u_i and $f_i u_i$ are calculated as follows.

Literacy rate (in %)	Number of cities f_i	x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	$f_i u_i$
45 – 55	3	50	– 20	– 2	– 6
55 – 65	10	60	– 10	– 1	– 10
65 – 75	11	70	0	0	0
75 – 85	8	80	10	1	8
85 – 95	3	90	20	2	6
Total	35				– 2

From the table, we obtain

$$\sum f_i = 35$$

$$\sum f_i u_i = -2$$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 70 + \left(\frac{-2}{35} \right) \times (10) \\ &= 70 - \frac{20}{35} \\ &= 70 - \frac{4}{7} \end{aligned}$$

$$= 70 - 0.57$$

$$= 69.43$$

Therefore, mean literacy rate is 69.43%.

Exercise 14.2

Question 1:

The following table shows the ages of the patients admitted in a hospital during a year:

age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Answer:

To find the class marks (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 30 as assumed mean (a), d_i and $f_i d_i$ are calculated as follows.

Age (in years)	Number of patients f_i	Class mark x_i	$d_i = x_i - 30$	$f_i d_i$
5 – 15	6	10	- 20	- 120
15 – 25	11	20	- 10	- 110
25 – 35	21	30	0	0
35 – 45	23	40	10	230
45 – 55	14	50	20	280
55 – 65	5	60	30	150
Total	80			430

From the table, we obtain

$$\sum f_i = 80$$

$$\sum f_i d_i = 430$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \frac{\sum f_i d_i}{\sum f_i} \\ &= 30 + \left(\frac{430}{80} \right) \\ &= 30 + 5.375 \\ &= 35.375 \\ &\approx 35.38\end{aligned}$$

Mean of this data is 35.38. It represents that on an average, the age of a patient admitted to hospital was 35.38 years.

It can be observed that the maximum class frequency is 23 belonging to class interval 35 – 45.

Modal class = 35 – 45

Lower limit (l) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal class = 21

Frequency (f_2) of class succeeding the modal class = 14

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 35 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10 \\ &= 35 + \left[\frac{2}{46 - 35} \right] \times 10 \\ &= 35 + \frac{20}{11} \\ &= 35 + 1.81 \\ &= 36.8\end{aligned}$$

Mode is 36.8. It represents that the age of maximum number of patients admitted in hospital was 36.8 years.



Question 2:

Question 2:

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Answer:

From the data given above, it can be observed that the maximum class frequency is 61, belonging to class interval 60 – 80.

Therefore, modal class = 60 – 80

Lower class limit (l) of modal class = 60

Frequency (f_1) of modal class = 61

Frequency (f_0) of class preceding the modal class = 52

Frequency (f_2) of class succeeding the modal class = 38

Class size (h) = 20

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 60 + \left(\frac{61 - 52}{2(61) - 52 - 38} \right) (20)\end{aligned}$$

$$= 60 + \left(\frac{9}{122 - 90} \right) (20)$$

$$= 60 + \left(\frac{9 \times 20}{32} \right)$$

$$= 60 + \frac{90}{16} = 60 + 5.625$$

$$= 65.625$$

Therefore, modal lifetime of electrical components is 65.625 hours.

Question 3:

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	Number of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Answer:

It can be observed from the given data that the maximum class frequency is 40, belonging to 1500 – 2000 intervals.

Therefore, modal class = 1500 – 2000

Lower limit (l) of modal class = 1500

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

Class size (h) = 500

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33} \right) \times 500 \\
 &= 1500 + \left(\frac{16}{20} \right) \times 500
 \end{aligned}$$

$$= 1500 + \left(\frac{80 - 57}{23} \right) \times 500$$

$$= 1500 + \frac{8000}{23}$$

$$= 1500 + 347.826$$

$$= 1847.826 = 1847.83$$

Therefore, modal monthly expenditure was Rs 1847.83.

To find the class mark, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of the given data = 500

Taking 2750 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated as follows.

Expenditure (in Rs)	Number of families f_i	x_i	$d_i = x_i - 2750$	$u_i = \frac{d_i}{500}$	$f_i u_i$
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1000 – 1500	24	1250	– 1500	– 3	– 72
1500 – 2000	40	1750	– 1000	– 2	– 80
2000 – 2500	33	2250	– 500	– 1	– 33
2500 – 3000	28	2750	0	0	0
3000 – 3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44
4000 – 4500	16	4250	1500	3	48
4500 – 5000	7	4750	2000	4	28
Total	200				– 35

From the table, we obtain

$$\sum f_i = 200$$

$$\sum f_i u_i = -35$$

$$\bar{x}(\text{mean}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$\bar{x} = 2750 + \left(\frac{-35}{200} \right) \times 500$$

$$= 2750 - 87.5$$

$$= 2662.5$$

Therefore, mean monthly expenditure was Rs 2662.50.

Question 4:

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states/U.T
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

Answer:

It can be observed from the given data that the maximum class frequency is 10 belonging to class interval 30 – 35.

Therefore, modal class = 30 – 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 30 + \left(\frac{10 - 9}{2(10) - 9 - 3} \right) \times (5) \\ &= 30 + \left(\frac{1}{20 - 12} \right) 5 \\ &= 30 + \frac{5}{8} = 30.625\end{aligned}$$

Mode = 30.6

It represents that most of the states/U.T have a teacher-student ratio as 30.6.

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 32.5 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated as follows.

Number of students per teacher	Number of states/U.T (f_i)	x_i	$d_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	$f_i u_i$
15 – 20	3	17.5	– 15	– 3	– 9
20 – 25	8	22.5	– 10	– 2	– 16
25 – 30	9	27.5	– 5	– 1	– 9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0

45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
Total	35				– 23

$$\begin{aligned}
 \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h \\
 &= 32.5 + \left(\frac{-23}{35} \right) \times 5 \\
 &= 32.5 - \frac{23}{7} = 32.5 - 3.28 \\
 &= 29.22
 \end{aligned}$$

Therefore, mean of the data is 29.2.

It represents that on an average, teacher–student ratio was 29.2.

Question 5:

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7

7000 – 8000	6
8000 – 9000	2

8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

Find the mode of the data.

Answer:

From the given data, it can be observed that the maximum class frequency is 18, belonging to class interval 4000 – 5000.

Therefore, modal class = 4000 – 5000

Lower limit (l) of modal class = 4000

Frequency (f_1) of modal class = 18

Frequency (f_0) of class preceding modal class = 4

Frequency (f_2) of class succeeding modal class = 9

Class size (h) = 1000

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9} \right) \times 1000 \\
 &= 4000 + \left(\frac{14000}{23} \right) \\
 &= 4000 + 608.695 \\
 &= 4608.695
 \end{aligned}$$

Therefore, mode of the given data is 4608.7 runs

Question 6:

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

Answer:

From the given data, it can be observed that the maximum class frequency is 20, belonging to 40 – 50 class intervals.

Therefore, modal class = 40 – 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left[\frac{20 - 12}{2(20) - 12 - 11} \right] \times 10 \\ &= 40 + \left(\frac{80}{40 - 23} \right) \\ &= 40 + \frac{80}{17} \\ &= 40 + 4.7 \\ &= 44.7\end{aligned}$$

Therefore, mode of this data is 44.7 cars.

Exercise 14.3

Question 1:

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 – 85	4
85 – 105	5
105 – 125	13

125 – 145	20
145 – 165	14
165 – 185	8
185 – 205	4

Answer:

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 135 as assumed mean (a), d_i , u_i , $f_i u_i$ are calculated according to step deviation method as follows.

Monthly consumption (in units)	Number of consumers (f_i)	x_i class mark	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 – 85	4	75	– 60	– 3	–

					12
85 – 105	5	95	– 40	– 2	– 10
105 – 125	13	115	– 20	– 1	– 13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
Total	68				7

From the table, we obtain

$$\sum f_i u_i = 7$$

$$\sum f_i u_i = 140$$

$$\sum f_i = 68$$

$$\text{Class size } (h) = 20$$

$$\begin{aligned}\text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 135 + \frac{7}{68} \times 20 \\ &= 135 + \frac{140}{68} \\ &= 137.058\end{aligned}$$

From the table, it can be observed that the maximum class frequency is 20, belonging to class interval 125 – 145.

Modal class = 125 – 145

Lower limit (l) of modal class = 125

Class size (h) = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

$$\begin{aligned}\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 125 + \left[\frac{20 - 13}{2(20) - 13 - 14} \right] \times 20 \\ &= 125 + \frac{7}{13} \times 20 \\ &= 125 + \frac{140}{13} = 135.76\end{aligned}$$

To find the median of the given data, cumulative frequency is calculated as follows.

Monthly consumption (in units)	Number of consumers	Cumulative frequency
65 – 85	4	4
85 – 105	5	4 + 5 = 9
105 – 125	13	9 + 13 = 22

125 – 145	20	22 + 20 = 42
145 – 165	14	42 + 14 = 56
165 – 185	8	56 + 8 = 64
185 – 205	4	64 + 4 = 68

From the table, we obtain

$$n = 68$$

Cumulative frequency (cf) just greater than $\frac{n}{2}$ (i.e., $\frac{68}{2} = 34$) is 42, belonging to interval 125 – 145.

Therefore, median class = 125 – 145

Lower limit (l) of median class = 125

Class size (h) = 20

Frequency (f) of median class = 20

Cumulative frequency (cf) of class preceding median class = 22

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\
 &= 125 + \left(\frac{34 - 22}{20} \right) \times 20 \\
 &= 125 + 12 \\
 &= 137
 \end{aligned}$$

Therefore, median, mode, mean of the given data is 137, 135.76, and 137.05 respectively.

The three measures are approximately the same in this case.

Question 2:

If the median of the distribution is given below is 28.5, find the values of x and y .

Class interval	Frequency
0 – 10	5
10 – 20	...

10 – 20	x
20 – 30	20

30 – 40	15
40 – 50	y
50 – 60	5
Total	60

Answer:

The cumulative frequency for the given data is calculated as follows.

Class interval	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total (n)	60	

From the table, it can be observed that $n = 60$

$$45 + x + y = 60$$

$$x + y = 15 \quad (1)$$

Median of the data is given as 28.5 which lies in interval 20 – 30.

Therefore, median class = 20 – 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20} \right] \times 10$$

$$8.5 = \left(\frac{25 - x}{2} \right)$$

$$17 = 25 - x$$

$$x = 8$$

From equation (1),

$$8 + y = 15$$

$$y = 7$$

Hence, the values of x and y are 8 and 7 respectively

Question 3:

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45

Below 40	78
Below 45	89
Below 50	92

Below 55	98
Below 60	100

Answer:

Here, class width is not the same. There is no requirement of adjusting the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	Number of policy holders (f_i)	Cumulative frequency (cf)
18 – 20	2	2
20 – 25	$6 - 2 = 4$	6
25 – 30	$24 - 6 = 18$	24
30 – 35	$45 - 24 = 21$	45
35 – 40	$78 - 45 = 33$	78
40 – 45	$89 - 78 = 11$	89
45 – 50	$92 - 89 = 3$	92
50 – 55	$98 - 92 = 6$	98

55 – 60	$100 - 98 = 2$	100
Total (n)		

From the table, it can be observed that $n = 100$.

Cumulative frequency (cf) just greater than $\frac{n}{2}$ (i.e., $\frac{100}{2} = 50$) is 78, belonging to interval 35 – 40.

Therefore, median class = 35 – 40

Lower limit (l) of median class = 35

∴ Median class = 35 – 40

Class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 35 + \left(\frac{50 - 45}{33} \right) \times 5 \\ &= 35 + \frac{25}{33} \\ &= 35.76\end{aligned}$$

Therefore, median age is 35.76 years.

Question 4:

The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves f_i
118 – 126	3

127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

(**Hint:** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5... 171.5 – 180.5)

Answer:

The given data does not have continuous class intervals. It can be observed that the

difference between two class intervals is 1. Therefore, $\frac{1}{2} = 0.5$ has to be added and subtracted to upper class limits and lower class limits respectively.

Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	Number or leaves f_i	Cumulative frequency
117.5 – 126.5	3	3
126.5 – 135.5	5	$3 + 5 = 8$
135.5 – 144.5	9	$8 + 9 = 17$
144.5 – 153.5	12	$17 + 12 = 29$

153.5 – 162.5	5	$29 + 5 = 34$
162.5 – 171.5	4	$34 + 4 = 38$
171.5 – 180.5	2	$38 + 2 = 40$

From the table, it can be observed that the cumulative frequency just greater than

$\frac{n}{2}$ (i.e., $\frac{40}{2} = 20$) is 29, belonging to class interval 144.5 – 153.5.

Median class = 144.5 – 153.5

Lower limit (l) of median class = 144.5

Class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 144.5 + \left(\frac{20 - 17}{12} \right) \times 9$$

$$= 144.5 + \frac{7}{4} = 146.75$$

Therefore, median length of leaves is 146.75 mm.

Question 5:

Find the following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500 – 2000	14

2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

Answer:

The cumulative frequencies with their respective class intervals are as follows.

Life time	Number of lamps (f_i)	Cumulative frequency
1500 – 2000	14	14
2000 – 2500	56	$14 + 56 = 70$
2500 – 3000	60	$70 + 60 = 130$
3000 – 3500	86	$130 + 86 = 216$
3500 – 4000	74	$216 + 74 = 290$
4000 – 4500	62	$290 + 62 = 352$

4500 – 5000	48	$352 + 48 = 400$
Total (n)	400	

It can be observed that the cumulative frequency just greater than

$\frac{n}{2}$ (i.e., $\frac{400}{2} = 200$) is 216, belonging to class interval 3000 – 3500.

Median class = 3000 – 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 3000 + \left(\frac{200 - 130}{86} \right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3406.976$$

Therefore, median life time of lamps is 3406.98 hours.

Question 6:

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	6	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Answer:

The cumulative frequencies with their respective class intervals are as follows.

Number of letters	Frequency (f_i)	Cumulative frequency
1 – 4	6	6
4 – 7	30	$30 + 6 = 36$
7 – 10	40	$36 + 40 = 76$
10 – 13	16	$76 + 16 = 92$
13 – 16	4	$92 + 4 = 96$
16 – 19	4	$96 + 4 = 100$
Total (n)	100	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{100}{2} = 50$) is 76, belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

$$= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

Median

$$= 7 + \left(\frac{50 - 36}{40} \right) \times 3$$

$$= 7 + \frac{14 \times 3}{40}$$

$$= 8.05$$

$$= 8.05$$

To find the class marks of the given class intervals, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 11.5 as assumed mean (a), d_i , u_i , and $f_i u_i$ are calculated according to step deviation method as follows.

Number of letters	Number of surnames f_i	x_i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
1 – 4	6	2.5	– 9	– 3	– 18
4 – 7	30	5.5	– 6	– 2	– 60
7 – 10	40	8.5	– 3	– 1	– 40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	100				– 106

From the table, we obtain

$$\sum f_i u_i = -106$$

$$\sum f_i = 100$$

$$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) h$$

Mean,

$$= 11.5 + \left(\frac{-106}{100} \right) \times 3$$

$$= 11.5 - 3.18 = 8.32$$

The data in the given table can be written as

Number of letters	Frequency (f_i)
-------------------	---------------------

1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4
Total (n)	100

From the table, it can be observed that the maximum class frequency is 40 belonging to class interval 7 – 10.

Modal class = 7 – 10

Lower limit (l) of modal class = 7

Class size (h) = 3

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding the modal class = 30

Frequency (f_2) of class succeeding the modal class = 16

$$\begin{aligned}
 \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
 &= 7 + \left[\frac{40 - 30}{2(40) - 30 - 16} \right] \times 3 \\
 &= 7 + \frac{10}{34} \times 3 \\
 &= 7 + \frac{30}{34} = 7.88
 \end{aligned}$$

Therefore, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.88.

Question 7:

The distribution below gives the weights of 30 students of a class. Find the median weight of the students

weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

Answer:

The cumulative frequencies with their respective class intervals are as follows.

Weight (in kg)	Frequency (f_i)	Cumulative frequency
40 – 45	2	2
45 – 50	3	$2 + 3 = 5$
50 – 55	8	$5 + 8 = 13$
55 – 60	6	$13 + 6 = 19$

60 – 65	6	$19 + 6 = 25$
65 – 70	3	$25 + 3 = 28$
70 – 75	2	$28 + 2 = 30$
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{30}{2} = 15$) is 19, belonging to class interval 55 – 60.

Median class = 55 – 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) of median class = 13

Class size (h) = 5

$$= l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$$

Median $\left(\frac{N}{2} \right)$

$$= 55 + \left(\frac{15-13}{6} \right) \times 5$$

$$= 55 + \frac{10}{6}$$

$$= 56.67$$

Therefore, median weight is 56.67 kg.

Exercise 14.4

Question 1:

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

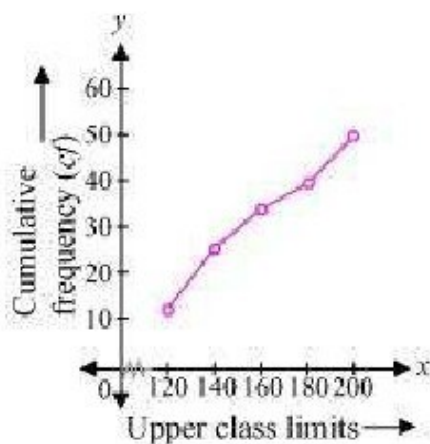
Answer:

The frequency distribution table of less than type is as follows.

Daily income (in Rs) (upper class limits)	Cumulative frequency
Less than 120	12
Less than 140	$12 + 14 = 26$

Less than 160	$26 + 8 = 34$
Less than 180	$34 + 6 = 40$
Less than 200	$40 + 10 = 50$

Taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis, its ogive can be drawn as follows.



Question 2:

During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

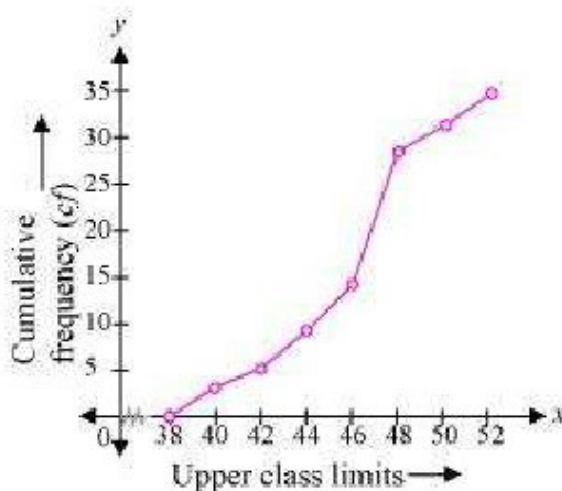
Draw a less than type ogive for the given data. Hence obtain the median weight from the graph verify the result by using the formula.

Answer:

The given cumulative frequency distributions of less than type are

Weight (in kg) upper class limits	Number of students (cumulative frequency)
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

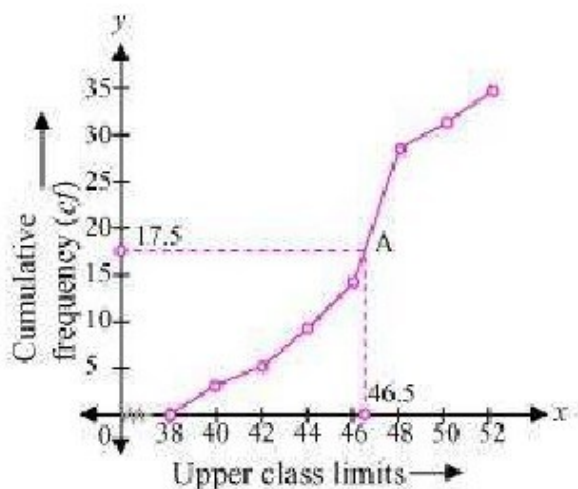
Taking upper class limits on x-axis and their respective cumulative frequencies on y-axis, its ogive can be drawn as follows.



Here, $n = 35$

So, $\frac{n}{2} = 17.5$

Mark the point A whose ordinate is 17.5 and its x-coordinate is 46.5. Therefore, median of this data is 46.5.



It can be observed that the difference between two consecutive upper class limits is 2. The class marks with their respective frequencies are obtained as below.

Weight (in kg)	Frequency (f)	Cumulative frequency
Less than 38	0	0
38 – 40	$3 - 0 = 3$	3
40 – 42	$5 - 3 = 2$	5
42 – 44	$9 - 5 = 4$	9
44 – 46	$14 - 9 = 5$	14
46 – 48	$28 - 14 = 14$	28
48 – 50	$32 - 28 = 4$	32

50 – 52	$35 - 32 = 3$	35
Total (n)	35	

The cumulative frequency just greater than $\frac{n}{2}$ (i.e., $\frac{35}{2} = 17.5$) is 28, belonging to

class interval 46 – 48.

Median class = 46 – 48

Lower class limit (l) of median class = 46

Frequency (f) of median class = 14

Cumulative frequency (cf) of class preceding median class = 14

Class size (h) = 2

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2 \\ &= 46 + \frac{3.5}{7} \\ &= 46.5\end{aligned}$$

Therefore, median of this data is 46.5.

Hence, the value of median is verified.

Question 3:

The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
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Number of farms	2	8	12	24	38	16
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Change the distribution to a more than type distribution and draw ogive.

Answer:

The cumulative frequency distribution of more than type can be obtained as follows.

Production yield (lower class limits)	Cumulative frequency
more than or equal to 50	100
more than or equal to 55	$100 - 2 = 98$

more than or equal to 60	$98 - 8 = 90$
more than or equal to 65	$90 - 12 = 78$
more than or equal to 70	$78 - 24 = 54$
more than or equal to 75	$54 - 38 = 16$

Taking the lower class limits on x -axis and their respective cumulative frequencies on y -axis, its ogive can be obtained as follows.

